

Bootstrap approach control limit for statistical quality control.

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ABSTRACT: This work is on non parametric approach to evaluating Cumulative Sum (CUSUM) or the Exponentially Weighted Moving Average (EWMA) control limits for a given data set, where the control limits are determined by the conditional distribution of the underlying data set. We applied the bootstrap method to evaluate the required control limits, detect the in – control and the out – of – control of the distribution, without rigid assumptions like the normality condition for the Statistical process control to be distributed.

Keyword: Large sample, Non Parametric, Bootstrap approach, Control limits, CUSUM, EWMA.

I. INTRODUCTION

Statistical quality control is important to all human endeavours. It makes uses of available data to elicit the required best decision for utmost profit. The theories and methods of Statistical Process Control (SPC) have been developed from industrial statistics roots, such as quality specifications. In modern times, while quality enhancement still remains a major field of applications like in healthcare monitoring [Sterner et al (1999)], detecting of genetic mutation [Krawezak et al (1999)], credit and financial fraud detecting [Bolton and Hand (2002)] to mention but a few. It has a wider range of applications. However, in this application, process distributions are often multimodal, skew or heavy tailed. So, the assumption of normality may not be apt in such circumstances. This work proposed bootstrap nonparametric approach to handle such cases.

According to Chatterjee and Qiu (2008), the SPC may be described as follows; A sequence of random variables $\{X_n, n \geq 1\}$ on the real line is observed, such that X_1, \dots, X_{t_0} follows a given distribution F (called an “in – control” distribution) and $X_{t_0+1}, X_{t_0+2}, \dots$ follows another distribution G (called an “out – of – control” distribution), where $F \neq G$. The major objective of SPC techniques is to detect such distribution F and G. When a shift in the mean of F is a major concern, the minimax sequential probability ratio test known as “cumulative sum control chart” (CUSUM chart hereafter) is the dominant technique for detecting such a shift, see Page (1954) and Van Dobben (1968). In order to detect the upward shift, the CUSUM C_n is defined by $C_0 = 0$ and

$$C_n = \max\{C_{n-1} + X_{n-k}, 0\} \text{ for } n \geq 1 \quad (1.1)$$

Where $k \geq 0$ is a pre-specified allowance constant. The process is stated to be out – of – control, if $C_n > h$, where the control limit h is determined by setting in – control “average run length” (ARL) at a certain nominal level and the in – control ARL is defined to be the expected time to signal under F. That is

$$ARL = E_F \inf\{n > 0: C_n > h\} \quad (1.2)$$

The probability of Type I error at a specific level in the hypothesis testing situation, with null hypothesis being that the process is in control is defined. If δ is the amount of shift in the mean from F to G, then according to Reynold (1975), we can choose $k = \frac{\sigma}{2}$ from equation (1.1) under certain regularity conditions for it to be optimal. Similarly, the CUSUMs exist for detecting downward shifts, two – sided shifts in mean or shift in variance – see Hawkins and Olwell (1998), Liu and Reynolds (1999).

The issue with the conventional CUSUM is its sensitivity to the assumption that both F and G be normal distributions with known in – control parameters. Resultant distributions from either F or G maybe left or right skewed, whether it is of heavy tailed, bimodal or multimodal. The CUSUM usually show two kinds of behaviour. It may have either a short or long actual in – control ARL, compared to the nominal in – control ARL value. When the actual in – control ARL value is smaller than ARL_0 , the CUSUM would be too sensitive to random noise, resulting in a large number of false alarms or out – of – control signals. The closeness of the actual in – control ARL value to ARL_0 is the robustness of this CUSUM to the various assumptions behind it. However, when explicit knowledge of F or G is not known or that distributions are not normally distributed, then, we need appropriate technique to handle this type of situation. Here, the bootstrap method may be used to get the control limit h , so that the actual in – control ARL value matches the nominal value ARL_0 . Since over thirty (30) years now, the bootstrap techniques have been successfully used in obtaining highly accurate

confidence intervals, estimates of the asymptotic variance, moments and probabilities, calibrations of different Statistics and so forth. In this study, we shall adopt the bootstrap technique to Statistical Quality Control (SQC).

ARL is a performance measure that is widely used to evaluate control charts. In this present work, the in – control ARL (ARL_0) will be used to compare the performance of the control charts. In classical Case, the upper control limit(UCL) and the lower control limit(LCL) of the EWMA are given below in equation 1.3

$$\left. \begin{aligned} UCL &= \bar{X} + L_\delta \sqrt{\frac{\lambda}{2-\lambda} [1 - (1-\lambda)^{2i}]} \\ CL &= \bar{X} \\ LCL &= \bar{X} - L_\delta \sqrt{\frac{\lambda}{2-\lambda} [1 - (1-\lambda)^{2i}]} \end{aligned} \right\} \quad (1.3)$$

Where L & λ are the design parameters of the EWMA chart. The EWMA control chart can be viewed as a weighted average of all the past and present observations, the distribution can be reasonably approximated by a normal distribution as a result of central limit theorem. But the bootstrap approach is not dependent on the type or condition of normality for data been considered, which are required for the CUSUM and EWMA control limits.

II. LITERATURE REVIEW

The need to avoid human and material errors, underscore the search for efficient methods to serve human needs. Statistical Process Control (SPC) and Statistical Quality Control (SQC) had been applied to these challenges, see Bolton and Hand (2002) and Good (2000). These researchers including Meulbrock (1992) used this approach to empirically analysis illegal insider trading in a financial deal (data). It must be noted that Bolton and Hand (2002) in their review of Statistical fraud detection studies, acknowledges not just the control limits for decision making, available methods for detecting changes in the underlying data distributions, they also review discrete and continuous data inspection schemes like the control charts for monitoring the mean and variance of autocorrelated processes, the Exponentially Weighted Moving Average (EWMA) types of control charts with varying time control limits with fast initial response. They were of the view that no simple basic method would do for comprehensive and efficient solutions to real data problem. Therefore, suggested that in addition to parametric approaches, non parametric approach could also be explored in this regard.

Chakraborti et al (2001) give an overview of how non parametric control charts could be applied to statistical quality control. Qiu and Hawkins (2001) used the rank based (non parametric method) CUSUM procedure for detecting shifts limit for decision making. Considering the efficiency of the bootstrap method as used by Bajgier (1992) to construct limits on control charts, following Wu and Wang (1996) and Good (2000) support for this approach due to its non parametric approach with less bias and variance structure, we therefore apply this resampling technique, with some modifications to suit not just the control limits but also to the resulting underlying density distribution of the data.

In 1979, Efron introduced this resampling technique called the bootstrap, since then a lot of studies and application had been done on this technique, particularly with the advent power and speed of the modern computer. See Efron (1979), Efron and Tibshirani (1993), Shao and Tu (1995), Good (2000) and Ogbonmwan (2008).

Generally, in the bootstrap method, we draw repeated samples with replacement from observed data and estimate the sample distribution of a related statistic of interest using these samples. It is implemented in an algorithm. It does not require the classical assumptions like the normality for it to operate. According to Efron and Tibshirani (1993), this method works under less stringent assumption or no assumptions. Resampling techniques for SPC are of considerable recent interest in the review literature. In the literature, efforts have been made to remove certain assumptions of the conventional CUSUM. Hawkins and Olwell (1998), suggested using the self – stating CUSUM when both F and G are normal but the in – control distribution parameters are unknown. Some non – parametric CUSUMs have been proposed, see Chakraborti et al (2001) for one – dimensional methods, Qiu (2008) and Qiu and Hawkins (2001, 2005) for the multivariate non – parametric CUSUMs. Wu and Wang (1996) and Wood et al (1999) design bootstrap – based control charts, though not for CUSUM. Steiner et al (1999) suggested using time varying control limits in the form of Exponentially Weighted Moving Average (EWMA) control chart. However, our bootstrap based SPC procedure is an attempt to be flexible on the choice of any assumption for F or G, particularly when we do even know them or even with the in – control mean μ and in- control standard deviation δ .

Basically, our method depends on the data set resampled to generate statistic of interest. This helps us to overcome the situation when the classical assumptions of F is unknown or misspecified, the results will not be reliable. We use the bootstrap approach for approximating the distribution of the CUSUM statistic and obtain a sequence of control limits. This procedure is distribution free. Also the difference between our method and that of Chatterjee and Qiu (2009) is that while they worked on the bootstrap CUSUM for detecting upward shift in the location of the parameter F . We modify and apply ours to the two – sided shifts CUSUM distribution. As in the literature, we state the statistics to facilitate our study. Let

$$T_n = \begin{cases} 0 & \text{if } C_n = 0 \\ j & \text{if } C_n \neq 0, \dots, C_{n-j+1} \neq 0 \\ & C_{n-j} = 0; j = 1, 2, \dots, n \end{cases}$$

(2.1)

Where T_n is the time elapsed after the last CUSUM C_n was zero. According to Qiu (2008), we shall call T_n the sprint length of the data.

Polansky (2005) provided a general framework on constructing control charts for both univariate and multivariate situation. We observed that univariate control charts have been devised to monitor the quality of a single process variable. It can contextually be extend to several variables. In particular, some studies on non parametric control charts favours the use of the bootstrap approach. This is due to the fact that it has the proven capabilities to effectively manage process data without making assumptions about their distribution. In Bajgier (1992), when he introduced a univariate control chart whose lower and upper control limits were estimated by the bootstrap technique. The Bajgier’s control charts tend to generate a wide gap between the lower and upper control limits when the in – control process is unstable. This observation where also noted in Seppala et al (1995) when they proposed a sub group bootstrap chart to compensate for the limitation of Bajgier’s control charts with too much ARL. The sub group bootstrap chart uses residuals which are the differences between the mean of j^{th} sub group obtained by a bootstrap technique and each observation in the j^{th} sub group. The lower and upper control limits are determined by adding the mean of the residuals to the grand mean.

Liu and Tang (1996) proposed the bootstrap control chart to monitor both independent and dependent observations in process control situation. Jones and Woodall (1998) compared the performance of the three above approaches control charts in the non normal situation and found that they did not perform significantly better than the traditional \bar{X} chart in terms of the in – control average run length (ARL_0). Suggesting a need for their enhancement or modification for better performance, this motivates our study. Recently, Lio and Park (2008) proposed a bootstrap control chart based on the Birnbaum – Saunder distribution. This chart fits tensile strength and breaking stress data. This approach uses a parametric bootstrap technique to establish control limit. They showed that their parametric bootstrap method can accurately estimate the control limits for the Birnbaum – Saunder’s percentiles. Parametric assumption here could be costly. However, Park (2009) proposed the median control charts whose control limits were determined by establishing the variance of the sample median via the bootstrap technique.

The Exponentially Weighted Moving Average (EWMA) control charting procedure is an alternative to the standard control chart and the cumulative sum (CUSUM) control chart. Since most real life data are multivariate in nature, another possible approach to monitor this data would be to use a multivariate distribution free control chart, such as the EWMA. For details, see Crowder (1987) and Edokpa et al (2009).

In this paper, we present a proposed bootstrap non parametric approach control charts as an alternative means of establishing the control limits in the univariate situation when the observed process may not even be normally distributed. The control limit of the bootstrap is based on the correlated/adjusted percentile statistic with a view to enhancing ARL.

III. METHODOLOGY

A modified bootstrap technique for statistical process control is presented for the situation under consideration when particularly the assumption of normality does not hold. The modified algorithm is as follows:

1. Compute the T statistic with n observations from the in – control data set using (1.1) that is from observed data of the estimate of F .
2. Let T_1, T_2, \dots, T_n be a set of T values from the bootstrap samples ($i = 1, \dots, B$) randomly drawn from the initial T statistic with replacement. Where B is a large number say $B \geq 1000$.

3. From the B bootstrap samples, estimate $[\hat{T}_{\% low}, \hat{T}_{\% upp}] = [\hat{G}_{(\alpha)}^{-1}, \hat{G}_{(1-\alpha)}^{-1}]$. Where \hat{G} is the cumulative distribution function of \hat{T} from the observed resampled data. Where α is a user – specified value α with the range between 0 and 1.
4. Determine the control limit by taking an average of $B100 \cdot \frac{th}{(1-\alpha)}$ percentile values $\bar{T}_{100 \cdot (1-\alpha)}$. Note that statistics other than the average like the median, variance, etc can be used. From the step 3 above and the control limits (lower and upper limits).
5. Use the establish control limit to obtain the distribution and monitor a new observation or the behavior of the observation. This is monitoring statistic of a new (other) observations that exceeds the upper and lower limits determined, which we shall declare as out of control from the ARL.

IV. APPLICATION AND DATA ANALYSIS

Given that Brown bean were grown under controlled conditions for a period of 10 weeks after germination, no measurement of mean height was recorded at the end of week one but the mean height recorded at the end of subsequent weeks. Source: Attwood and Dyer (1995,p.161). See Appendix I. the established mean growth of Bean is 13, this incidentally coincide with Osanaiye and Talabi (1989)), Appendix II. where $k = 13$, $h = 11$. Table I (extracted from British Standards 5703, Part 4) in Kemp (1962) and Goel and Wu (1971) is used as it is relevant (as used by We conduct a control limit to decide where the growth is undergrowth or overgrowth.

Recall: with this data at $\chi^2_7(5\%) = 14.067$, $\chi^2 = 4.16$ via goodness of fit test. This distribution can be modeled by a continuous uniform distribution and not a normal distribution. This affects the assumption of normality by the classical CUSUM or EWMA controls model development but not that of the Bootstrap method.

We have below, illustration using the brown beans data.

Table 1: CUSUM tabulation for brown beans growth data.

Weeks	X_i	$d = X_i - X_{i-1}$	$X_{i-1} - k$	$\sum d = \sum (X_{i-1} - k)$	S_i
Week2	20	20	8	8	8*
Week3	31	11	-2	6	0
Week4	48	17	4	10	6*
Week5	63	15	2	12	12*
Week6	73	10	-3	9	0
Week7	78	13	0	9	0
Week8	101	13	0	9	0
Week9	112	11	-2	7	0
Week10	120	8	-6	-1	0

Where $S = \max(0, X_{i-1} - 13 + S_{i-1}) > 11$.

From table, the mean growth rate for brown Beans is 13, an out – of – control sign is given when the growth rate shifts upwards to an unexpected level. This can be seen at week2, week4 and week5. Week3, week6, week9 and week10 .At the same vein, when the mean growth rate is lower than the unexpected, such as in week7 and week8. We say that the mean growth rate is out-of-control. Below is the table of computed CUSUM, and Bootstrap (B=1000 and B=5000) mean, control limits, ARL and variances.

Table 2: Computed CUSUM, and Bootstrap (B=1000 and B=5000) for the Brown beans data.

Approach	\bar{X}	<i>UCL</i>	<i>LCL</i>	<i>ARL</i>	δ
CUSUM	12.5	15.9641	9.0359	6.9282	3.464
Bootstrap B=1000	13.333	16.6997	9.9669	6.7328	3.3664
Bootstrap B=5000	13.3232	16.6892	9.9572	6.7320	3.3660

The bootstrap approach variance is less than the classical case and the mean of the Bootstrap is more closer to the targeted mean of $k = 13$ than the CUSUM approach, see Kemp(1962) and Goel and Wu (1971). Our

proposed Bootstrap approach without the assumption of normality of F (of the data) as in the classical case and perform comparatively with lower δ and ARL. This is seen in the achieved mean and ARL in Table 2.

It is worth noting that the Bootstrap approach like the CUSUM scheme is applicable to counted data. When we apply the approach to the data in Osanaiye and Talabi (1989). See Appendix II. For $n = 146$ we assumed that F to be a standard normal distribution and the allowance constants $k = 0.5, \lambda = 0.12, L = 2.75$ which is optimal for the shifts from $F = N(0,1)$ to $G = N(1,1)$.

We first approximate the distributions of $[C_n/T_{n=j}]$ for $j = 1, 2, \dots, 10$ by their empirical distributions. Obtained from a preliminary run of 1000 independent bootstrap replications of the sampling X_1, \dots, X_n from F is performed. That is $B = 1000$. We also Bootstrap $B = 5000$ replications of the samples with replacement from the data and perform the EWMA and ARL from the resampled data. Below are estimated results from the various approaches.

Table 3: Computed EWMA, and Bootstrap (B=1000 and B=5000) for diabetic disease data.

Approach	\bar{X}	UCL	LCL	ARL	δ
EWMA	17.93	18.36	17.5	0.86	7.5
Bootstrap B=1000	17.9935	18.5854	17.4015	0.5919	4.2631
Bootstrap B=5000	17.9997	18.1717	17.8276	0.3441	4.2604

Where the acceptable mean level is 18 as $n \rightarrow \infty$. This means that the local means (classical and the Bootstrap are approximately 18). The closest value (17.9997) with B=5000 has the closest value to the target value 18. Our proposed Bootstrap approach without the assumption of normality of F (of the data) as in the classical case, perform comparatively with lower δ and ARL. This is seen in the achieved mean and ARL in Table 3. We achieved at B=5000, $ARL = 0.3441$ which is even smaller than the classical EWMA. This has an obvious implication for in-control and out-of control decision. This approach do not require prior information about both the F and G . Consequently, it is robust to distributional assumptions.

V. CONCLUSION

We have presented the nonparametric bootstrap approach to obtain control limits which depend on the distribution of resampling with replacement of the original data and computing of vital statistics. This method does not use the classical assumption in obtaining these results. It has reduce variance estimates and estimate comparatively the mean and ARL of the data set to the classical case. It does not require prior information about both the F and G . Consequently, it is robust to distributional assumptions.

The general picture that emerges from the above results is that, if both F and G are normal, then the conventional classical method is a good performer.

REFERENCES

- [1]. Attwood, G. and Dyer, G. (1995): Statistics. Heinemann modular Mathematics for London AS and A-level.
- [2]. Heinemann Publishers, Oxford. London.
- [3]. Bajgier, S.M. (1992): The use of bootstrapping to construct limits on control charts in Process Decision Science Institute, San Diego, 1611 – 1613.
- [4]. Bolton, R.J. and Hand, D.J. (2002): Statistical Fraud Detection: A review Statistics Science, 17, 235 – 255.
- [5]. Bowman, A.W. and Azzalini, A. (1997): Applied smoothing techniques for data analysis. Oxford University Press, 31 – 32.
- [6]. Chakraborti, S., Van Der Laan, P. and Bakir, S.T. (2001): Non Parametric Control Chart: An overview and some results. Journal of Quality Technology, 33(3), 304 - 315.
- [7]. Chatterjee, S. and Qiu, P. (2009): Distribution free cumulative sum control charts using Bootstrap based control limits. Annals of Applied Statistics. Vol. 3, 349 – 369.
- [8]. Crosier, R.B. (1986): A new two - sided cumulative sum quality control scheme. Technometrics, 28, 187 – 194.
- [9]. Crowder, S. V. (1987): A simple method for studying run-length distribution of Exponentially Weighted Moving Average chart/ Technometrics. Vol. 29. No. 4.
- [10]. Edokpa, I W. Ikpotokin, O and Erimafa, J.T (2009): A study of average run length distribution of an Exponentially weighted moving average control chart using Diabetic data from Oyo State of Nigeria from January 1974-February 1986. Journal of Mathematical Sciences. Vol.20. No3. 171-179.
- [11]. Efron, B. (1979): Bootstrap Methods: Another look at the jackknife. Ann. Statistics. 7, 1 – 26.
- [12]. Efron, B. and Tibshirani, R.J. (1993): An Introduction to the Bootstrap. Chapman and Hall, Boca Raton, FL. USA.
- [13]. Good, P. (2000): Permutation test. A practical guide to sampling method for testing hypothesis. 2nd Edition. Spring Publisher, New York.
- [14]. Goel, A.L. and Wu, (1971): Determination of ARL and a contour nomograph for CUSUM charts to control normal means. Technometrics, 13, 221-230.
- [15]. Hawkins, D.M. and Olwell, D.H. (1999): Cumulative sum chart and charting for Quality improvement. Springe, New York.

[21]. Kemp, K. W. (1962): The use of cumulative sums for sampling inspection schemes. *Applied Statistics*. 11,16-31.

[22]. Krawezak, M., Ball, E., Fenton, I., Stenson, P., Abeysinghe, S., Thomas, N. and Cooper, D.N. (1999): Human gene mutation database. A biomedical information and research resources. *Human mutation*, 15, 45 – 51.

[23]. Jones, L.A. and Woodall, W.H. (1998): The performance of Bootstrap Control Charts. *Journal of Quality Technology*, 30, 362 – 375.

[24]. Liu, C.W. and Reynolds, M.R. (Jnr) (1999): Control Chart for monitoring the mean and variance of autocorrelation processes. *J. Quality Technology*, 31, 259 – 274.

[25]. Liu, R.Y. and Tang, J. (1996): Control chart dependent and independent measures based on bootstrap methods. *J. Amerstatist Association*, 91, 1694 – 1700.

[26]. Lio, Y.L. and Park, C. (2008): A Bootstrap Control Chart for Birnbaum – Saunders Percentile. *Quality and Reliability Engineering International*, 24, 585 – 600.

[27]. Meulbroek, L.K. (1992): An empirical analysis of illegal insider trading. *J. Finance* 47, 1661 – 1699.

[28]. Ogbuide, E.M. and Ogbonmwan, S.M. (2008): Estimating confidence interval: Bootstrap and Permutation approaches. *Proceedings of the Mathematical Association of Nigeria (MAN) in Asaba*. MAN Illoin. Pp16-22.

[29]. Osanaiye, P.A. and Talabi, C.O. (1989): On some non-manufacturing application of counted data cumulative sum (CUSUM) control chart scheme. *The Statistician*. 38:251-257.

[30]. Page, E.S. (1954): Continuous inspection schemes. *Biometrika*, 41, 100 – 114.

[31]. Park, H.I. (2009): Median Control Charts based on Bootstrap method. *Communications in Statistics – Simulation and Communication*, 38, 558 – 570.

[32]. Polansky, A.M. (2005): A general framework for constructing control charts. *Quality and Reliability Engineering International*, 21, 633 – 653.

[33]. Qiu, P. (2008): Distribution free multivariate process control based on log –linear modeling. *11E Transaction* 40, 664 - 677.

[34]. Qiu, P. and Hawkins, D. (2001): A rank based multivariate CUSUM procedure *Technometrics*, 43, 120-132.

[35]. Qiu, P. and Hawkins, D. (2003): A non parametric multivariate CUSUM procedure for detecting shift in all directions. *J.Roy Statistical Social Sciences*, D52, 151-161.

[36]. Shao, J. and Tu, D. (1995): *The Jackknife and Bootstrap*. Springer, New York.

[37]. Reynolds, M.R. (1975): Approximations to the average run length in Cumulative Sum Control Chart. *Technometrics*, 17, 65 – 71.

[38]. Silverman, B.W. (1986): *Density estimation for statistics and data analysis*. Chapman and Hall. London.

[39]. Seppala, T., Moskowitz, H., Plante, R and Jang, I. (1995): Statistical process control via the subgroup bootstrap. *Journal of Quality Technology*, 27, 139 – 153.

[40]. Steiner, S.H., Cook, R. and Farewell (1999): Monitoring paired binary surgical outcomes using Cumulative Sum Chart. *Statistics in Medicine*, 8, 69 – 86.

[41]. Van Dobben De Bruyn (1968): *Cumulative Sum Test: Theory and Practice*, Griffin London.

[42]. Wood, M., Kaye, M. and Capon, N. (1999): The use of resampling for estimating control chart limits. *J. Operational Research Soc.* 50, 651 – 659.

[43]. Wu, Z. and Wang, Q. (1996): Bootstrap control charts. *Quality Engineering*, 9, 143-150.

APPENDIX I

Table 1: Recorded Brown bean data grown under controlled conditions for a period of 10 weeks after germination.

Week	2	3	4	5	6	7	8	9	10
Height	20	31	48	63	75	88	101	112	120

Source: Attwood and Dyer (1995, P.161).

APPENDIX II

Table 2: The diabetic disease data in Osanaiye and Talabi (1989).

Year	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sept	Oct	Nov	Dec
1974	20	23	16	19	23	16	22	12	9	17	20	18
1975	2	8	8	23	14	25	16	25	7	2	3	13
1976	20	18	30	17	23	21	14	22	18	18	13	27
1977	25	20	31	22	15	26	21	23	14	13	58	15
1978	29	27	25	10	17	17	30	22	14	15	14	14
1979	24	14	19	9	11	7	19	8	19	22	11	22
1980	25	22	19	23	17	17	10	23	24	15	41	16
1981	15	7	10	26	9	17	23	22	30	32	22	27
1982	25	20	35	17	19	19	27	29	11	23	25	16
1983	24	20	19	12	16	10	9	16	7	9	18	9
1984	18	17	14	14	19	23	12	20	7	17	9	14
1985	18	2	6	18	14	17	22	12	18	13	6	18
1986	21	17										