Extracting wavelet filters using mathematical methods

Maha Ammar Mustafa
Department of Applied Sciences, University of Technology

ABSTRACT: *In this work, the origins of wavelets were studied mathematically and how to derive the algorithms for preparing the proposed wavelets for use in many areas, that will help to derived the new laws and algorithms and it makes the new wavelet to take its role in the image processing.*
Some examples and algorithms also included in this paper. including numerical solutions to solve many mathematical problems and image processing such as image compression and noise , physical samples were selected that were compressed using the proposed wavelets, and good results were obtained that prove the efficiency of the method used.
KEYWORDS - *New filter, Peak MATLAB , Tree desertion, color Image processing, .*

I. INTRODUCTION

The theory of integrated transformation. The inverse of analysis is the integration of the signal multiplied by the basic analysis functions but the reconstruction process starts from, [11], [12].

If g=T(f) is the transform of f.

The question here, is whether it is possible to reconstruct f knowing g?

In the appropriate circumstances depends on the function used with the features that qualify for that process then the answer is yes.[1-6]. It is possible to reconstruct or form desecrate f conversion, which requires many advantage in numerical terms. So on to reduce the number of separate information for reconstruction, leading to the concept of wavelet base after the introduction of the converted wave used this applicable to discrete Laguerre wavelet transform starting   from the concept of multi resolution analysis of the norm space of the finite energy signals, [10]. The previous results period setting for signal analysis in the form of a succession of approximation with low of resolution supplemented by a series of details, [7-9]. In this paper it is possible to determine the best decomposition of a specific signal, which relates to the entropy criterion or standard by displaying the bases of the waveforms where the orthogonal rules are subject to orthonormal at the same time in order to improve the frequency accuracy of the wavelet analysis then suggest a richer associated with a large group of decomposition [13-16].

II. WAVELET TRANSFORM

$$\rho_{s,r}(t) = |s|^{\frac{-1}{2}} \rho\left(\frac{t-r}{s}\right) \qquad s,r \in R, \quad s \neq 0 \tag{1}$$

where $\qquad \rho(t) = [\rho_0(t), \rho_1(t),..., \rho_{M-1}(t)]^T$

The elements $\rho_0(t), \rho_1(t),..., \rho_{M-1}(t)$ are the basis functions, orthogonal on the [0,1].

Laguerre   wavelet $\rho_{n,m}(t) = \rho_{t,n,m,k}$ have four arguments; $k = 1,2,...,$ $n = 1,2,..., 2^{k-1}$, m is order for Laguerre   polynomials and t is normalized time.

If we dilation by parameter $s = 2^{-(k+1)}$ and translation by parameter $r = 2^{-(k+1)}(2n-1)$ and use transform x in (1).

$$x = 2^{-(k+1)}(2^k t)$$

Then we will get the following equation

$$\rho_{n,m}(t) = \begin{cases} 2^{\frac{k+1}{2}} \tilde{L}_m(2^k t - 2n + 1) & \frac{n-1}{2^{k-1}} \le t \le \frac{n}{2^{k-1}} \\ 0 & otherwise \end{cases} \tag{2}$$

Where $\tilde{L}_m = \dfrac{1}{m!} L_m$ for k=2

$$\rho_{1,0}(t) = 2\sqrt{2}$$
$$\rho_{1,1}(t) = 2\sqrt{2} \qquad (2-4t) \qquad \Big\} \; t \in [0,0.5),$$
$$\rho_{1,2}(t) = \sqrt{2} \qquad (16\,t^2 - 24\,t + 7)$$

$\vdots$

$\vdots$

$\vdots$

$$\rho_{2,0}(t) = 2\sqrt{2}$$
$$\rho_{2,1}(t) = 2\sqrt{2} \qquad (4-4t) \qquad \Big\} \; t \in [0.5,1)$$
$$\rho_{2,2}(t) = \sqrt{2} \qquad (16\,t^2 - 40\,t + 23)$$

$\vdots$

$\vdots$

$\vdots$

A function approximation $f(t) \in L^2[0,1]$ may be expanded as

$$f(t) = \sum_{n=1}^{\infty} \sum_{m=0}^{\infty} A_{n,m}\, \rho_{n,m}(t) \tag{3}$$

where

$$A_{n,m} = \langle\, f(t), \rho_{n,m}(t)\,\rangle \tag{4}$$

In equation (4), $\langle .,. \rangle$ denoted the inner product with weight function $w_n(t)$ on the Hilbert Space $[1,0)$. If the infinite series in above equation is truncated, then equation (3) can be written as

$$f(t) = \sum_{n=1}^{2^{k-1}} \sum_{m=0}^{M-1} A_{n,m}\, \rho_{n,m}(t) = A^T \rho_{n,m}(t) \tag{5}$$

where A and $\rho(t)$ are $2^{k-1}M \times 1$ matrices given by:

$$A = \left[A_{1,0}, A_{1,1}, ..., A_{2,0}, ..., A_{2,(M-1)}, ..., A_{2^{k-1},0}, ..., A_{2^{k-1},M-1} \right]^T \tag{6}$$

$$\rho(t) = \left[\begin{array}{c} \rho_{1,0}, \rho_{1,1}(t), ..., \rho_{1,M-1}(t), \rho_{2,0}(t), ..., \\ \rho_{2^{k-1},M-1}(t), ..., \rho_{2^{k-1},0}(t), ..., \rho_{2^{k-1},M-1}(t) \end{array} \right]^T \tag{7}$$

In equation (2) for M=3 The six basis functions are given by

If we applied equations (1-7) with levels 0,1,2 that mean take j=0,1,2 we have

$$\begin{cases} j = 0 \\ \vartheta_{0,0}(t) = L_0(t) = 1 \subset V_0 \\ j = 1 \\ \rho_{1,0}(t) = \dfrac{1}{\sqrt{2}} L_0(2t) = \dfrac{1}{\sqrt{2}} \\ \rho_{1,1}(t) = \dfrac{1}{\sqrt{2}} L_1(2t - 1) = \dfrac{1}{\sqrt{2}}\left[1 - (2t - 1)\right] \\ j = 2 \\ \rho_{2,0}(t) = \dfrac{1}{2} L_0(4t) = \dfrac{1}{2} \\ \rho_{2,1}(t) = \dfrac{1}{2} L_1(4t - 1) = \dfrac{1}{2}\left[1 - (4t - 1)\right] \\ \rho_{2,2}(t) = \dfrac{1}{2} L_2(4t - 2) = \dfrac{1}{2}\left[(4t - 2)^2 - 4(4t - 2) + 2\right] \\ \rho_{2,3}(t) = \dfrac{1}{2} L_3(4t - 3) = \dfrac{1}{2}\left[6 - 18(4t - 3) + 9(4t - 3)^2 - (4t - 3)^3\right] \end{cases}$$

$n = 0, 1, 2, ..., 2^j - 1 \quad and \quad j = 0, 1, 2, ...$

Then

$$\vartheta_{j,n}(t) = \begin{cases} 1 & t \in [0,1) \\ 0 & otherwise \end{cases} \qquad \text{the scale function of DLWT}$$

$$f(t) = A_0 + \sum_{j=0}^{\infty} \sum_{n=0}^{2^j - 1} A_{j,n} \rho_{j,n}(t) \tag{8}$$

III.   ADD DLWT IN MATLAB PROGRAM

In this section, an algorithm was developed to show how to add the proposed wave in the Matlab program, which requires this work before the start of the algorithm classification of any wavelet family mentioned see [11], which enters the field Wavelet Toolbox (WT) using the "wavemngr" function in Matlab program.

 The Stages of Creating a Waveform Table contains DLWT

-        **Algorithm of Add New DLWT** : create an orthogonal wavelet family containing only one wavelet.

Step 1: Create a Wavelet Family with One Type 1 Wavelet Orthogonal wavelet
            With scale function

Step 2: Define the five parameters associated with the wavelet family. And use the
            "wavemngr" function

Step 3: Write the following steps in order to determine the wave of the first type is
            the orthogonal type using the following function
             >> familyName   = 'MyWAVE T1';

Step 4: in this step short name must be specified
                familyShortName = 'lgw';
                familyWaveType = 1;
                 familyNums   = '';
                 fileWaveName   = ' lgw.mat';

Step 5: in this step need greate a filter associated with the wavelet and save it in
             the MAT-file mywa.mat. This step must be done before using the new
             wavelet family.This filter will be used by the "wfilters" function to build
              four Filters used by the Discrete Laguerre Wavelet Transform.   (DLWT)
              and save it
              >> save lgw lgw

Step 6: In this step, the new wavelet family will be added to a pile of captive
             families.

Step 7: Finally, make sure to add the new wave through the following instructions
             wavemngr('read')

IV.   FOUR FILTERS ASSOCIATED WITH DLWT

From wavelet toolbox by using Matlab program using the "wfilters" function

[LO_D,HI_D,LO_R,HI_R] = wfilters('lgw');

Will get four filters by using (8×8) matrix in $[0,1]$

LO_D = 0.4527   0.3402 0.2459   0.1693   0.1087   0.0623   0.0288   0.0064
HI_D = -0.0064   0.0288 -0.0623   0.1087   -0.1693   0.2459 -0.3402 0.4527
LO_R = 0.0064   0.0288   0.0623   0.1087   0.1693   0.2459   0.3402 0.4527
HI_R = 0.4527   -0.3402 0.2459   -0.1693   0.1087 -0.0623   0.0288   -0.0064

The following figure illustrates the scaling and wavelet functions by using the following functions

wname = 'mywa';

wavefun(wname,'plot',7);

a = findobj(gcf,'Type','axes');

    axis(a,'tight')

Applied Method

    In this section, the proposed wavelets are used in processing the color image using the Matlab program, which activated the wavelets by designing programs to analyze the image into approximation parameters and details where the image features are studied.
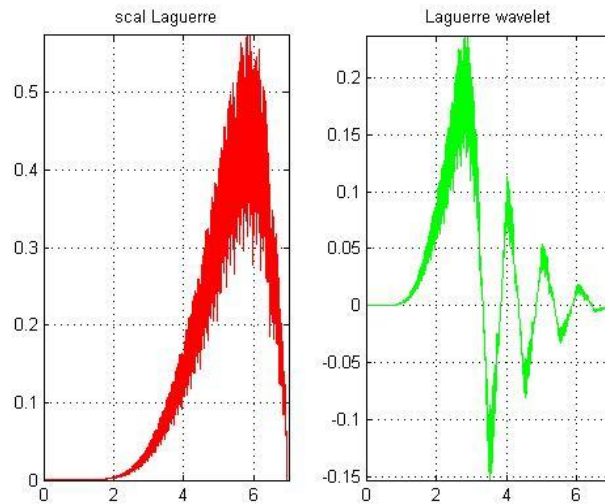
Fig.1 analysis DLWT (8×8)
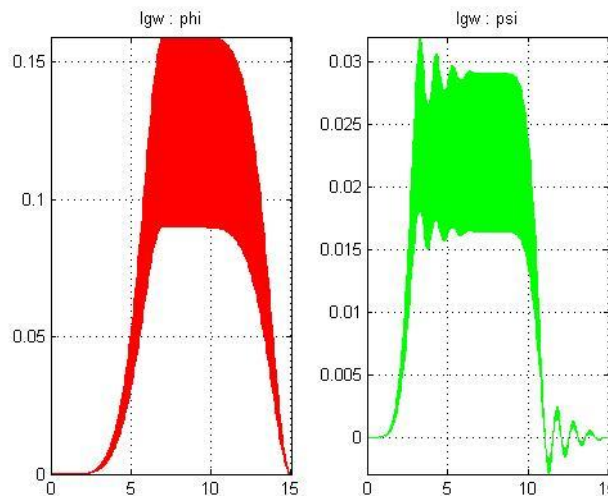
Fig.2 analysis DLWT (16×16)

Obtain the first 4 wavelet packets for the Discrete Laguerre wavelet with 4 vanishing moments.
By using
```
[wpws,x] = wpfun('lgw',4,10);
for nn = 1:size(wpws,1)
 subplot(3,2,nn)
 plot(x,wpws(nn,:)); axis tight;
 title(['W',num2str(nn-1)]);
end
```
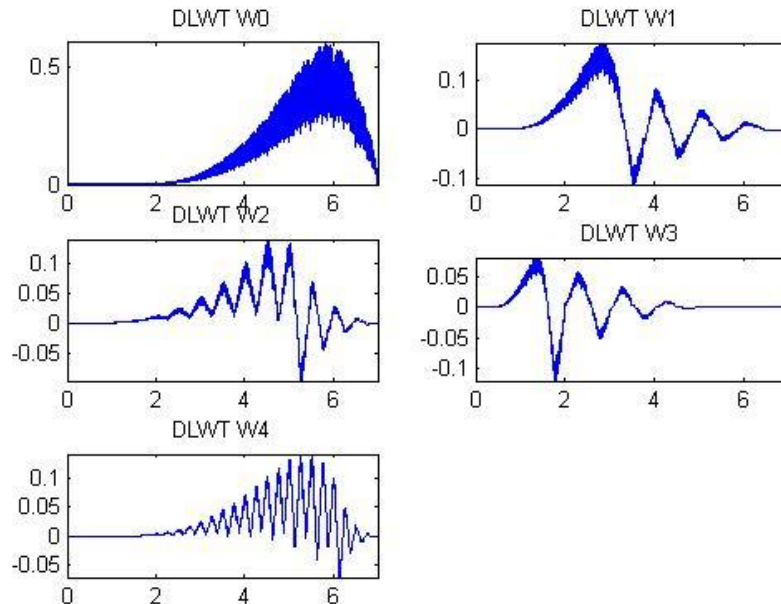

Fig.3 analysis DLWT level 4

When using the Wavelet and the Wavelet Packet Display tools from the "wavemenu" WT, the new wavelet is also available. Here is a screen shot of the Wavelet Display Tool.
The following figure illustrates all the DLWT analysis functions by using load signal in matlab program in level 3
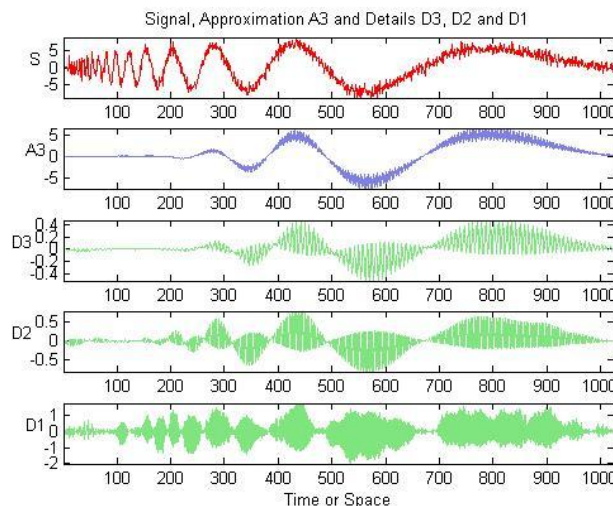
Fig.4 analysis DLWT with load signal

The signals from top to bottom in the figure above are S (the signal), A2 (Approximation of level 2), D2 (Detail of level 2) and D1 (Detail of level 1).If the wavelet is an orthogonal wavelet, the reconstruction must be perfect, which is, S = A2 + D2 + D1

This is the case as we can see below level 2 with norm $\left(\dfrac{1}{\sqrt{2}}\right)$ and without norm Respectively .

```
  maxdiff = max(abs(x-(A2+D1+D2)))
```
1. maxdiff =
     1.1421
2. maxdiff =
     0.8093

We can now use this new DLWT orthogonal wavelet to analyze a signal.

```
load noisdopp; % Load a signal
x = noisdopp;
scales = 1:1:128;
coefs = cwt(x,scales,wname);
clf; wscalogram('image',coefs,'scales',scales,'ydata',x);
```
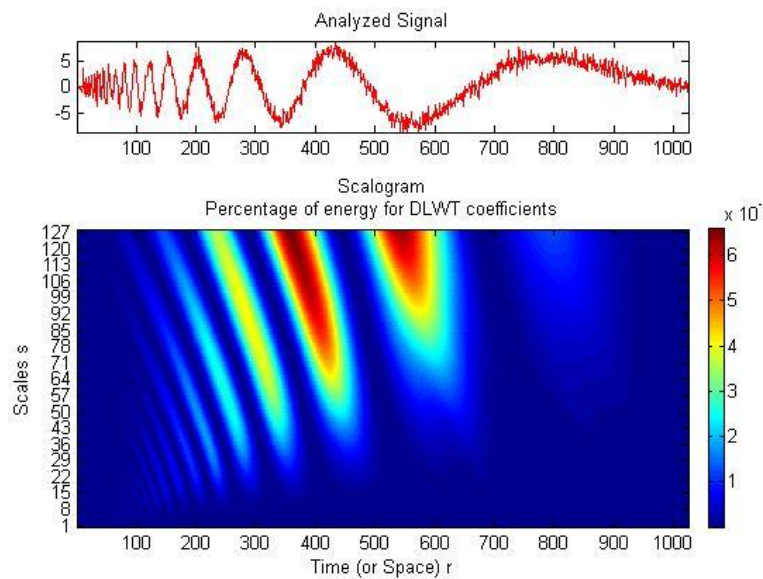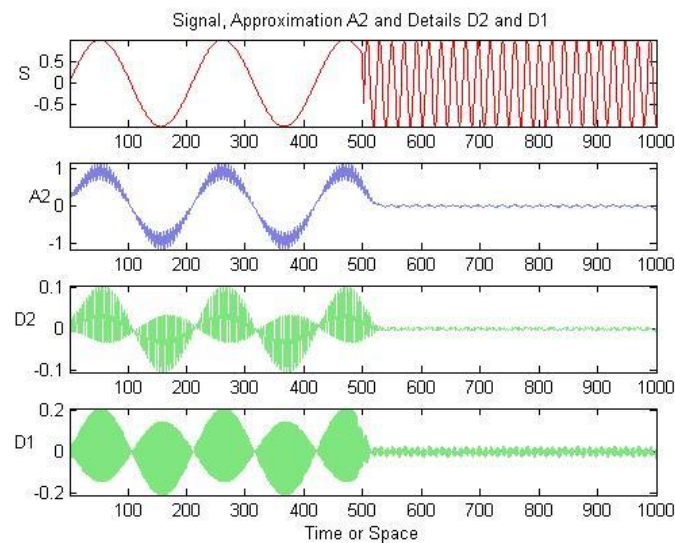

Fig.5 Doppler signal analysis by DLWT

Fig.6 analysis DLWT level 2 without norm with scalar function and load signal

---
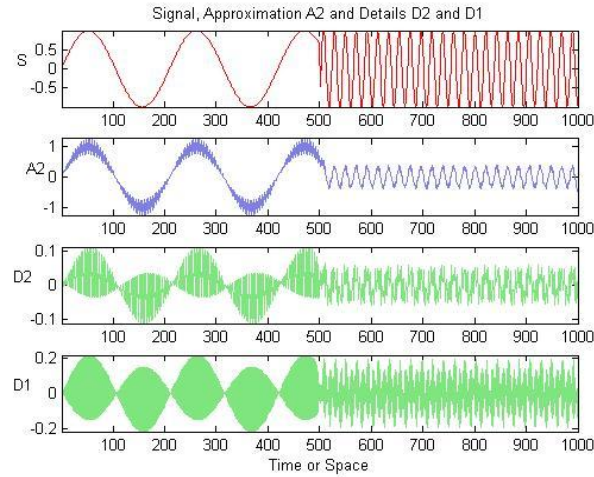
Fig.7 analysis DLWT level 2 with norm with scalar function and load signal

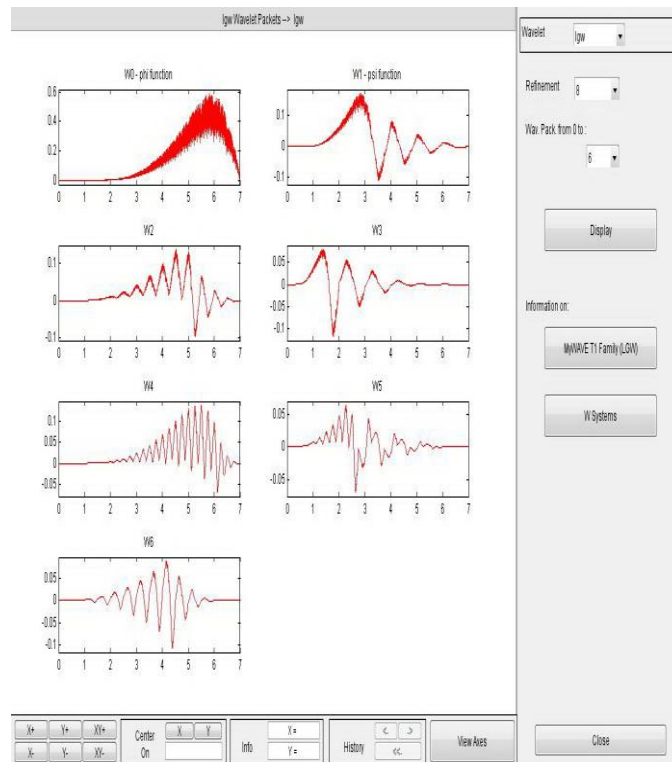We named DLWT in Matlab program is (lgw) is used it in all programming

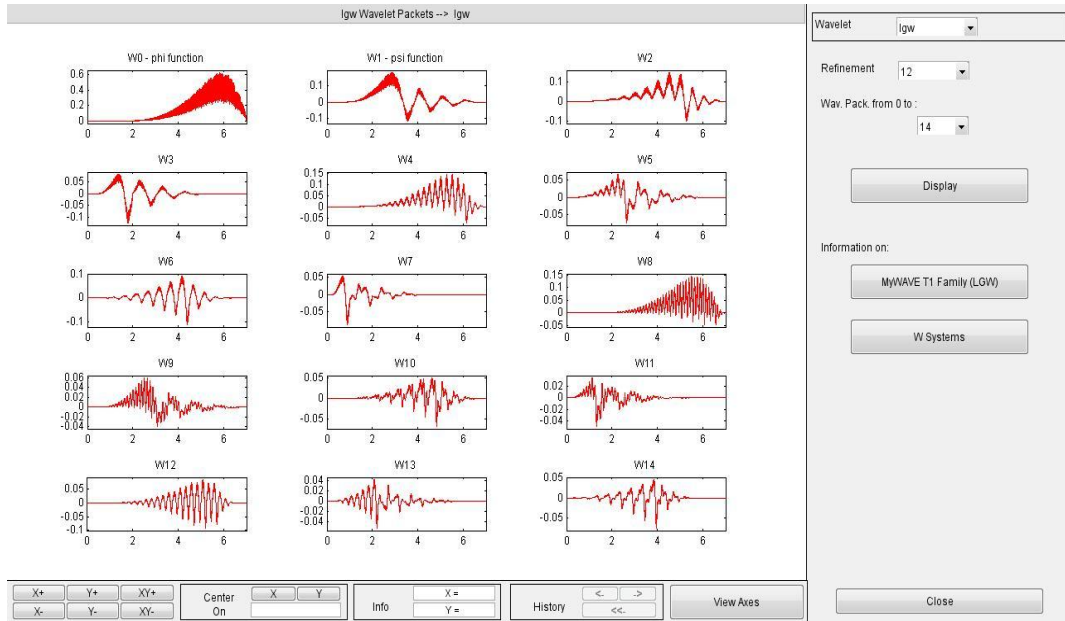Fig.8 analysis DLWT level 5 with norm with scalar function and load signal

Fig.9 analysis DLWT level 14 with norm with scalar function and load signal

V.   THE ANALYSIS OF DLWT WITH SIGNAL S IN 3D

All the continuous analysis functions, including DLWT, and the corresponding WT, can also operate on the new DLWT. The following figure illustrates DLWT IN 3D analysis signal

scales = 1:1:128;
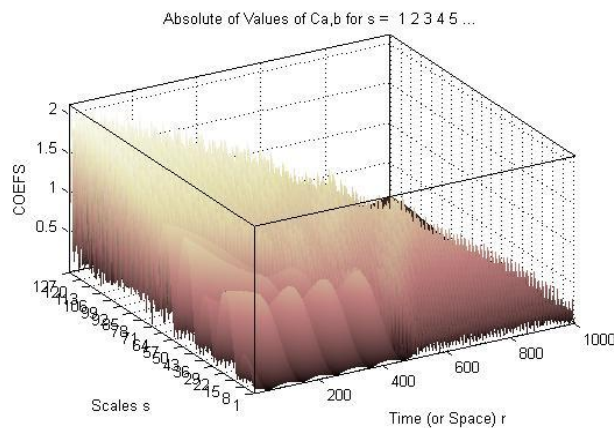clf; cwt(x,scales,wname,'3Dplot');

Fig.10 analysis DLWT with load signal in 3D

In the second scales as
scales = 1:1:265;
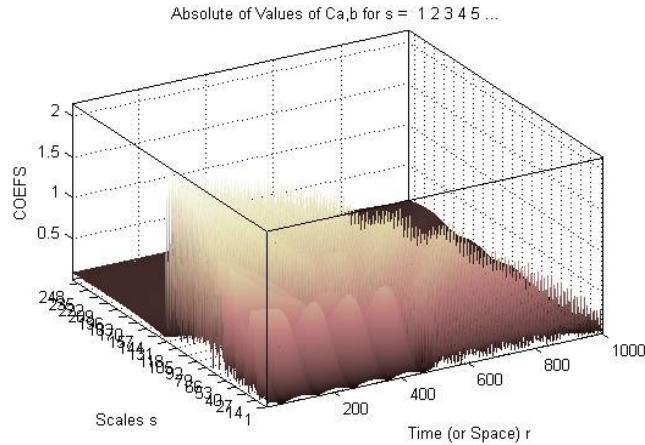clf; cwt(x,scales,wname,'3Dplot');

Fig.11 analysis DLWT with load signal in 3D with another scales

VI.   DLWT USED TO ANALYZE AN IMAGE

The DLWT can also be used to analyze an image by using

```
load mask; % Load an image
[cA,cH,cV,cD] = dlwt2(X,wname);
clf; imagesc(abs([cA cH ; cV cD]));
clear cA cH cV cD
```
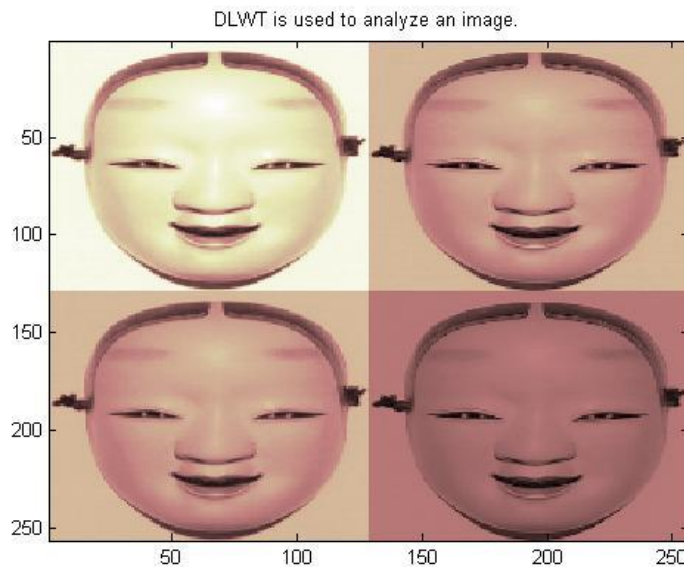

Fig.16 analysis DLWT with load Mask

VII.   CONCLUSION

In this work, the normal color image and the medical images that were examined with Magnetic resonance imaging (MRI) were compressed using DLWT and the results indicated in the tables above proved the efficiency of the theory used, allowing it to be used in many fields such as medicine, Engineering and Sciences.

REFRENCES

[1].   Andreas Savakis and Richard Carbone, 'Discrete Wavelet Transform Core for Image Processing Application' SPIE, Vol 5671, 142-151, (2005) .

[2].   B.R. Ambedkar & Sangeeta Arora 'Numerical Solution of Wave Equation Using Haar Wavelet', International Journal of Pure and Applied Mathematics (IJPAM), VOL (98), NO (4), PP 457-469, (2015).

[3].   B. Satyanarayan, Y. Pragathi Kumar, Asma Abdulelah, 'Laguerre Wavelet and its Programming' , International Journal of Mathematics Trends and Technology (IJMTT), VOL(49), NO(2), 129-137), (2017).

[4].   B. Satyanarayan, Asma Abdulelah 'Mathematical Aspects of Laguerre Wavelets Transformation', Annals of Pure and Applied Mathematics, (APAM), VOL(16), NO(1), PP(53-61), (2018).

[5]. Debayan Goswami, 'A Discrete Wavelet Transform based Cryptographic algorithm', International Journal of Computer Science and Network Security (IJCSNS), Vol,11,No4,(2011)

[6]. Dipalee Gupta, Siddhartha Choubey 'Discrete Wavelet Transform for Image Processing', International Journal of Emerging Technology and Advanced Engineering, (IJETAE), VOL(4), PP(598-602), (2015).

[7]. Juanli HU, Jiabin Deng, 'Image Compression Based on Improved FFT Algorithm'Journal of Networks, VOL(6), NO(7), PP(1041-1048), (2011).

[8]. Kamrul Hasan, Koichi Harada, 'Haar Wavelet Based Approach for Image Compression and Quality Assessment of Compressed Image', International Journal of Applied Mathematics (IJAM), VOL(36), NO(1), PP(1-8), (2007).

[9]. Lenka Kormanikora, 'Shape Design and Analysis of Adaptive Structures', Structural and Physical Aspects of Construction Engineering (ELSEVIER), VOL(190), PP(7-14), (2017).

[10]. Michael Berry, 'An Introduction to Wavelet Analysis', AMERICAN MATHEMATICAL SOCIETY, VOL(40), NO(3), PP(421-427), (2003).

[11]. Mridul Kumar, Gunjan Mathur, 'Image Compression Using DFT Through Fast Fourier Transform Technique', International Journal of Emerging Trends And Technology In Computer Science (IJETTCS), VOL(1), PP(129-133), (2012).

[12]. Michel Misiti, Y.Ves Misiti, 'Wavelets and Their Applications', Published by Hermes Science/ LAVOISER, (2003).

[13]. Muhammad Asad Iqbal & Umer Saeed, 'Modified Laguerre Wavelets Method for Delay Differentail Equations of Fractional-Order', (Science Direct), VOL(2), PP(50-54), (2015).

[14]. Prabhjotot kour, 'Image Processing Using Discrete Wavelet Transform', International Journal of Electronics and Communication (IIJEC), VOL(3), NO(1), (2015).

[15]. Sanjeev Kumar, Varun Sood, 'Quality Assessment of Colour Image Compression Using Haar Wavelet Transform', International Journal of Engineering Trends and Technology (IJETT), VOL(3), NO(3), (2012).

[16]. Trevor.C. Bailey, 'Signal Detection in Underwater Sound Using Wavelets', Journal of the Ameririran Statistican Statistical Association, Vol. 93, No. 441. 73-83. (1998).