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Some types of generalized closed and generalized star closed sets in topological ordered spaces

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Abstract
*In the present work our intention is to establish relationship between new types of closed sets namely g*b-closed sets (resp.gb-closed) and g*i-closed sets(resp.gi-closed) and g b- closed sets(resp.gb-closed) and g*d-closed sets(resp.gd-closed). We also established the independency between the notions g*i-closedness (resp.gi-closedness) and g*d-closedness (resp.gd-closedness).*

Key Words:*Topological ordered space, increasing set, Decreasing set, Balanced set, gi-closed set, gd-closed set, gb-closed set, g i-closed set, g*d-closed set, g*b-closed set. 2010*
MATHEMATICS SUBJECT CLASSIFICATION: 54A05,54A06.

I. Introduction

The notion of topological ordered space was first studied by L. Nachbin [9]. A triple (X, τ, ≤) where X is a non-empty set, τ is a topology and ≤ is a partial order on X called as a topological ordered space. A subset A of topological ordered space (X, τ, ≤) is said to d(A) where be an increasing set if A i(A) and is a decreasing set if A- (A)=$\cup_{a \in A}$ [a. →] and d(A)= $\cup_{a \in A}$ [a. ←]. The sets [x, →] = {y∈X / x ≤ y} and [←, x] = (y∈X / y ≤ x) are defined for any x∈X. The complement of an increasing set is a decreasing set and vice versa. A subsetof a topological ordered space (X, τ, ≤) is a balanced set if it is both increasing and decreasing set.

The study of Increasing closed set, Decreasing closed set and Balanced closed set(briefly i-closed, d-closed and b-closed) in topological ordered spaces was initialized by M. K. R. S. Veerakumar [12]. The notion of generalized closed set (briefly g-closed set) was introduced by N. Levin [7]. Later Bhattacharya and Lahiri [7] introduced and studied semi generalized closed sets (briefly sg-closed sets) in topological spaces. Also, generalized star closed sets (briefly g-closed sets) were introduced by Veerakumar [13]. In the later years some Authors [11] introduced and studied g*i-closed sets, g*d-closed sets and g*b-closed sets in topological ordered spaces.

In the present work, we established that every g*b-closed (resp.gb-closed) set is both g i-closed (resp. gi-closed) set and g*d-closed (resp. gd-closed) set. We also provided examples for the independency of the notions namely g*i-closedness and g*d-closedness.

II. Preliminaries

Unless otherwise mentioned, (X. τ) represent non-empty topological space on which no separation axioms are assumed. The usual notations, cl(A), int (A) and C(A) denote the closure, the interior of A and the complement of A respectively for a subset A.
We recall the following definitions which are useful in the sequel.

DEFINITION 2.1. A subset A of a topological space (X. τ) is called
1. a generalized closed set (briefly g-closed [8]) if cl(A)⊆U whenever A⊆U and U is open in (x, τ). The compliment of a g-closed is a g-open set.
2. a g-closed [13] if cl(A)⊆ U whenever A⊆U and U is g-open in (X,t).

DEFINITION 2.2. [11] In a topological ordered space (X, τ,≤), a subset A is a g*i-closed (resp. gi-closed) set if A is both increasing and g-closed (resp. g-closed) set.
DEFINITION 2.3. [11] In a topological ordered space (X, τ, ≤), a subset A is a g'd-closed (resp.gd-closed) set if A is both decreasing and g-closed (resp. g-closed) set.
DEFINITION 2.4. [11] In a topological ordered space (X, τ, ≤), a subset A is gb-closed (resp. gb-closed) set if A is both balanced and g*-closed (resp.g-closed) set.

III. SOME APPLICATIONS.

THEOREM 3.1. Every g b-closed set is a g*i-closed set.

Proof. Let A be a gb-closed set in the TOS (X, τ, ≤). Then, A is an increasing set and is a g-closed set. Thus, A is a g*i-closed set.

The converse of the above theorem is not true. This can be seen in the following example.

EXAMPLE 3.2. Let X = {a, b, c{, τ_6={φ. X. {a}, {b}, {a, b}, {a, c}{ and ordering $\leq_9$=((a, a), (b, b), (c, c), (a, c)}. Then, (X, τ_6, $\leq_9$) is a topological ordered space. In this space, the g*i-closed sets are 4, X. (b), (c), (a, c). (b, c) and the g b-closed sets are 4, X, (b), (a, c). Then, A (b, c) is a g*i-closed set but it is not a g*b-closed set.

THEOREM 3.3. Every g b-closed set is a g*d-closed set.

Proof. Let A be a g*b-closed set in the topological ordered space (X, τ,≤). Then A is a decreasing set and is a g*-closed set. Thus, A is g*d-closed set.

The converse of the above theorem is not true. This can be seen in the following example.

EXAMPLE 3.3. Let X = (a, b, c), τ_6 = { φ. X. {a}, {b}, {a, b}, {a, c}} and the ordering $\leq_{10}$= {(a, a), (b, b), (c, c), (b, a), (b, c), (c, a)). Then, (X,τ_6,$\leq_{10}$) is a topological ordered space. In this space the 3*d-closed sets are φ, X. (b), (b, c) and the g*b- closed sets are φ, X. Then, A = {b, c) is a g* d-closed set but it is not a g*b-closed set.

THEOREM 3.4. Every gb-closed set is a gi-closed set.

Proof. Let A be a gb-closed set in the topological ordered space (X, τ, ≤). Then, A is an increasing set and is a g-closed set. Thus, A is a gi-closed set.

The converse of the above theorem is not true. This can be seen in the following example.

EXAMPLE 3.5. Let X (a, b, c), τ_6 ={φ, X. {a}, {b}, {a, b}, {a, c}} and the ordering $\leq_9$= ((a, a), (b, b), (c, c), (a, c)). Then, (X,τ_6,$\leq_9$) is a topological ordered space. In this space, the gi-closed sets are φ, X,{b},{c},{a, c},{b, c}and the gb-closed sets are φ, X,{b},{a, c}. Then, the subset A = {b, c} is a gi-closed set but it is not a gb-closed set.

THEOREM 3.6. Every gb-closed set is a gd-closed set.

Proof. Let A be a gb-closed set in the topological ordered space (X, τ, ≤). Then A is a decreasing set and is a g-closed set. Thus, A is gd-closed set.

The converse of the above theorem is not true. This can be seen in the following example.

EXAMPLE 3.7. Let X ={a, b, c}, τ_6 = {φ, X. {a}, {b}, {a, b}, {a, c}} and the ordering $\leq_{10}$= ((a, a), (b, b), (c. c). (b, c), (c, a). (b. a)). Then, (X, τ_6, $\leq_{10}$) is a topological ordered space. In this space, the gd-closed sets are φ, X,{b},{b, c} and the gb-closed sets are φ, X. Then, the subset A= {b, c} is a gd-closed set but it is not a gb-closed set.

IV. INDEPENDENT NOTIONS

THEOREM 4.1. The notions g*i-closedness and g*d-closedness are independent.

Proof. Follows form the following examples.

EXAMPLE 4.2. Let X={a, b, c}, τ_1= {φ, X, {a}, {b}, {a, b}, {a, c}} and the ordering $\leq_2$= {(a, a), (b, b), (c, c), (a, b), (c, b)}. Then, (X, ti, s) is a topological ordered space. In this space the g*i-closed sets are φ, X. {b, c} and the g*d-closed sets are φ, X, {c} {a, c}. Then, the subset A (b. c) is a g*i-closed set but it is not a g-d-closed set. On the other hand, the subset B-fa. c) is a g*d-closed set but it is not a g*i-closed set.

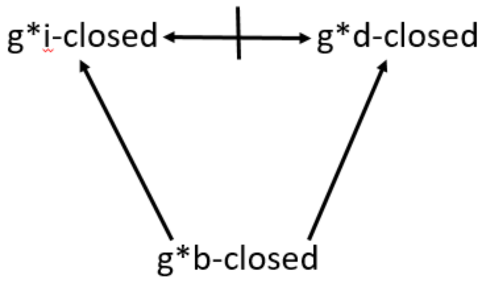
THEOREM 4.3. The notions gi-closedness and gd-closedness are independent. Proof. Follows form the following examples.

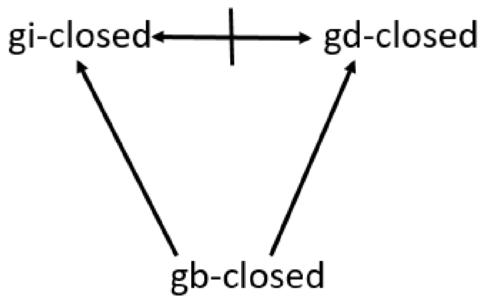
EXAMPLE 4.4. Let X = {a. b. c}τ_1 = {φ, X, {a}, {b}, {a, b}}and the ordering $\leq_2$= {(a, a), (b, b), (c, e), (a, b), (c, b)}. Then, (X,τ_1,$\leq_2$) is a topological ordered the gi-closed sets are φ, X,{b. c} and the gd-closed sets are φ, X. {c}{a, c}. Then, the subset A = {b, c} is a gi-closed set out it is not a gd-closed set. On the other hand, the subset B = {a, c} is a gd-closed set but not a gi-closed set.

The following diagram shows the relationships established in the present work.

Here. A→B means A implies B but not conversely and A ↔ B denote A and B are independentnotions.

Diagram





V. CONCLUSION

In the present work, we established some relationships between g-closed type independency of the two types of closedness. As a further study we will focus on the relationships sets and g-closed type sets in topological ordered spaces. We also provided examples for the of semi generalized closed type sets with other types of closed sets in topological ordered spaces.

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