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Generalized Permuting Tri-(f,g)-Derivations of Incline Algebras

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Abstract : *In This paper we introduce the concept of a generalized permuting tri-(f,g) -derivation which is generalization of permuting tri-f-derivation on incline algebra and investigate some properties of Based on this concept .*

Keywords: *Incline algebra,Derivation on incline algebra,Permuting tri-(f,g) derivation,distributive lattice,ideal .*

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I. Introduction

In our real life various problems related with medical sciences, engineering, political, financial, social deciplines and numerous different arenas involve provisional data which are not always necessarily in crisp, appropriate and conclusive forms due to uncertainty associated with these problems. Such problems are usually being handled with the help of the topics like probability theory , fuzzy set theory, intuitionistic fuzzy sets, interval mathematics and rough sets

Derivation on various algebraic structures is very active area of research since last 20 years due to their various application in different field of mathematics.Incline algebra is generalization of both Boolean and fuzzy algebra and it is a special type of semiring which follows both a semiring structre and a poset structure.The notion of derivation on a lattice and their properties is firstly studied by Szasz [3].Ozturk [4] introduced the notion of a permuting tri–derivation and proved some properties.later as generalization of derivations of,f-derivation,(f,g)-derivations,symmetric bi-derivations,permuting tri-derivations,symmetric f-derivations on prime ,semiring and lattice are studied by lot of researchers[5,6,].also in [2] permuting tri –f-derivation studied by K.H Kim and Y.H Lee. We take the basic notion and definitions based on Cao,Kim and Rough[1] .In this paper, as a generalization of permuting tri-(f,g) derivation of an incline algebra is introduced .Further using this notion some related properties of an incline algebra are investigated .

II. Prelimniries

Definition 2.1.*An incline is an algebraic structure ($\mathfrak{I}$,+,*) having a non-empty set $\mathfrak{I}$ and two binary operations + and * such that for all x,y,z in $\mathfrak{I}$, if the following laws hold*

[K1] *Associative laws*

(i) $x + (y + z) = (x + y) + z$,

(ii) $x * (y * z) = (x * y) * z$.

[K2] *Commutative laws*

(i) $x + y = y + x$,

(ii) $x * y = y * x$.

[K3] *Distributive laws*

(i) $x * (y + z) = (x * y) + (x * z)$,

(ii) $(y + z) * x = (y * x) + (z * x)$.

[K4] *Idempotent law:* $x + x = x$.

[K5] *Incline law*

(i) $x + (x * y) = x$,

(ii) $y + (x * y) = y$.

In an incline algebra $\mathfrak{I}$, the following properties also hold.

[K6] $x * y \leq x$ and $y * x \leq x$ for all $x, y \in \mathfrak{I}$,

[K7] $x \leq x + y$ and $y \leq x + y$, for all $x, y \in \mathfrak{I}$,

[K8] $y \leq z$ implies $x * y \leq x * z$ and $y * x \leq z * x$, for all $x, y, z \in \mathfrak{I}$,

[K9] If $x \leq y$ and $a \leq b$, then $x+a \leq y+b$, and $x*a \leq y*b$ for all $x, y, a, b \in \mathfrak{I}$

For convenience, we pronounce " + " (resp. " $*$ ") as addition (resp. multiplication). Every distributive lattice is an incline algebra. An incline algebra is a distributive lattice if and only if $x * x = x$ for all $x \in \mathfrak{I}$,. Note that $x \leq y \Leftrightarrow x + y = y$ for all $x, y \in \mathfrak{I}$,. It is easy to see that " $\leq$ " is a partial order on K and that for any $x, y \in \mathfrak{I}$,, the element $x + y$ is the least upper bound of $\{x, y\}$. We say that $\leq$ is induced by operation +

We shall also use the following definitions and properties in our further investigations.

Definition 2.2. *Let $x,y \in \mathfrak{I}$. The incline order relation denoted as "$\leq$" and is defined as $x \leq y \leftrightarrow x + y = y$.*

From the incline axiom (K5) obiviously, we have

I. *$x + y \geq x$ and $x + y \geq y$ for $x,y \in \mathfrak{I}$,*

II. *$xy \leq x$ and $xy \leq y$ for $x,y \in \mathfrak{I}$.*

which are known as incline properties.

Furthermore, an incline algebra $\mathfrak{I}$ is said to be commutative if $x * y = y * x$ for all $x, y \in \mathfrak{I}$.

A subincline of an incline algebra $\mathfrak{I}$ is a non-empty subset N of $\mathfrak{I}$ which is closed under the addition and multiplication. A subincline N is called an ideal if $x \in N$ and $y \leq x$ then $y \in N$. An element "0" in an incline algebra $\mathfrak{I}$ is a zero element if $x + 0=x = 0 + x$ and $x * 0 = 0 = 0 * x$ for any $x \in \mathfrak{I}$.

An non-zero element "1" is called a multiplicative identity if $x * 1 = 1 * x = x$ for any $x \in \mathfrak{I}$. A non-zero element $a \in \mathfrak{I}$ is said to be a left (resp. right) zero divisor if there exists a non-zero $b \in \mathfrak{I}$ such hat $a * b = 0$ (resp. $b * a = 0$). A zero divisor is an element of $\mathfrak{I}$ which is both a left zero divisor and a right zero divisor. An incline algebra $\mathfrak{I}$ with multiplicative identity 1 and zero element 0 is called an integral incline if it has no zero divisors. By a homomorphism of inclines, we mean a mapping f from an incline algebra $\mathfrak{I}$ into an incline algebra L such that

$$f(x + y) = f(x) + f(y) \text{ and } f(x * y) = f(x) * f(y) \text{ for all } x, y \in \mathfrak{I}.$$

for any $x \in \mathfrak{I}$. A non-zero element $a \in \mathfrak{I}$ is said to be a left (resp. right) zero divisor if there exists a non-zero $b \in \mathfrak{I}$ such hat $a * b = 0$ (resp. $b * a = 0$). A zero divisor is an element of $\mathfrak{I}$ which is both a left zero divisor and a right zero divisor. An incline algebra $\mathfrak{I}$ with multiplicative identity 1 and zero element 0 is called an integral incline if it has no zero divisors. By a homomorphism of inclines, we mean a mapping f from an incline algebra $\mathfrak{I}$ into an incline algebra L such that

$$f(x + y) = f(x) + f(y) \text{ and } f(x * y) = f(x) * f(y) \text{ for all } x, y \in \mathfrak{I}.$$

Definition 2.3. Let $\mathfrak{I}$ be an incline algebra. A mapping $D: \mathfrak{I} \times \mathfrak{I} \times \mathfrak{I} \rightarrow \mathfrak{I}$ is said to be permuting if it satisfies the following condition:

$D(x, y, z) = D(x, z, y) = D(y, x, z) = D(y, z, x) = D(z, x, y) = D(z, y, x)$ for all $x, y, z \in \mathfrak{I}$.

A mapping $d : \mathfrak{I} \rightarrow \mathfrak{I}$ defined by $d(x) = D(x, x, x)$ is called a trace of D where D is a permuting mapping.

Definition 2.4. Let $\mathfrak{I}$ be an incline algebra. A permuting mapping $D : \mathfrak{I} \times \mathfrak{I} \times \mathfrak{I} \rightarrow \mathfrak{I}$ is said permuting tri-derivation if it satisfies the following condition

$D(x * w, y, z) = (D(x, y, z) * w) + (x * D(w, y, z))$, for all $x, y, z, w \in \mathfrak{I}$.

Definition 2.5. Let $\mathfrak{I}$ be an incline algebra. A permuting mapping $D : \mathfrak{I} \times \mathfrak{I} \times \mathfrak{I} \rightarrow \mathfrak{I}$ is called a permuting tri-f -derivation if there exists a function $f : \mathfrak{I} \rightarrow \mathfrak{I}$ such that

$D(x * w, y, z) = (D(x, y, z) * f(w)) + (f(x) * D(w, y, z))$, for all $x, y, z, w \in \mathfrak{I}$.

It is obvious that if D is a permuting tri-f -derivation of $\mathfrak{I}$, then it is satisfies the relations $D(x, y * w, z) = (D(x, y, z) * w) + (y * D(x, w, z))$and $D(x, y, z * w) = (D(x, y, z) * w) + (z * D(x, y, w))$, for all $x, y, z, w \in \mathfrak{I}$.

III. Generalized Permuting tri-(f,g)-derivations of incline algebra.

Let $\mathfrak{I}$ be an incline algebra. A permuting mapping $D : \mathfrak{I} \times \mathfrak{I} \times \mathfrak{I} \rightarrow \mathfrak{I}$ is called generalized permuting tri-(f,g) -derivation if there exists a function $f : \mathfrak{I} \rightarrow \mathfrak{I}$ and $g : \mathfrak{I} \rightarrow \mathfrak{I}$ such that

$$D(x * w, y, z) = (D(x, y, z) * f(w)) + (g(x) * D(w, y, z)), \text{ and}$$
$$D(x * w, y, z) = (D(x, y, z) * g(w)) + (f(x) * D(w, y, z)), \text{ for all } x, y, z, w \in \mathfrak{I}.$$

It is obvious that if D is a generalized permuting tri-(f,g)-derivation of $\mathfrak{I}$, then it is satisfies the relations:$D(x, y * w, z) = (D(x, y, z) * w) + (y * D(x, w, z))$,

$$D(x, y, z * w) = (D(x, y, z) * w) + (z * D(x, y, w)),$$
$$D(x, y * w, z) = (D(x, w, z) * y) + (w * D(x, y, z)) \text{and}$$
$$D(x, y, z * w) = (D(x, y, w) * z) + (w * D(x, y, z)),$$
$$\text{for all } x, y, z, w \in \mathfrak{I}.$$

Example 3.1. Let $\mathfrak{I}$ be a commutative incline algebra and $a \in \mathfrak{I}$. Define a function D on $\mathfrak{I}$ by

$$D(x, y, z) = (f(x) * f(y)) * g(z), \text{ and } (g(x) * g(y)) * f(z),$$

where $f : \mathfrak{I} \rightarrow \mathfrak{I}$ and $g : \mathfrak{I} \rightarrow \mathfrak{I}$ satisfies $d(x * y) = d(x) * f(y) + g(x) * d(y) \leq d(x)+d(y)=d(x+y)$ for all $x, y \in \mathfrak{I}$. Then D is a generalized permuting tri-(f,g) -derivation of $\mathfrak{I}$.

Example 3.2. Let $\mathfrak{I}$ be a commutative incline algebra and $a \in \mathfrak{I}$. Define a function D on $\mathfrak{I}$ by

$$D(x, y, z) = a * ((f(x) * f(y)) * g(z)),$$

where $f : \mathfrak{I} \rightarrow \mathfrak{I}$ and $g : \mathfrak{I} \rightarrow \mathfrak{I}$ satisfies

$$d(x * y) = d(x) * f(y) + g(x) * d(y) \text{ for all } x, y \in \mathfrak{I}.$$

Then D is a generalized permuting tri-(f,g) -derivation of $\mathfrak{I}$.

Proposition 3.1. Let $\mathfrak{I}$ be an incline algebra and let D be a generalized permuting tri-(f ,g)-derivation of $\mathfrak{I}$. Then the following properties hold for all $x, y, z, w \in \mathfrak{I}$.

(1) $D(x * w, y, z) \leq D(x, y, z) + D(w, y, z)$,

(2) If $x \leq w$, then $D(x * w, y, z) \leq f(w)+g(w) \leq g(w)$

Proof. (1) Let $x, y, z, w \in \mathfrak{I}$. By using (K6), we have $D(x, y, z) * f(w) \leq D(x, y, z)$ and $g(x) * D(w, y, z) \leq D(w, y, z)$. By using (K9), we get

$$(D(x, y, z) * f(w)) + (g(x) * D(w, y, z)) \leq D(x, y, z) + D(w, y, z),$$

and so $D(x * w, y, z) \leq D(x, y, z) + D(w, y, z)$.

(2) Using (K6), we get the relation $D(x, y, z) * f(w) \leq f(w)$. Let $x \leq w$. Then by using (K6) and (K9), we get $g(x) * D(w, y, z) \leq f(w) * D(x, y, z) \leq f(w)$.

That is, $D(x * w, y, z) = (D(x, y, z) * f(w)) + (g(x) * D(w, y, z)) \leq f(w) + g(w) = g(w)$, which implies $D(x * w, y, z) \leq g(w)$

Proposition 3.2. Let $\mathfrak{I}$ be an incline algebra and let d be the trace of a Generalized permuting tri-(f,g) - derivation D of $\mathfrak{I}$. If $f(0) = g(0)=0$, then $d(0) = 0$.

Proof. Let $f(0) = g(0)=0$ and $x \in \mathfrak{I}$. Then we get

$$d(0) = D(0, 0, 0) = D(x * 0, 0, 0)= (D(x, 0, 0) * f(0)) + (g(x) * D(x, 0, 0))$$
$$\text{and } (D(x, 0, 0) * g(0)) + (f(x) * D(x, 0, 0))$$

$\Longrightarrow (D(x, 0, 0) * 0) + (f(x) * D(x, 0, 0))$ and $(D(x, 0, 0) * 0) + (g(x) * D(x, 0, 0))$

$\Longrightarrow 0 + f(x) * D(x, 0, 0)$and $0 + g(x) * D(x, 0, 0)$

$\Longrightarrow f(x) * D(x, 0, 0)$. and $g(x) * D(x, 0, 0)$

Taking $x = 0$, we get $d(0) = f(0) * D(0, 0, 0)$ and $g(0) * D(0, 0, 0) = 0 * D(x, 0, 0) = 0$.

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IV. Conclusion

In this paper, we introduced on generalized permuting tri-(f,g) derivation on incline algebra .Further I studied some properties related to this notion and many more scope to investigate properties of incline algebra based on this notion.

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