

## Narrow Band Decomposition of Audio Signal using FN-BMFLC and Kalman Filter

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**ABSTRACT:** The paper presents a relatively new method for time frequency decomposition of audio signal based on Adaptive Spectral Estimation of Fourier Coefficients. The method adopted is referred as Frequency Normalized Band-limited Multiple Fourier Linear Combiner (FN-BMFLC), a newer version of BMFLC with Kalman Filter (KF). A brief comparison with existing time frequency analysis methods, Study on the efficiency of the new system and finally running the system for an audio input in WAV format and verifying the results form an overview of the paper. The results show good estimation accuracy, optimal time frequency resolution, stability and acceptable computation time.

**KEYWORDS:** Adaptive Fourier Coefficient Estimation, Audio Analysis, Estimation Accuracy, Frequency Normalized Band limited Multiple Fourier Linear Combiner, Kalman Filter

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### I. INTRODUCTION

The transient nature of audio signals makes the time frequency analysis a tedious job. Non parametric methods such as Continuous Wavelet Transform (CWT) and Short Time Fourier Transform (STFT) are used extensively. But both the methods suffer a tradeoff between temporal and spectral resolution. Apart from the best performance, computational requirement for CWT is higher [1]. These points to the need of parametric methods such as Adaptive Spectral Estimation.

Fourier Coefficient Estimation using Fourier Linear Combiner (FLC) is an efficient method when used along with least Mean Square algorithm (LMS) or fast Recursive Least Square algorithm (fRLS) [3], [6], [7]. But both fail in application when the input contains multiple dominant frequencies in a given band. LMS require long data segments and more time to converge [1]. The narrow bands of transient signal states show a variation described by a random walk model when no priori information is given. This give rise to a relatively new concept of Adaptive Spectral Estimation called Band limited Multiple Fourier Linear Combiner with Kalman Filter (BMFLC-KF). For a raw audio data, the Estimation Accuracy of the system is around 39%. To improve the Estimation Accuracy, certain modifications are suggested and have been tested here. The major challenges include Estimation Accuracy, Computation time and Memory usage.

Experiments conducted on the existing system, evolved a new system called Frequency Normalized BMFLC-KF (FN-BMFLC-KF) which has improved estimation accuracy, reduced computation time, tuned parameters and automatic fixation of frequency resolution based on input statistics.

### II. FREQUENCY NORMALIZED BMFLC WITH KALMAN FILTER

Any band limited signal audio Sample  $y_k$  at instant  $k$  can be represented as:

$$y_k = \sum_{r=1}^n a_{rk} \sin\left(\left(\frac{f_r}{f_s}\right) k\right) + b_{rk} \cos\left(\left(\frac{f_r}{f_s}\right) k\right) \quad (1)$$

Where  $a_{rk}$ ,  $b_{rk}$  are the adaptive weights corresponding to the frequency  $f_r$  at time instant  $k$ . Let  $f_s$  be the sampling frequency of input. Then the FN-BMFLC-KF System can be modeled as follows. Where  $\Delta f$  represents the frequency resolution

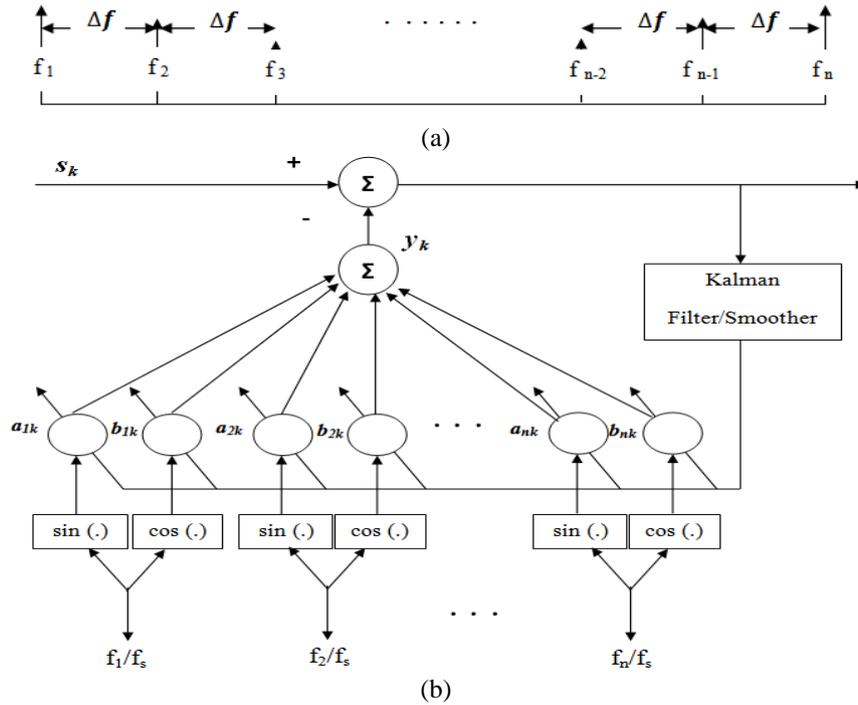


Fig. 1. (a) frequency components distribution; (b) FN-BMFLC-KF model.

Let the Normalized Frequency Component Matrix be as follows:

$$X_k = \begin{bmatrix} \left[ \sin\left(\frac{f_1}{f_s} k\right) \quad \sin\left(\frac{f_2}{f_s} k\right) \quad \dots \quad \sin\left(\frac{f_n}{f_s} k\right) \right]^T \\ \left[ \cos\left(\frac{f_1}{f_s} k\right) \quad \cos\left(\frac{f_2}{f_s} k\right) \quad \dots \quad \cos\left(\frac{f_n}{f_s} k\right) \right]^T \end{bmatrix} \quad (2)$$

The Optimizing Coefficient Matrix be:

$$W_k = \begin{bmatrix} [a_{1k} \quad a_{2k} \quad \dots \quad a_{nk}]^T \\ [b_{1k} \quad b_{2k} \quad \dots \quad b_{nk}]^T \end{bmatrix} \quad (3)$$

Let the Measurement Error  $v$  and State Error  $\eta$  be uncorrelated, zero mean, WGN follows:

$$v \sim N(0, R) \quad (4)$$

$$\eta \sim N(0, Q) \quad (5)$$

When no priori information is available the State Space Model can be assumed to be a Random Walk Model as follows. The Reconstructed Sample can be represented as:

$$y_k = X_k^T W_k + v_k \quad (6)$$

Then the State update Equation is:

$$W_{k+1} = W_k + \eta_k \quad (7)$$

A Kalman Filter is employed along with FN-BMFLC. It can provide the Minimum Mean Square Estimators within the class of linear estimators, with the conditional expected weight given by:

$$\hat{w}_k = E[w_k | y_{k-1}] \quad (8)$$

If  $K_k$  be the Kalman Gain at  $k$  and  $P_k$  is the State Error Covariance at  $k$  then with initial conditions  $\hat{W}_0$  and  $P_0$ , the KF can be implemented as Follows:

$$K_k = P_k X_k^T [X_k P_k X_k^T + R]^{-1} \quad (9)$$

$$\hat{W}_{k+1} = \hat{W}_0 + K_k [y_k - X_k^T \hat{W}_k] \quad (10)$$

$$P_{k+1} = [1 - K_k X_k] P_k + Q \quad (11)$$

Where  $Q$  and  $R$  are the State Error Covariance and the Measurement Error Covariance respectively. The Absolute Weight Vector of Frequency Components at time instant  $k$  is given by:

$$W_k^f = \left[ \sqrt{a_{1k}^2 + b_{1k}^2} \quad \dots \quad \sqrt{a_{nk}^2 + b_{nk}^2} \right]^T \quad (12)$$

The Time-Frequency Decomposed Matrix  $D$  for  $m$  Samples which contain  $n$  Frequency Components is given by:

$$D = \begin{bmatrix} \sqrt{a_{11}^2 + b_{11}^2} & \dots & \sqrt{a_{1m}^2 + b_{1m}^2} \\ \sqrt{a_{21}^2 + b_{21}^2} & \dots & \sqrt{a_{2m}^2 + b_{2m}^2} \\ \vdots & \ddots & \vdots \\ \sqrt{a_{n1}^2 + b_{n1}^2} & \dots & \sqrt{a_{nm}^2 + b_{nm}^2} \end{bmatrix} \quad (14)$$

The Energy Distribution in Time-Frequency Mapping can be obtained by:

$$Power = D \otimes D \quad (15)$$

Where  $\otimes$  represents the element by element multiplication. The Estimation Accuracy is given by:

$$RMS\% = \frac{RMS(s) - RMS(e)}{RMS(s)} \times 100 \quad (16)$$

Where  $RMS(s)$  is the RMS value of the original signal and  $RMS(e)$  is the RMS value of Estimation Error.  $RMS(s)$  can be given as:

$$RMS(s) = \sqrt{\left( \sum_{k=1}^n s_k^2 \right) / n} \quad (17)$$

### III. RESULTS

In this section, an audio signal is decomposed in time-frequency for a narrow band of frequencies, an estimate of the signal is made from the adaptively estimated spectral coefficients, and energy spectral of the signal is also determined. The dataset used and the parameter selection for the procedure is given below. The experiment has been evaluated in MATLAB environment.

#### 1.1 Data Set

An Audio signal with Sampling Frequency 8000 Hz in WAV format is taken as the input. This has to be analyzed for 3494Hz to 3497 Hz frequency band. Fig.2 represents the frequency spectrum of the input obtained using FFT.

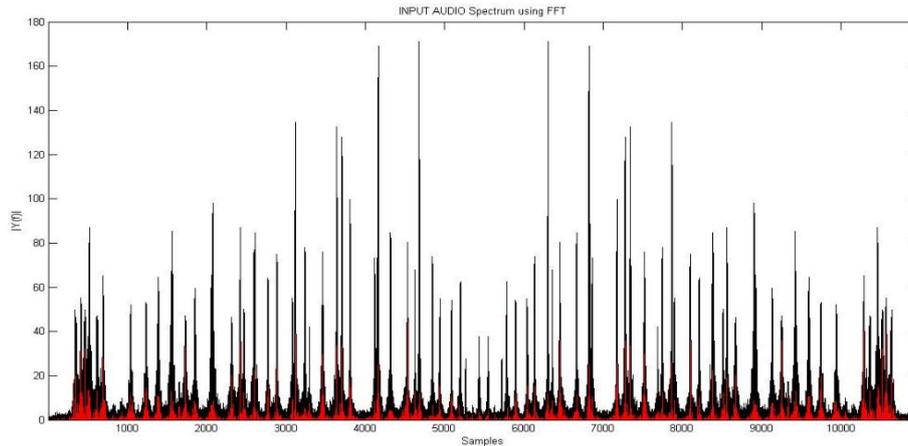


Fig. 2. frequency spectrum of audio input

Using the equation (18), we can identify any of the major frequency bands. Here we have taken 3494 Hz to 3497Hz:

$$f = \frac{k * f_s}{N} \quad (18)$$

### 1.2 Parameter Selection

Initial Prediction Error Covariance P0 is taken as a low value say, 0.001 as experimentally obtained for EEG [1]. A tuning procedure is done for AUTO and MANUAL mode to obtain the State Error Covariance Q and Measurement Error Covariance R. Assuming a low R of 0.0001 the optimum Q is obtained as X for the given audio signal. Initially the weight vector  $\hat{w}_0$  is assumed to be zero. Initially the State Error and Measurement error are also taken as zero. Both the above errors are assumed to be WGN, uncorrelated and zero mean processes. .

In the existing system of BMFLC-KF the estimation of the decomposition band width (frequency resolution)  $\Delta f$  is not done, and the user has to feed the value. So, for an unknown signal it is difficult to obtain an optimum value. In the proposed method it is achieved from the rough frequency Spectrum obtained using N point FFT of input by the following algorithm. The dominant indices k of the spectrum have been selected using the mean of the spectrum as the threshold. The corresponding frequencies in Hz are obtained by equation (18). A Thresholding has been done again to avoid the delay in computation. Now the minimum of such frequency list is taken as the decomposition frequency  $\Delta f$ . This selection is done automatically by analyzing the statistics of the input signal. This ensures a reduction in computational complexity while retaining an optimum frequency resolution. A user defined sample reduction algorithm based on input statistics can be applied to reduce number of samples at the cost of Estimation Accuracy.

**Table 1. Parameter Selection**

Frequency Band (Hz)	3494-3497
Resolution (Hz)	0.7286
Number of Samples	10980
State Error Cov. (Q)	20000
Measurement Error Cov.(R)	0.00001
Initial Prediction Error P0	0.001
Initial Weights W0	0

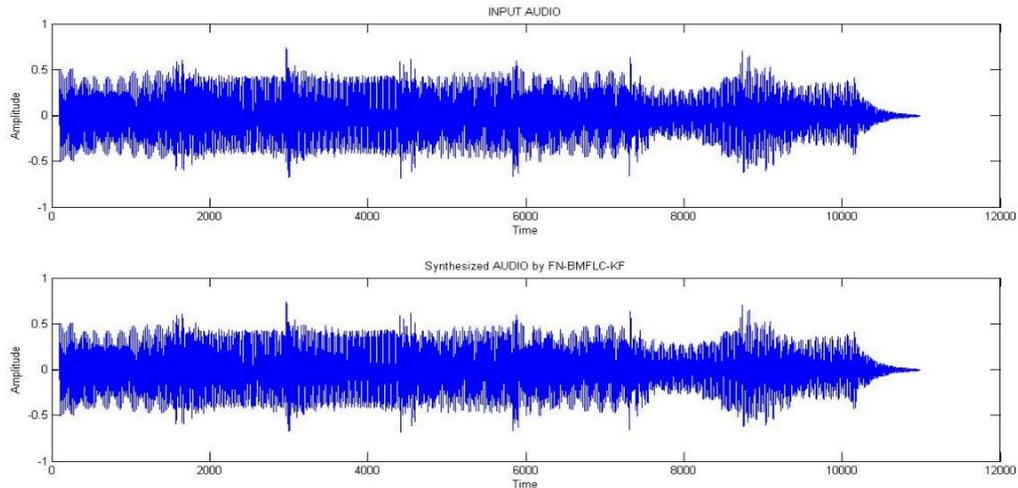


Fig. 3. Audio input and Synthesized Signal using FN-BMFLC and KF

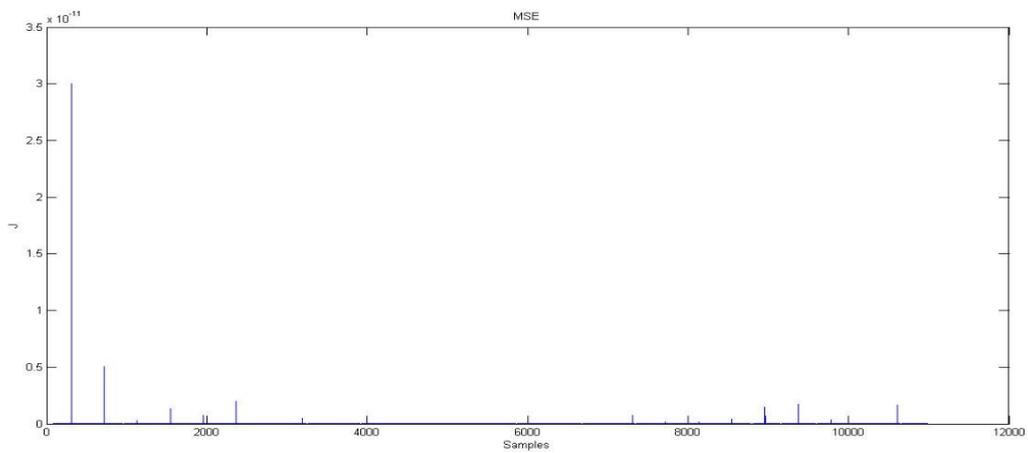


Fig. 4. Mean Square Error (MSE)

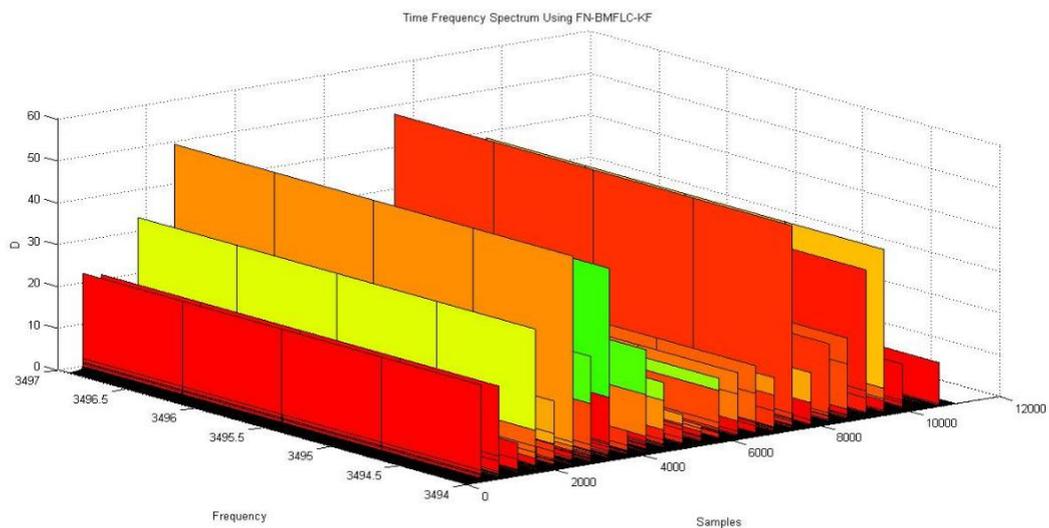


Fig. 5. Time –Frequency Decomposition of Audio input using FN-BMFLC and KF

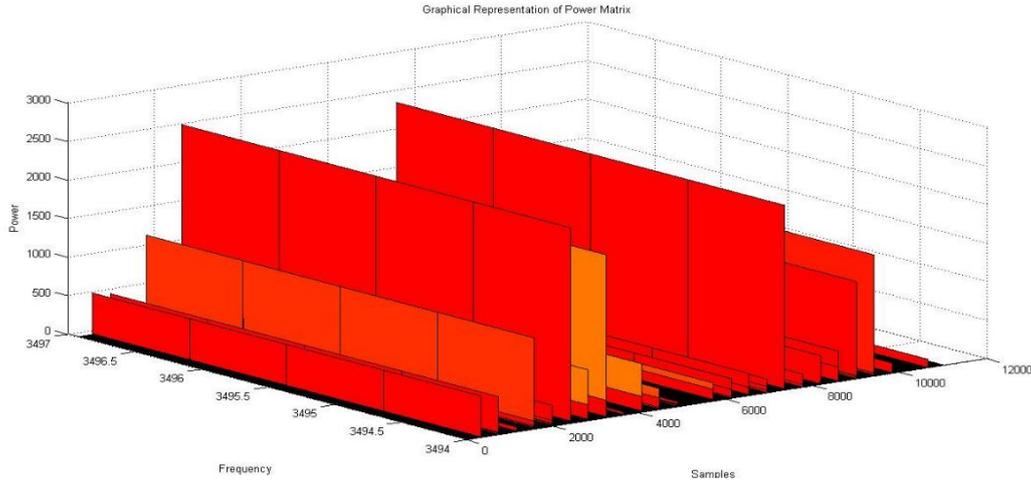


Fig. 6. Energy Distribution of AUDIO input using FN-BMFLC and KF

#### IV. DISCUSSIONS & CONCLUSIONS

The effects of samples and resolution on Estimation Accuracy have been studied . Results showing Good Estimation Accuracy while providing better stability. Also MSE is negligible. Advantages include the following things. It functions as an Adaptive MMSE Spectral Estimator for transient signals. Simple and user friendly. Computational Complexity is reduced using input Statistics. Input Versatility is achieved, i.e the system can be generalized for inputs other than audio . Low MSE (Mean Square Error) and Estimation Accuracy above 99% generally observed. Manual mode can be used for user-defined offline estimation of important signals. Limitations include Tradeoff between Resolution and Computational Complexity. It also possesses Tradeoff between Samples and Computational Complexity. The following figures and tables give an idea about the characteristics of the system.

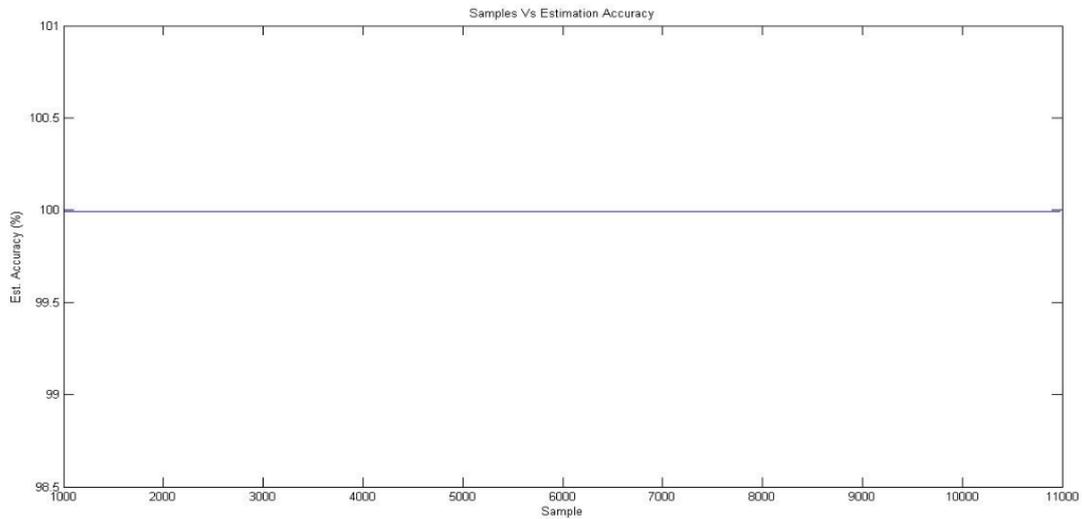


Fig. 7. Sample Vs. Estimation Accuracy for  $\Delta f = 0.7286\text{Hz}$  and in the band 3494-3497Hz

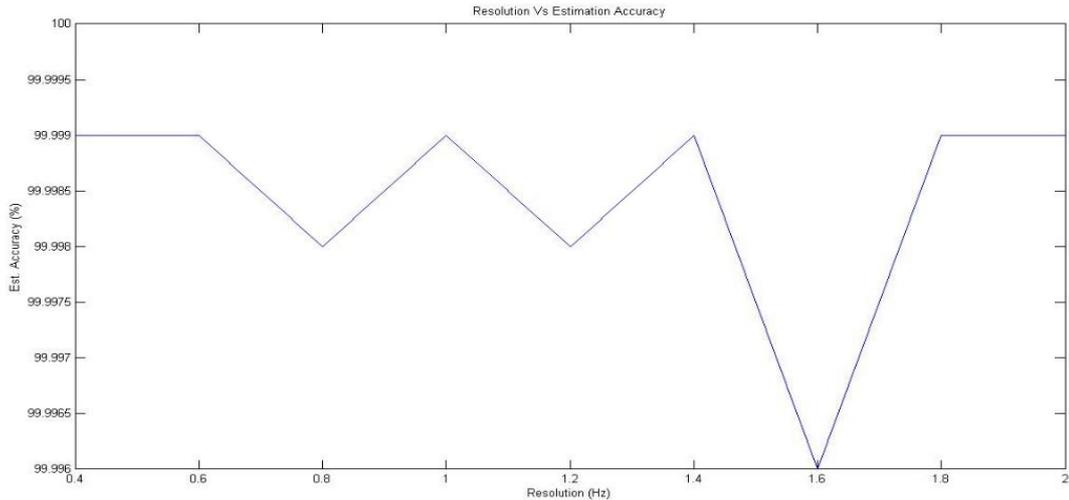


Fig. 8. Resolution Vs. Estimation Accuracy for 10980 samples in the band 3494-3497Hz

**Table 2. General Characteristics**

Signal	Frequency Band(Hz)	Samples	Frequency Resolution(Hz)	Estimation Accuracy(%)	MSE
Audio (WAV)	3494-3497	10980	0.7286	99.99	$4.39 \times 10^{-15}$

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