A Potent MIMO–OFDM System Designed for Optimum BER and its Performance Analysis in AWGN channel

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Abstract—Multiple-input Multiple-output (MIMO) systems use multiple antennas at the transmitter and receiver end, are a key technology to meet the growing demand for high data rate wireless systems. MIMO communication systems have the potential to provide increased capacity and reduced bit error rate and reliability without increasing the bandwidth or transmitted power. Obtaining this information, It’s paramount to design a knacky MIMO system to fulfill the growing demand for high data rate. In this purpose, Our research done and turned into complexion of the paper. The aim of this paper is to design MIMO-OFDM system with the aim of achieving the lowest and optimum Bit Error Rate (BER) while increasing the system capacity using multicarrier delay diversity modulation (MDDM), proposed for fifth generation systems. To implement the design Binary Phase Shift Keying (BPSK) chosen as constellation mapper and demapper rather than QAM and simulated using MATLAB, which is examined in associated Additive White Gaussian Noise (AWGN) channel models with no fading channel.

Keywords—SISO, MIMO, MDDM, MRC, AWGN channel, Spatial Correlation, MIMO-OFDM;

1. INTRODUCTION

One of the challenges faced by future wireless communication systems as 5G (Fifth Generation) is to provide high data rates at high quality of service (QoS) [1]. Combined with the fact that spectrum is a scarce resource and propagation conditions are hostile this requires radical increase in spectral efficiency and link reliability. The system requirements can be met by the combination of two powerful technologies in the physical layer design: multi input and multi output (MIMO) antennas and orthogonal frequency division multiplexing (OFDM) modulation [2]. These two are considered to be the cardinal enabling technologies that will help the next generation networks exceed the current system performance. Use of multiple antennas can offer significant increase in data throughput, spectral efficiency and link range without additional bandwidth or transmit power [3]. Link reliability and diversity are improved but this also increases frequency selective fading in the channels. The primary advantage of OFDM is its ability to cope with severe channel conditions like frequency selective fading due to multipath propagation, attenuation of high frequency in long copper wire and narrowband interference [2]. Multicarrier Delay Diversity Modulation (MDDM) technique has been recognized as the MIMO technique to increase transmission capacity [4]. Simulated performance of MIMO is strongly influenced by the choice of the channel model. Modeling of the radio channel is essential for system design and performance evaluation. Channel conditions like spatial correlation have been noticed to substantially impair the performance of MIMO wireless communication systems. Most of the research in MIMO OFDM performance has been concentrated on using independent and identically distributed Rayleigh channel model which will also be considered along with more realistic channel models with spatial correlation. Based on all information and considerations, this work is done to evaluate and improve the performance of MIMO OFDM system with Delay Diversity Modulation (MDDM).

Why is OFDM used in the system?

MIMO wireless communication systems operating in a rich scattering environment usually face unacceptable inter-symbol interference (ISI) from multipath propagation and their inherent delay spread. Thus, the channel exhibits frequency selective fading. Orthogonal Frequency Division Multiplexing (OFDM) is a powerful, multicarrier technique for combating ISI that operates with specific orthogonality constraints between the subcarriers, which enables it to achieve a very high spectral efficiency. In effect, it transforms a frequency selective channel into a set of parallel flat fading channels and hence, the signals on each subcarrier undergo narrowband fading. In order to deal with frequency selective nature of broadband wireless channels, MIMO can be combined with OFDM technique as shown in the block diagram of Figure 1.
In OFDM, to maintain the orthogonality of the subcarrier channels, the correlation between signals transmitted on subcarriers must be zero. Assume that the available bandwidth for the OFDM system is $\Delta W$ and it is to be divided into $K$ subcarriers. The input serial data stream is to be converted into $K$ parallel data streams which are assigned to the $K$ subcarriers [5]. The symbol duration of the input serial data is $T_s$ with serial data rate of $f_s = \frac{1}{T_s}$. Therefore, if the number of parallel data streams is equal to the number of OFDM subcarriers, the symbol duration for OFDM will be $T_s = KT_s$, that indicates that the symbol duration of an OFDM signal is $K$ times larger than that of single serial stream symbol duration. Therefore, the OFDM scheme has the inherited advantage over single carrier modulation techniques to mitigate ISI and frequency selectivity of the channel.

The OFDM transmitted signal $S(t)$ can be written as

$$S(t) = \sum_{m=0}^{K-1} A \left( d_{tm} \cos(2\pi f_m t) - d_{qm} \sin(2\pi f_m t) \right)$$

Using the trigonometric identity

$$\cos(\alpha + \beta) = \cos(\alpha) \cos(\beta) - \sin(\alpha) \sin(\beta)$$

$S(t)$ can be written as

$$S(t) = \sum_{m=0}^{K-1} \alpha_m \left( \frac{d_{tm}}{\sqrt{d_{tm}^2 + d_{qm}^2}} \cos(2\pi f_m t) - \frac{d_{qm}}{\sqrt{d_{tm}^2 + d_{qm}^2}} \sin(2\pi f_m t) \right)$$

Where $\alpha_m = A \sqrt{d_{tm}^2 + d_{qm}^2}$. Using the trigonometric identity

$$\cos(\alpha + \beta) = \cos(\alpha) \cos(\beta) - \sin(\alpha) \sin(\beta)$$

The correlation between any two symbols transmitted on separate subcarriers, represented as $R_{ij}$ must be equal to zero to maintain the orthogonality of subcarriers [5].

$$R_{ij} = \int_{-\infty}^{\infty} s_i(t) s_j(t) \, dt = \int_0^T a_i \cos(2\pi f_i t + \theta_i) a_j \cos(2\pi f_j t + \theta_j) \, dt$$

Using the trigonometric identity, $\cos(\alpha) \cos(\beta) = \frac{1}{2} \cos(\alpha + \beta) + \frac{1}{2} \cos(\alpha - \beta)$

Equation (3) can be written as,

$$R_{ij} = \frac{a_i a_j}{2} \int_0^T \left( \cos(2\pi f_i t + \theta_i) + \cos(2\pi f_j t + \theta_j) \right) dt$$

(5)

where $a_i, a_j$ are constant for the symbol duration. For $2\pi (f_i + f_j) / T_s \gg \frac{1}{T_s}$, Equation (5) can be written as

$$R_{ij} = \frac{a_i a_j}{2} \int_0^{T_s} 2\pi (f_i - f_j) dt$$

(6)

From Equation (6), $R_{ij} = 0$ if $(f_i - f_j)T_s = M$ where $M \in \text{set of positive integers}$, $\Rightarrow (f_i - f_j) = \frac{M}{T_s}$

(7)

Therefore, minimum frequency separation between two consecutive subcarriers to maintain orthogonality must be
\[ \Delta f = \frac{1}{T_s} = f_s \]  

(8)

Where \( f_s \) is the rate of OFDM symbols.

II. SISO System

The SISO system model is shown in Figure 2. The signal transmitted from the antenna is denoted as \( x(t) \). The signal received at the receiving end, \( r(t) \), passes through the channel with impulse response \( h(t) \) in an additive white Gaussian noise (AWGN) environment. It is assumed that the bandwidth of the signal is small enough such that the frequency response of the channel is flat and the channel response can be given as

\[ h = |h| e^{i \phi} \]  

(9)

where \( i = \sqrt{-1} \). The relationship between the transmitted signal and receive signal is given by

\[ r(t) = \int_{-\infty}^{\infty} x(\tau) h(t-\tau) d\tau + n(t) = \int_{-\infty}^{\infty} x(\tau) |h| e^{i \phi} \delta(t-\tau) d\tau + n(t) = x(t) h + n(t) \]  

(10)

where \( \delta(t) \) is the Dirac delta function and \( n(t) \) is the AWGN. The received signal is the transmitted signal convolved with the channel impulse response plus added noise [6].

Energy per bit is paramount parameter to measure BER of a channel is defined as

\[ E_b = \frac{1}{T_s} |X_k|^2 dt = A^2 T_b \]  

(11)

Where \( E_b \) represents the average energy per bit at the receiver of the SISO system and \( k \) is the time index of the BPSK symbol (before serial to parallel and after parallel to serial conversions) and \( X_k \) is the BPSK modulated symbol as defined in Table 1. For simulation and analysis, the amplitude \( A = 1 \) is maintained. Equation (11) shows that the energy per bit for BPSK modulation is the same for bit 0 and bit 1. Thus the transmitted power for the SISO system can be written as

\[ P_{SISO} = \frac{E_b}{T_b} = A^2 \]  

(12)

The probability of bit error \( P_b \) for a baseband equivalent SISO BPSK system in discrete domain for correlation demodulator can be represented as [7].

\[ P_b = Q \left( \frac{E_b}{\sqrt{2} N_0} \right) \]  

(13)

In this case \( Z^+ \) is represented as \( Z^+ = A \) \n
And \( \sigma_z^2 = \frac{N_0}{2T_b} \) \n
(14)

(15)

where \( \frac{N_0}{2} \) represents the two sided noise power spectral density for the real part of the AWGN. Substituting Equation (14) and Equation (15) into Equation (13) yields,

\[ P_b = Q \left( \sqrt{\frac{2T_b A}{N_0}} \right) = Q \left( \sqrt{\frac{2E_b}{N_0}} \right) \]  

(16)

The bit error probability of the SISO system as given by Equation (16) will serve as a benchmark for comparison of the MISO and MIMO systems with MDDM in the AWGN channel. In this paper the number of transmit antennas is three but, examined several number of antennas individually and all the transmit antennas transmit equal power. The total energy per bit transmitted for MISO and MIMO systems using BPSK symbols can be defined as

\[ E_b' = P T_s' \]  

(17)

And, \( P_{SISO} = P_{MISO} = P_{MIMO} \) \n
(18)

Therefore, the energy transmitted per antenna \( P' \) can be given as

\[ P' = \frac{P_{SISO}}{3} = \frac{P_{MISO}}{3} = \frac{P_{MIMO}}{3} \]  

(19)

Now, Equation (17) is rewritten as, \( E_b' = P' T_s' = \frac{P_{SISO} T_s}{3} = \frac{E_b}{3} \) \n
(20)

III. MIMO channel capacity

Capacity is a fundamental limit on the spectral efficiency that a communication channel can support reliably. In contrast to the capacity of the scalar additive white Gaussian noise (AWGN) channel that was first derived in [8]. Throughout this paper work, the MIMO channel capacity is used as a fundamental performance measure because it captures both the SNR and the multipath spatial characteristics. For a given channel realization, the channel capacity is given by

\[ c = \max_{\tau, R_x} \log_2 \det \left( I_N + \frac{1}{\sigma_x^2} H R_x H^H \right) b/s/Hz \]  

(21)

where \( T_x(\cdot) \) denotes the trace of the matrix, and \( R_x = E(x x^H) \) is the transmitted signal covariance matrix. The channel capacity \( c \) is the maximum data rate per unit bandwidth that can be transmitted with arbitrarily low
probability of error. For a given bandwidth $W$ the achievable data rate is $Wc$ b/s. Figure 3 illustrates increasing the MIMO capacity with increasing number of antennas.

### 3.1 MIMO System

A multiple-input multiple-output (MIMO) system with multiple ($L$) transmit antennas and multiple ($J$) receive antennas is illustrated in Figure 4. The representation of the model is largely based on [2,8,9]. For a faded channel, it is assumed that channel responses from each transmit antenna to each receive antenna are independent.

Figure 2: SISO system

Figure 4: MIMO system

\[
 r^j = h^j_1 x^1 + h^j_2 x^2 + \cdots + h^j_L x^L + n^j
\]

where $r$ and $n$ represent the $J \times 1$ received signal and noise column vectors, $H$ is a $J \times L$ complex channel matrix and $x$ is the $L \times 1$ transmitted column matrix. All the elements in the channel matrix are considered independent identically distributed (IID) complex Gaussian Random Variables (GRVs), and similarly, the elements of the noise vector are also complex Gaussian random variables.

### 4.1 Simulation of MDDM Transmitter

The MDDM transmitter was simulated in MATLAB with equivalent baseband BPSK in the discrete time domain. The simulation was implemented with one sample for each BPSK symbol. The MDDM transmitter scheme was simulated without added guard interval, D/A converter and RF modulator blocks to facilitate the simulations. Equal power was transmitted from each antenna. To achieve the same total power transmission as that of a single antenna BPSK transmitter, the signal at each branch of MIMO transmitter was multiplied with a gain factor of $g$ for normalization. The transmitted energy per symbol for BPSK is same whether a binary 1 or 0 is transmitted. With the gain factor $g$ the energy per symbol is represented as

\[
 E_s = \frac{1}{2} \int_{-T_s}^{T_s} \left| g x(t) \right|^2 dt = g^2 A^2 T_s
\]

So, the total energy per bit transmitted for MISO and MIMO systems using BPSK symbols can be defined as

\[
 E_s = P T_s
\]

And

\[
 P_{MISO} = P_{MIMO} = P_{MDDM}
\]

Therefore, the energy transmitted per antenna $P'$ can be given as

\[
 P' = \frac{P_{MISO}}{2} = \frac{P_{MDDM}}{2}
\]

Now, Equation (25) is rewritten as

\[
 E_s' = \frac{1}{2} \int_{-T_s}^{T_s} \left| g x(t) \right|^2 dt = \frac{g^2 A^2}{2} T_s
\]

Substituting Equation (28) into Equation (24) yields

\[
 \frac{P_{MISO}}{2} T_s = \frac{g^2 A^2}{2}
\]

Thus the transmitted power for the SISO system can be written as

\[
 P_{MISO} = \frac{g^2 A^2}{T_s} = A^2
\]

Using Equation (30), Equation (29) can be rewritten as

\[
 g^2 = \frac{1}{2} \Rightarrow g = \frac{1}{\sqrt{2}}
\]
The gain factor \( g = 1/\sqrt{2} \) is maintained for all subsequent simulations to transmit the same power as that of a single antenna BPSK system. Thus the effective amplitude of the BPSK modulated signal for each antenna is given as

\[
|X_k| = \frac{A}{\sqrt{2}} = \frac{1}{\sqrt{2}}
\]  

\( (32) \)

4.2 Simulation of MDDM Receiver

The MDDM receiver was also simulated in MATLAB with equivalent baseband BPSK in the discrete time domain. After multicarrier delay diversity demodulation, the space diversity receptions of MIMO systems were combined by using the optimum MRC technique. After the BPSK correlation demodulator as illustrated in Figure 5, \( \xi_k \) (the real part of the random variable \( Z_k \)) was compared with the threshold level according to Table 2. It is assumed that the transmitter and receiver frequencies are synchronized for analysis and simulation purposes and, in the case of MIMO systems, all the diversity receptions are also synchronized.

<table>
<thead>
<tr>
<th>( \xi_k )</th>
<th>Output Binary Bit</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \geq 0 )</td>
<td>0</td>
</tr>
<tr>
<td>( &lt; 0 )</td>
<td>1</td>
</tr>
</tbody>
</table>

**Table 2: Demodulation of BPSK signal**

5. Simulation and Performance Analysis of MDDM in AWGN

The performance of MDDM was simulated and analyzed in AWGN only. In this simulation and analysis no fading is assumed. This means that each channel response coefficient \( h_{ij} \) equals one. A block diagram of the MIMO system in AWGN with three receiving antennas is illustrated in Figure 6. Both the MDDM transmitter and receiver are collapsed into one block each to facilitate presentation. In this simulation two gain blocks each with normalizing gain factor of \( g \) are shown at the outputs of the transmitter for each transmitting antenna. The AWGN channel blocks add white Gaussian noise to the signal at respective receiving antennas. The noise power is increased progressively with each simulation run to calculate the bit error rate at different signal to noise ratios. Bit error rate is calculated by comparing the input binary data stream at the input to the transmitter \( b_k \) and output binary data stream at the output of the receiver \( \hat{b}_k \).

5.1 Performance Analysis of MIMO System with Three Transmitting and Three Receiving Antennas

Similarly, in this section, the performance analysis of MDDM MIMO system with three transmit and three receive antennas is discussed in equation term and simulation result shown in figure 8. As \( 3 \times 3 \) MIMO simulated, SISO, MISO, and SIMO are also simulated and illustrated in figure 9 so that it can be easily compared and analyzed the result as well as realized the performance of the system. It is also assumed that the signal receptions at all antennas are uncorrelated and signals are received at the same time without any relative delay. The received signals at the three receive antennas are given by

\[
\begin{align*}
\mathbf{r}_m^1 & = x_{m1}^1 + x_{m2}^1 + x_{m3}^1 + n_m^1 \\
\mathbf{r}_m^2 & = x_{m1}^2 + x_{m2}^2 + x_{m3}^2 + n_m^2 \\
\mathbf{r}_m^3 & = x_{m1}^3 + x_{m2}^3 + x_{m3}^3 + n_m^3
\end{align*}
\]

\( (33) \)

After the FFT operation the signals are represented as

\[
\begin{align*}
\mathbf{R}_k^1 & = X_k^1 + X_k^2 + X_k^3 + N_k^1 \\
\mathbf{R}_k^2 & = X_k^1 + X_k^2 + X_k^3 + N_k^2
\end{align*}
\]

\( (34) \)
Based on the discussion in previous sections, the components of the variables from all the space diversity receptions are
\[
\begin{align*}
Z_k^1 &= R_k^1(1 + e^{j\theta_k}) = [3 + 2\cos(\phi_k)]X_k^1 + (1 + e^{j\theta_k})N_k^1 \\
Z_k^2 &= R_k^2(1 + e^{j\theta_k}) = [3 + 2\cos(\phi_k)]X_k^2 + (1 + e^{j\theta_k})N_k^2
\end{align*}
\]
(35)
Now all the space diversity receptions are combined to form random variable \(Z_k\) which is given by
\[
Z_k = Z_k^1 + Z_k^2
\]
(36)
If correlation demodulator conditions are assumed, then
\[
E[Z_k | "0" was transmitted] = E\left[\frac{1}{T_b} \int_0^{T_b} [3\sqrt{2}A(1 + \cos(\phi_k)) + N_k(1 + e^{j\theta_k})] \, dt\right] = 3\sqrt{2}A(1 + \cos(\phi_k))
\]
(37)
The variance of \(\zeta_k = Re[Z_k]\) is only due to the variance of real part of noise components [7, 10]. Noise components at both receive antennas are IID Gaussian random variables. Therefore the total variance of the sum of two noise components is the sum of their individual variances [4]. The total noise variance is given as
\[
\sigma_{\zeta_k}^2 = \sigma_{\zeta_k}^2 + \sigma_{\zeta_k}^2 + \sigma_{\zeta_k}^2
\]
(38)
Where \(\zeta_k^1 = Re[Z_k^1]\), \(\zeta_k^2 = Re[Z_k^2]\) and \(\zeta_k^3 = Re[Z_k^3]\) Following the derivation of noise variance, Equation (38) is represented as
\[
\sigma_{\zeta_k}^2 = \frac{2(1 + \cos(\phi_k))N_k}{T_b}
\]
(39)
and \(\sigma_{\zeta_k}\) is given as
\[
\sigma_{\zeta_k} = \sqrt{\frac{2(1 + \cos(\phi_k))N_k}{T_b}}
\]
(40)
Following the derivation of Equation \(P_b = Pr\left\{\frac{\zeta_k - \overline{\zeta_k}}{\sigma_{\zeta_k}} > \frac{1}{\sigma_{\zeta_k}}\right\}\) the bit error probability \(P_b\) conditioned on the value \(\phi_k\) is given by
\[
P_b = \Phi\left(\frac{\sqrt{\frac{2(1 + \cos(\phi_k))N_k}{T_b}}}{\sigma_{\zeta_k}}\right) = \Phi\left(\frac{\sqrt{\frac{2(1 + \cos(\phi_k))N_k}{T_b}}}{\sigma_{\zeta_k}}\right)= Q\left(\frac{\sqrt{\frac{2(1 + \cos(\phi_k))N_k}{T_b}}}{\sigma_{\zeta_k}}\right)
\]
(41)
The average probability of bit error \(P_b\) for this case can be obtained by
\[
P_b = \frac{1}{2} + \sum_{k=1}^{K} \Phi\left(\frac{\sqrt{\frac{2(1 + \cos(\phi_k))N_k}{T_b}}}{\sigma_{\zeta_k}}\right) + \sum_{k=1}^{K} Q\left(\frac{\sqrt{\frac{2(1 + \cos(\phi_k))N_k}{T_b}}}{\sigma_{\zeta_k}}\right)
\]
(42)
The simulation was conducted for the same number of OFDM frames or data symbols. The simulated bit error rate (BER) for the MIMO system is plotted in Figure 7, 8, 9 with compared to SISO, MISO, and SIMO. The simulated results follow theoretical results very closely.

Figure 3: Increasing the MIMO capacity with increasing number of antennas.

Figure 7: Simulation result of MDDM MISO vs. SISO system in AWGN
5.2. Result Analysis

Using multicarrier delay diversity modulation (MDDM), the simulation of MIMO-OFDM system with SISO achieved bit-error-rate performance results coherent with the theoretical analysis in an AWGN channel with no fading. The performance metric of bit error probability versus $E_b/N_0$ (energy per bit to noise power spectral density ratio) was used. To set up a fair comparison, the data rate and transmitted power for SISO, MISO and MIMO systems were equal. The comparison of performances in AWGN shown that the MIMO-OFDM system with MDDM performed better than the SISO system for low $E_b/N_0$ with up to a 6.7 dB performance gain and performed worse for higher $E_b/N_0$ values. The performances of the MIMO-MDDM systems were better than that of SISO system for all values of $E_b/N_0$. MIMO-OFDM systems with three receive antennas and three receive antennas, outshine a SISO system by 1.6 dB and 3.3 dB performance gain. The improvement in performances of the MIMO-OFDM and MISO systems over the SISO system can be exploited for future wireless communication by using MDDM technique.

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