Heat transfer to down flowing viscous films with Evaporation from the interface

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Abstract
Introduction: *Within long vertical boiling tubes thermo hydrodynamic processes in liquid film take place at a regime of cycling mixing in the bulk of the film by the powerful waves. The existing Heat Transfer models of such phenomena do not take info account such intermittent mixing of fluid*
Materials and methods: *The heat transfer and hydrodynamic processes that take place in down flowing film of water and sugar solutions at the regimes of evaporation from the interface have been studied. There were developed mathematical models, which then were compared with the result of direct experimentation of heat transfer in tubes at the regimes of solutions concentrations.*
Results and discussion: *A mathematical model of heat transfer in laminar, heated to the saturation temperatures liquid films with the developed wavy structures on the free interface have been developed. The model takes into consideration cyclic relaxation of transient temperature field which happens right after the passage of a powerful big wave. The developed mathematical model describes the time history of the two dimensional temperature fields as a function of the Peclet number and the core characteristic of the wavy motion (the length of big waves). Based upon the proposed model a set of correlations have been obtained. These are proposed as a means for the generalization of heat transfer experimental data, obtained within the experimental studies of liquid films, heated to the saturation temperatures and evaporating from the interface. A generalized equation has been derived, which can be used for the calculations of Heat Transfer Coefficients (HTC) to the saturated sugar solutions liquid films. This equation contains wavy characteristics of down flowing films and valid within the range of parameters characteristic for the sugar industry evaporators, namely: concentrations – 0…70 % dry matter; liquid mass flow rate density–0.01×10^{-3}…0.6×10^{-3} m^2/sec, the Peclet number range – 400…25000. The mathematical model of the temperature field cyclic relaxation turned out efficient for generalization of heat transfer experimental data not only laminar, but turbulent liquid films either, despite of the fact that the transport equations do not contain turbulent characteristics.*
Conclusions: *A correlation between the liquid film wavy structure with the heat transfer has been established. The correlation is based upon the model of temperature field cyclic relaxation after passage of big waves. The respective correlations have been presented.*
Key words: *Films Waves Heat transfer Temperature Evaporation*

I. Introduction

A wavy structure is being formed on the interface of the liquid film down flowing over the vertical surface even at a lowest density of liquid flow rate. At a distance to2…2.5 from the liquid film inlet a so called regime of the wavy motion saturations takes place [1]. This regime is characterized by a completely structure of big waves which move on the interface at a so called "bulldozer" regime. On top of this, big waves even in case of viscose concentrated solutions contain a swirl [2]. Therefore, their movement on the surface entails to the intensive mixture of liquid and, respectively, if accompanies by a noticeable deformation of velocity and temperature profiles, and thus by equalization of the concentrations over the film thickness. Studies into the wavy structures of liquid films – as an adequate form of liquid film movement is important and becomes a point of interest for many modern foreign publications [3…12]. It is clear that within the time period between the passage of two consecutive big waves a transient process of temperature and velocity fields relaxation takes place. It is quite natural that exactly this process has a decisive influence on the heat transfer. This statement corroborated by the deviation of the heat transfer coefficient curve from the Nusselt line [13]. The fact that the said deviation happens at laminar regime of film flow substantiates a decisive effect of the interface wavy structures on the film heat transfer.

A major number of theoretical studies dedicated to the film wavy motion [14…16] directed at the aspect of the development and structures of regular waves. The portion of these waves on the length of heat transfer tubes of industrial evaporator is small when compared to the regions at which big waves take place. A big number of scientific publications aimed at the studies of heat transfer in down flowing films, the effect of wavy structures upon the heat transfer have being taken into account indirectly by the turbulence parameters and liquid film

interface shear stress. A majority of experimental results obtained with evaporating films heated to the saturation temperatures at the conditions which model industrial evaporators [17…19]. The obtained data therefore contain information of the heat transfer at the simultaneous action of a number of influencing factors. It is impossible to separate the effect of some individual factor in these conditions. That is why the existing correlations for the calculations of heat transfer coefficients to the saturated liquid films in industrial evaporators are sufficiently limited within the range of regime parameters in which the process had been modeled. Similarly, the said correlations are limited in terms of the geometry of the experimental unit. The proposed project is aimed at the development of the theoretical description of temperature field relaxation which takes place in the film after passing of a big wave.

As a result of this, an interrelation between the liquid film wavy structure with the heat transfer at a regime of liquid film evaporation from the free interface is established.

II.   Materials and methods.

A direct experimentation of heat transfer in down flowing liquid films heated to the saturation temperatures with sugar solutions as model liquids has been carried out at the experimental unit with the independent formation of phases' mass flow rates and heat flux. The main core of the experimental unit was represented by a stainless still pipe with the inside diameter of 20 mm and 1.8 m long. The experimental tube was separated into the initial 1.5 m stabilization section and 0.3 m measurement section. The down flowing of water (sugar solutions) film has been formed by means of overflowing over the tube's upper rim. In the event of steam-liquid flow modeling, dry saturated steam has been supplied in co-current regime. The liquid falling film has been heated by dry saturated steam which was supplied into outside heating sections attached to the experimental tube. The heating chambers were designed in a such way as to provide an individual heating of the stabilization section and the experimental one. The said sections were hooked up to the individual vacuum-condensation sections which allowed for the keeping of different pressures in each chamber. Such arrangement allowed also maintaining vacuum down to 0.8 bars and thus, vary the temperature head between the heating steam temperature and evaporation temperature. Special probes for taking samples of liquid to determine its concentration and measurements of temperatures were positioned directly after the measurement section. A detailed description of the experimental unit is given in [21] and its schematic is given in fig.1.
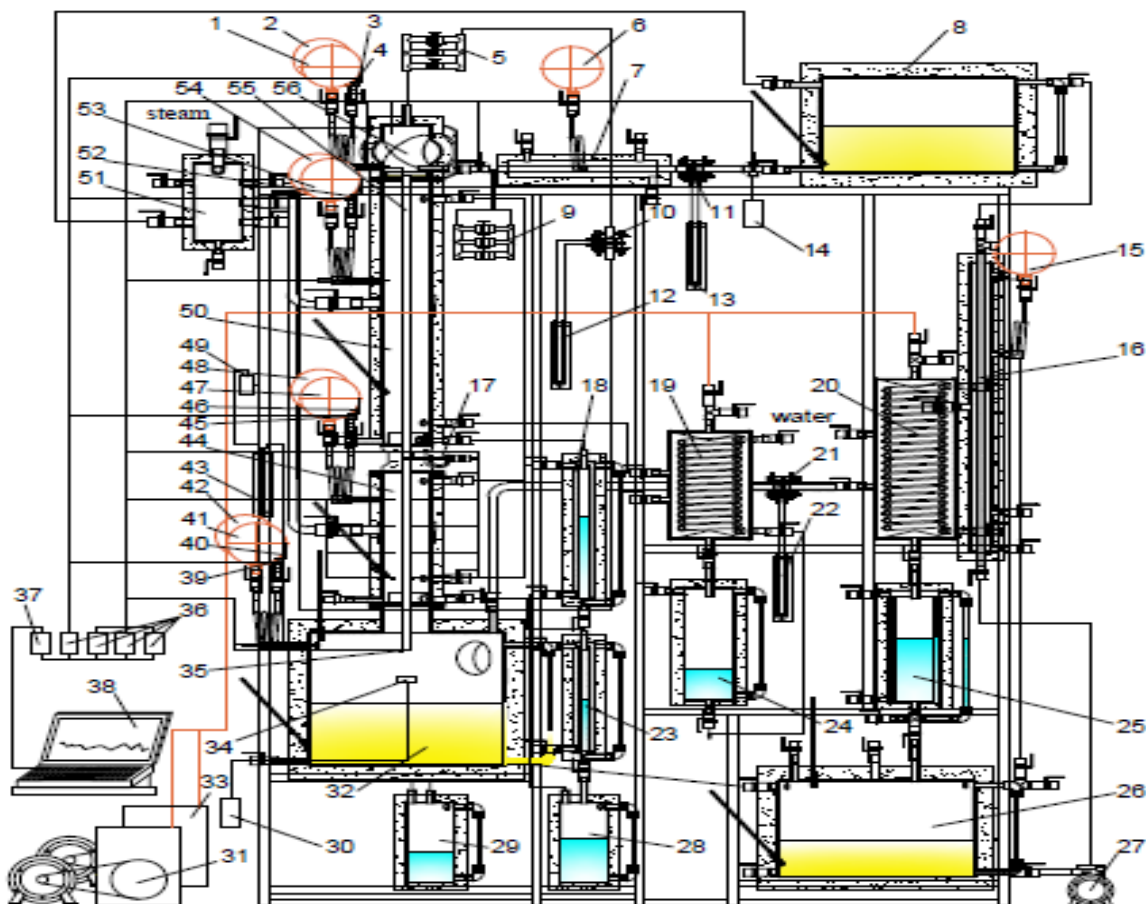
Fig.1. Scheme of the experimental stand

1,6,15,41,47,53 – manometer; 2,42,48,54 – vacuum manometer; 3,39,45 – sensor for vacuum control; 4,40,46,52 – sensor for pressure control; 5,9 – collector flow control; 7,16 – heat exchanger; 8,26,32 – tank; 10,11,21 – diaphragm; 12,13,22,43 – differential pressure gage; 14,30 – sampler; 17 – gage probe; 18,23,24,25,28,29 – hotwell; 19,20 – condenser; 27 – pump; 31,33 – vacuum pump; 34 – tank for sampling; 35 – tank of solution's temperature measurement on the exit of pipe; 36 – module analog input 7018P; 37 – module 7520; 38 – computer; 44,50 – heating chamber; 49 – equaling tank; 51 – separator of heating steam; 55 – experimental pipe; 56 – top chamber.

III.   Results and discussions

The model of temperature field relaxation has been designed aiming at the following: to develop an analytical expression for the calculation of heat transfer to the liquid film, containing a sought parameter, which can be determined by the comparison of experimental data with those found analytically.

Thus, the developed expression could be used for the generalization of the experimental data, obtained within a wide range of regime parameters, and for the engineering calculations of heat transfer coefficients, either.

The main suggestions of the development of the proposed model are following: the big waves rolling over the liquid film surface are associated with the vortexes, which mix the liquid in the film; therefore we assume that right after the passage of a strong wave the velocity profile will be constant and the temperature profile is curved in a such way, hat the bulk of the liquid will have temperature equal to the saturation temperature, but the layers adjacent to the wall will be heated and assume the wall temperature.

This mode of velocity distribution will keep existing along the first regime of flow. At the same time there will be the development of temperature field in the liquid film. It is clear, that since a bulk of the liquid will have the temperature equal to that of saturation, there will be no temperature gradient on the liquid film interface and thus there will be no evaporation on this section. Along with the liquid film movement the development of the temperature field will take place. The temperature front will move gradually from the heated wall towards the film interface. Than the time will came when the sensitive temperature gradient appears on the film interface. This moment signifies the beginning of the second regime. This period we assume that the velocity profile keeps parabolic and constant. And the further development of the temperature field takes place. Within this period of flow the heat flux on the wall will go down and that on the interface goes up, until the temperature profile becomes linear. Despite this simplification this model looks quite reasonable, because the relative lengths of the mentioned above regimes is quite small, especially of the first one, and heating of the film until it starts evaporate takes very short time due to the small film thickness. It is worth mentioning that these suggestions allowed a significant simplification of the differential equations describing heat transfer. Thus, the suggestions of the constant value of mean film velocity within the two regimes permitted to substitute the temperature-time derivatives by the temperature-longitudinal distance derivatives. The model developed under such suggestions might be turned as a quasi-transient.
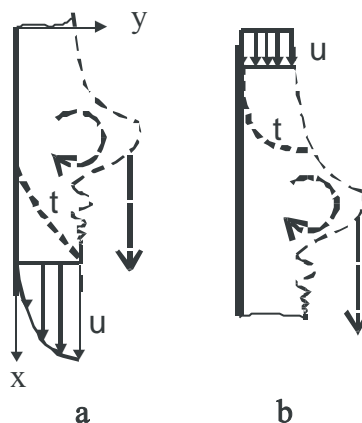
Fig. 2. Model of liquid film structure

a – temperature and velocity distribution before a strong wave passage
b – after.

According to the fig. 2b which depicts the moment right after the passage of big wave assuming the constant velocity distribution which is $\overline{u} = \dfrac{g \delta^2}{3 \nu}$, the heat transfer equation in the dimensionless form can be written as:

$$\frac{\partial \theta(\eta,\xi)}{\partial \xi} = \frac{4}{Pe} \frac{\partial^2 \theta(\eta,\xi)}{\partial \eta^2}, \tag{1}$$

here $\theta(\eta,\xi) = \dfrac{t(\eta,\xi) - t_l}{t_w - t_l}$ – dimensionless temperature; t_w, t_l – temperature of wall and saturation, respectively; $\eta = \dfrac{y}{\delta}, \xi = \dfrac{x}{\delta}$ – dimensionless crossways and longitudinal coordinates; $Pe = \dfrac{4\Gamma_v}{a} = \dfrac{4\overline{u}\delta}{a}$ – the Peclet number; $\delta = \sqrt[3]{\dfrac{3\Gamma_v \nu}{g}}$ – liquid film thickness; Γ_v – volumetric flow rate; a – temperature conductivity; ν – cinematic velocity.

The solution of(1) with boundary conditions:

$$\theta(0,0) = 0, \ \theta(0,\xi) = 1, \ \theta(1,0) = 0, \ \frac{\partial \theta(\infty,\xi)}{\partial \eta} = 0, \tag{2}$$

will be

$$\theta(\eta,\xi) = erfc\left(\frac{\eta}{4}\sqrt{\frac{Pe}{\xi}}\right). \tag{3}$$

The profile(3) will be developing until a significant temperature gradient on the interface, appears which corresponds to the ξ_m. Further on, as a result of evaporation, the temperature of the film surface will remain constant. In this case the temperature, which corresponds to the equation (3) at $\xi = \xi_m$ will be

$$\theta(\eta,\xi_m) = erfc\left(\frac{\eta}{4}\sqrt{\frac{Pe}{\xi_m}}\right) \tag{4}$$

The curve (4) represents a limiting case, after which the boundary conditions (2) become meaningless. Thus the first regime ceases to exist and the second regime begins. As it was mentioned above for the second regime, we assume a parabolic velocity distribution. Then, the heat transfer equation will read:

$$\frac{g\delta^3}{2\nu a}\left(2\eta - \eta^2\right)\frac{\partial \theta(\eta,\xi)}{\partial \xi} = \frac{\partial^2 \theta(\eta,\xi)}{\partial \eta^2}, \tag{5}$$

and the boundary conditions

$$\eta = 0, \ \theta = 1; \quad \eta = 1, \ \frac{\partial \theta}{\partial \eta} = 0. \tag{6}$$

Substituting the left side of (5) by the mean velocity value, one obtains

$$\frac{g\delta^3}{2\nu a}\int_0^1 \left(2\eta - \eta^2\right)\frac{\partial \theta(\eta,\xi)}{\partial \xi}d\eta = \frac{g\delta^3}{2\nu a}\frac{2}{3}\frac{\partial \theta_{av}(\xi)}{\partial \xi} = \frac{Pe}{4}\frac{\partial \theta_{av}(\xi)}{\partial \xi},$$

and the equation (5) can be rewritten as:

$$\frac{Pe}{4}\frac{\partial \theta_{av}(\xi)}{\partial \xi} = \frac{\partial^2 \theta(\eta,\xi)}{\partial \eta^2}. \tag{7}$$

Double integration of (7) with boundary condition (6) yields

$$\theta(\eta,\xi) = \frac{Pe}{4}\frac{\partial \theta_{av}(\xi)}{\partial \xi}\left(\frac{\eta^2}{2} - \eta\right) + 1. \tag{8}$$

From the equation for mean temperature value

$$\theta_{av} = \int_0^1 \theta(\eta,\xi)\frac{u(\eta)}{\overline{u}}d\eta = \int_0^1 \theta(\eta,\xi)\left(2\eta - \eta^2\right)\frac{3}{2}d\eta = 1 - \frac{Pe}{10}\frac{\partial \theta_{av}(\xi)}{\partial \xi},$$

taking into account, that $\xi = 0$, $\theta_{av} = 0$ one obtains

$$\theta_{av} = 1 - \exp\left(-\frac{10}{Pe}\xi\right). \tag{9}$$

Substituting derivative of (9) by ξ into (8), one obtains limiting temperature curve at $\xi = \xi_m$

$$\theta(\eta,\xi_m) = \frac{5}{2} exp\left(-\frac{10}{Pe}\xi_m\right)\left(\frac{\eta^2}{2} - \eta\right) + 1 \, , \qquad (10)$$

The coordinate ξ_m can be found from (10) at a condition, that at $\xi = \xi_m$ the dimensionless temperature equals 0 ($\theta(1,\xi_m) = 0$)

$$\xi_m = 0,0223 \ Pe$$

(11)

Within the region $\xi \geq \xi_m$ the liquid temperature on the interface remains constant, since evaporation takes place. There fore the boundary conditions (6) will change in to

$$\eta = 0 \, , \, \theta = 1 \, ; \quad \eta = 1 \, , \, \theta = 0 \, ,$$

(12)

and the initial conditions at $\xi = \xi_m$ will be as for equation (10).

Integration of (7) with the boundary conditions (12) yields

$$\theta(\eta,\xi) = \frac{Pe}{8} \frac{\partial \theta_{av}(\xi)}{\partial \xi}(\eta^2 - \eta) - \eta + 1 \cdot \qquad (13)$$

In order to find the derivative $\dfrac{\partial \theta_{av}(\xi)}{\partial \xi}$, we determine a mean temperature within the region $\xi \geq \xi_m$

$$\theta_{av}(\xi) = \int_0^1 \theta(\eta,\xi)(2\eta - \eta^2)\frac{3}{2}d\eta = \frac{3}{8} - \frac{7}{320}Pe\frac{\partial \theta_{av}(\xi)}{\partial \xi} \cdot \qquad (14)$$

This, in turn, yields

$$\theta_{av} = \frac{3}{8} + C \ exp\left(-\frac{320}{7Pe}\xi\right) \cdot \qquad (15)$$

The integration constant C can be determined from the initial condition (10). Having determining the mean temperature $\theta_{m.av}$ at $\xi = \xi_m$

$$\theta_{m.av}(\xi_m) = \int_0^1 \theta(\eta,\xi_m)(2\eta - \eta^2)\frac{3}{2}d\eta = 1 - exp\left(-\frac{10}{Pe}\xi_m\right),$$

and substituting this expression into (15), having θ_{av} in mind that $\xi \to \xi_m$, we obtain the constant C

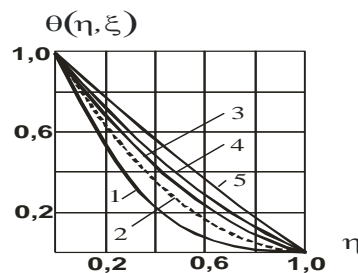$$C = \left(\frac{5}{8} - exp\left(-\frac{10}{Pe}\xi_m\right)\right)exp\left(\frac{320}{7Pe}\xi_m\right) \cdot$$

Then

$$\theta_{av} = \frac{3}{8} + \left(\frac{5}{8} - exp\left(-\frac{10}{Pe}\xi_m\right)\right)exp\left(\frac{320}{7Pe}(\xi_m - \xi)\right) \cdot \qquad (16)$$

Substituting a derivative of (16) by ξ into (13), one obtains a temperature distribution in the film at $\xi \geq \xi_m$

$$\theta(\eta,\xi) = \left[\frac{40}{7}exp\left(-\frac{10}{Pe}\xi_m\right) - \frac{25}{7}\right]exp\left(\frac{320}{7Pe}(\xi_m - \xi)\right)(\eta^2 - \eta) - \eta + 1 \cdot \qquad (17)$$

Thus obtained temperature profiles calculated by the equations (3, 10, 17) are given at fig. 3

Fig. 3 Dimensionless temperature distribution across the water film as per

(3, 10, 17) at t = 100 °C; $\square_v = 0,5 \cdot 10^{-3}$ $\square^2/\square$; $\xi_m = 267$;

$1 - \xi = 167$, equation(3); $2 - \xi = \xi_m$, equation(10); $3 - \xi = 367$; $4 - \xi = 467$;

$5 - \xi = 767$, 3, 4, 5 – equation (17).

The heat flux on the wall ($\eta = 0$) within the region $\xi \geq \xi_m$ is obtained from (17)

$$q_2(\xi)_{\eta=0} = -\lambda \frac{t_w - t_l}{\delta} \frac{d\theta}{d\eta}\bigg|_{\eta=0} = \lambda \frac{t_w - t_l}{\delta}\left\{\left[\frac{40}{7}\exp\left(-\frac{10}{Pe}\xi_m\right) - \frac{25}{7}\right]\exp\left(\frac{320}{7Pe}(\xi_m - \xi)\right) + 1\right\}, \quad (18)$$

and within the region $\xi \leq \xi_m$ from (3)

$$q_1(\xi)_{\eta=0} = -\lambda \frac{t_w - t_l}{\delta} \frac{d\theta}{d\eta}\bigg|_{\eta=0} = \lambda \frac{t_w - t_l}{2\delta\sqrt{\pi}}\sqrt{\frac{Pe}{\xi}}. \quad (19)$$

As stated above, a cyclic process of heat transfer in the liquid film is subdivided into two characteristic zones: the first one $\xi \leq \xi_m$ which is characterized by the gradual development of the temperature field until the temperature curve riches limiting curve ξ_m. It has been mentioned that within this region only heating at the liquid bulk in the film takes place. Since there is no temperature gradient on the interface in this region there is no evaporation from the film. Further on within the region $\xi \geq \xi_m$ the linearization of the temperature profile takes place. Here the process keeps going at a constant temperature on the interface. Respectively, we con determine the values of mean temperature fluxes within these regions $(\xi_m - \xi_o)$, $(\xi_v - \xi_m)$

$$q_{1av} = \frac{1}{\xi_m - \xi_o}\int_{\xi_o}^{\xi_m} q_1(\xi)_{\eta=0}\,d\xi = \lambda \frac{t_w - t_l}{\delta\sqrt{\pi}(\xi_m - \xi_o)}\left(\xi_m\sqrt{\frac{Pe}{\xi_m}} - \xi_o\sqrt{\frac{Pe}{\xi_o}}\right) \quad (20)$$

$$q_{2av} = \frac{1}{\xi_v - \xi_m}\int_{\xi_m}^{\xi_v} q_2(\xi)_{\eta=0}\,d\xi = \frac{\lambda(t_w - t_l)}{\delta(\xi_v - \xi_m)} \times$$

$$\left[\left(\frac{Pe}{8}\exp\left(-\frac{10\xi_m}{Pe}\right)\right)\left[\frac{5}{8}\exp\left(\frac{390\xi_m - 320\xi_v}{7Pe}\right) - \exp\left(-\frac{320}{7}\frac{(\xi_v - \xi_m)}{Pe}\right) + 1\right]\right] \\ -\frac{5Pe}{64} + \xi_v - \xi_m. \quad (21)$$

where $\xi_v - \xi_o = \frac{L_w}{\delta}$ is dimensionless distance between the consequent peaks of big waves; L_w - is distance between the consequent peaks of big waves.

Thus ξ_o determines the depth of the temperature profile "lowering" right after the passage of a big wave. The lower value of ξ_o, the bigger will be temperature gradient and, respectively, the lower deviation of the temperature curve depicted on fig. 3.

The length of big waves on the surface of water films according [20] within the Re numbers range 40…400, remains constant at the level of 100…120 mm and at further growth of Re number to 4000 gradually increases to the value of 140 mm at a distance approximately 2.4 m from the film distributor.

Therefore, the mean temperature flux could be given as a mean weighted

$$q_{av} = \frac{q_{1av}(\xi_m - \xi_o) + q_{2av}(\xi_v - \xi_m)}{\xi_v - \xi_o}, \quad (22)$$

and the Nusselt number will be determined as:

$$Nu = \left(\frac{\sqrt{Pe\,\xi_m} - \sqrt{Pe\,\xi_o}}{\sqrt{\pi}(\xi_v - \xi_o)}\right) +$$

$$+ \frac{1}{\xi_v - \xi_o} \left[\left[\left(\frac{Pe}{8} exp \left(-\frac{10\,\xi_m}{Pe} \right) \right) \left[\frac{5}{8} exp \left(\frac{390\,\xi_m - 320\,\xi_v}{7\,Pe} \right) - exp \left(-\frac{320}{7} \frac{(\xi_v - \xi_m)}{Pe} \right) + 1 \right] \right] - \frac{5\,Pe}{64} + \xi_v - \xi_m \right] \quad (23)$$

where $\alpha = \dfrac{q_{av}}{t_w - t_i}$, $Nu = \dfrac{\alpha\,\delta}{\lambda}$.

The temperature of interface surface t_i is determined as the saturation temperature at a mean liquid concentration in the film.

Comparison of the calculated by (23) Nusselt numbers with the experimental data of heat transfer to the saturated liquid films of water and sugar solutions [13] allows to determine the value of ξ_o

$$\xi_o = 43\,Y + 0{,}2 \text{ at } Y \leq 0{,}115$$
$$\xi_o = 1150\,Y^{0{,}98} - 133 \text{ , at } Y \geq 0{,}115 \text{ ,} \quad (24)$$

here $Y = \left(\dfrac{\delta}{\delta^*} \right)^{0{,}9} \left(\dfrac{v_{wat}}{v} \right)^{0{,}01}$, v_{wat} – water cinematic viscosity; $\delta^* = \sqrt{\dfrac{\sigma}{g\,\rho}}$.

At $Pe \geq Pe_o$, $Y_o = \left(\sqrt[3]{\dfrac{3\,Pe_o\,a\,v}{4\,g}} / \delta^* \right)^{0{,}9} \left(\dfrac{v_{wat}}{v} \right)^{0{,}01}$, here $Pe_o = 10300$.

$$\xi_o = 43\,Y_o + 0{,}2 \text{ at } Y \leq 0{,}115 \text{ ,}$$
$$\xi_o = 1150\,Y_o^{0{,}98} - 133 \text{ , at } Y \geq 0{,}115$$

The generalization of experimentally obtained heat transfer coefficients to the water and sugar solutions up to the concentration of $70\,\%(v = 3{,}77 \cdot 10^{-6}\,m^2/sec)$ at free flowing down films with the results of calculations are given on fig. 4.
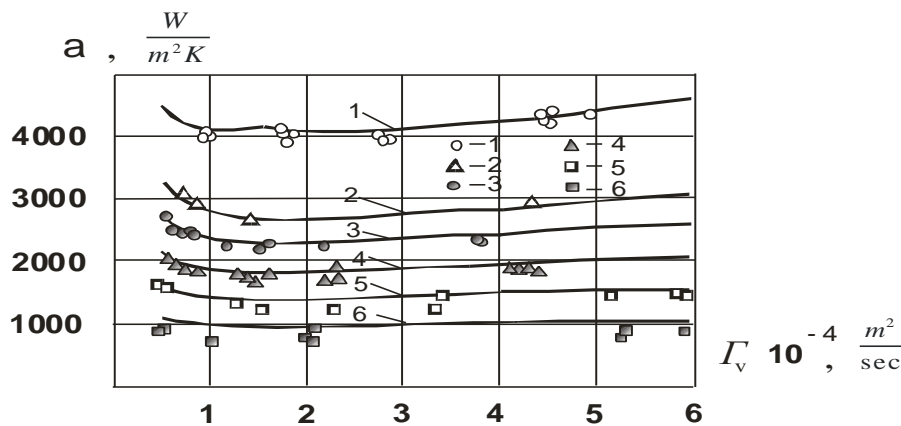
Fig. 4 Comparison of experimentally determined heat transfer coefficient to water and sugar solutions at free flowing down over the vertical surface films at a regime of evaporation from the interface with the calculated values. The lines are calculated as per (23, 24) at $L_w = 120$ mm. 1 – water, t = 100 C; 2 – sugar solution, DM = 30 %; 3 – 40 %; 4 – 50 %; 5 – 60 %; 6 – 70 %.

As it might be seen from the fig.4, the calculations as per (23, 24) closely correlate the experimental data within the volumetric flow rates from $0...6\times10^{-4}$ m^2/sec. It is worth mentioning, that the close correlation takes place not only within the laminar regime of flow, but also in the turbulent region despite the fact that, whilst deriving the model the parameters of turbulence were not explicitly taken into consideration.

IV.   Conclusions

The proposed model of heat transfer with the cyclic relaxation of temperature field in the down flowing liquid films adequately displays the physical processes that take place. Particularly the proposed model allows to interpret the deviation of the experimental data on heat transfer within the laminar flow regime from the theoretical Nusselt curve.

It has been proven that the proposed model can be successfully applied the generalization of experimental data of heat transfer to the down flowing liquid films of water and viscose water solutions at a regime of evaporation from the free surface.

Equates(23, 24) are recommended to be used for the engineering calculations of heat transfer coefficients to the liquid films of water and sugar solutions in the regimes of industrial multi-effect industrial evaporators within the wide range of liquid mass flow rates.

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