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Unit Graphs and Subgraphs of Finite Groups U_n and K_4

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Abstract: *We represent finite group in the form of graphs. These graphs are called unit graphs. In this paper we shall study unit graphs of finite groups U_n (multiplicative group of integers modulo n) and K_4 (Klein's four group). Also study of different properties like the subgroups of a group are carried out using the unit graph of the group.*
Keywords: *Finite Group, Graph, Subgraph*

I. INTRODUCTION

The phenomenon of representing Groups using Graphs has been studied theoretically by number of researchers [6-8, 12, 13]. In a series of investigations S. Akbari and A. Mohammadian [1] discussed the zero divisor graphs of finite rings. In this paper, we give unit graphs and subgraphs of finite groups U_n (multiplicative group of integers modulo n) and K_4 (Klein's four group).

II. UNIT GRAPH OF A GROUP

2.1. Definition
A graph G (V, E) is Unit Graph of a group (G,.) if
(i) Distinct elements v_i and v_j are adjacent in graph G (V, E) if $v_i.v_j = e$ in group (G, .).
(ii) Every element of group (G,.) is adjoined with the unity of group (G,.).

2.2. Unit Graph of U_n
The multiplicative group of integers modulo n is given by $U_n = \{x \in Z | 1 \leq x < n, (x, n) = 1\}$.The unit graphs of U_n for some n are given as follows:

2.2.1. Unit Graph of U_2
The multiplicative group of integers modulo 2 is $U_2 = \{1\}$. The unit graph of U_2 is:

Fig. 1: Unit Graph of U_2

We have some following properties of this graph:-
(i) This graph is finite graph.
(ii) This graph is complete graph.
(iii) This graph is connected graph
(iv) The chromatic number is $\chi(U_2) = 1$.

2.2.2. Unit Graph of U_3
The multiplicative group of integers modulo 3 is $U_3 = \{1,2\}$. The unit graph of U_3 is:

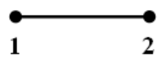

Fig. 2: Unit Graph of U_3

We have some following properties of this graph:-
(i) This graph is Bipartite because the vertex set V can be decomposed into two disjoint subsets V_1 and V_2 such that every edge in U_3 has one end point in V_1 and one end point in V_2.
(ii) This graph is Regular because every vertex is of same degree i.e., every vertex is of degree one.
(iii) This graph is finite because there are finite numbers of vertices and edges.
(iv) This graph is complete graph.
(v) This graph is connected graph
(vi) This graph is planar graph.
(vii) The chromatic number is $\chi(U_3) = 2$.

2.2.3. Unit Graph of U_5

The multiplicative group of integers modulo 5 is $U_5 = \{1,2,3,4\}$. The unit graph of U_5 is:

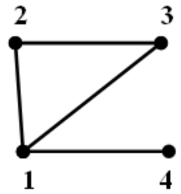

Fig. 3: Unit Graph of U_5

We have some following properties of this graph:-
(i) This graph is finite graph.
(ii) This graph is connected graph
(iii) The chromatic number is $\chi(U_5) = 3$.

2.2.4. Unit Graph of U_9

The multiplicative group of integers modulo 9 is $U_9 = \{1,2,4,5,7,8\}$. The unit graph of U_9 is:

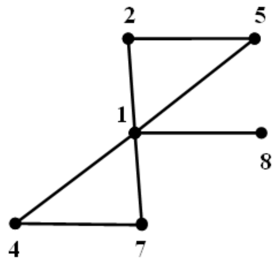

Fig. 4: Unit Graph of U_9

We have some following properties of this graph:-
(i) This graph is finite graph.
(ii) This graph is connected graph.
(iii) The chromatic number is $\chi(U_9) = 3$.

2.2.5. Unit Graph of U_{13}

The multiplicative group of integers modulo 13 is $U_{13} = \{1,2,3,4,5,6,7,8,9,10,11,12\}$. The unit graph of U_{13} is:

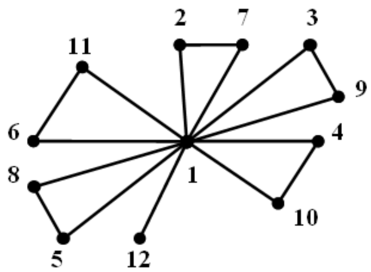

Fig. 5: Unit Graph of U_{13}

We have some following properties of this graph:-
(i) This graph is finite graph.
(ii) This graph is connected graph.
(iii) The chromatic number is $\chi(U_{13}) = 3$.

2.2.6. Unit Graph of U_{16}

The multiplicative group of integers modulo 16 is $U_{16} = \{1,3,5,7,9,11,13,15\}$. The unit graph of U_{16} is:

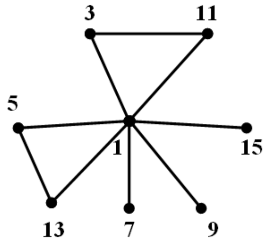

Fig. 6: Unit Graph of U_{16}

We have some following properties of this graph:-

(iv) This graph is finite graph.
(v) This graph is connected graph.
(vi) The chromatic number is $\chi(U_{16})$ =3.

2.2.7. Unit Graph of U_{17}

The multiplicative group of integers modulo 17 is $U_{17} = \{1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16\}$. The unit graph of U_{17} is:

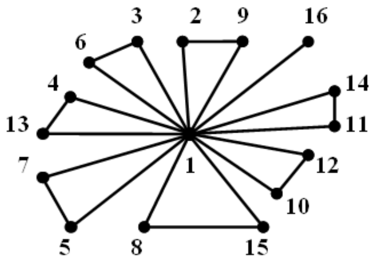

Fig. 7: Unit Graph of U_{17}

We have some following properties of this graph:-

(iv) This graph is finite graph.
(v) This graph is connected graph.
(vi) The chromatic number is $\chi(U_{17})$ =3.

2.2.8. Unit Graph of U_{19}

The multiplicative group of integers modulo 19 is $U_{19} = \{1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18\}$. The unit graph of U_{19} is:

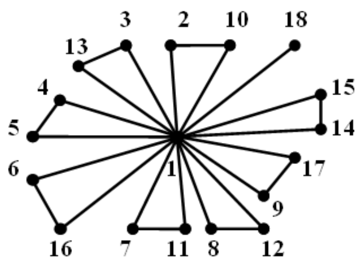

Fig. 8: Unit Graph of U_{19}

We have some following properties of this graph:-

(i) This graph is finite graph.
(ii) This graph is connected graph.
(iii) The chromatic number is $\chi(U_{19})$ =3.

2.2.9. Unit Graph of U_{21}

The multiplicative group of integers modulo 21 is $U_{21} = \{1,2,4,5,8,10,11,13,16,17,19,20\}$. The unit graph of U_{21} is:

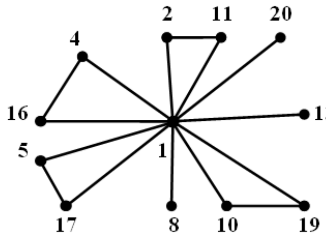

Fig. 9: Unit Graph of U_{21}

We have some following properties of this graph:-

(i) This graph is finite graph.
(ii) This graph is connected graph.
(iii) The chromatic number is $\chi(U_{21})$ =3.

2.2.10. Unit Graph of U_{22}

The multiplicative group of integers modulo 22 is $U_{22} = \{1,3,5,7,9,13,15,17,19,21\}$. The unit graph of U_{22} is:

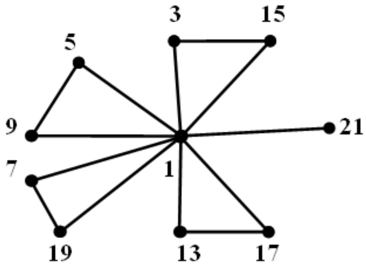

Fig. 10: Unit Graph of U_{22}

We have some following properties of this graph:-

(i) This graph is finite graph.
(ii) This graph is connected graph.
(iii) The chromatic number is $\chi(U_{22})$ =3.

2.3. Unit Graph of K_4

The Klein's four group K_4 is given by $K_4 = \{e, a, b, c\}$, where $a^2 = b^2 = c^2 = e$ and $ab = ba = c$, $bc = cb = a$ and $ca = ac = b$. The unit graph of K_4 is:

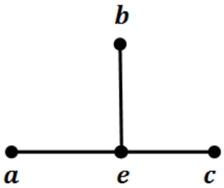

Fig. 11: Unit Graph of K_4

We have some following properties of this graph:-

(i) This graph is finite graph.
(ii) This graph is connected graph.
(iii) This graph is planar graph.
(iv) The chromatic number is $\chi(K_4)$ =2.

III. UNIT SUBGRAPH OF A GROUP

3.1. Definition

Let graph G (V, E) be an unit graph of a group (G,.). If H is a subgroup of the group G, then the unit graph drawn for the subgroup H is known as the unit subgraph of the group G.

3.2. Unit Subgraphs of U_n

The unit subgraphs of U_n for some n are given as follows:

3.2.1. Unit Subgraphs of U_2

The multiplicative group of integers modulo 2 is $U_2 = \{1\}$. Subgroup of U_2 is $H_1 = \{1\} = U_2$ only.
The unit subgraph of U_2 is:

Fig. 12: Unit Graph of H_1

3.2.2. Unit Subgraphs of U_3

The multiplicative group of integers modulo 3 is $U_3 = \{1,2\}$. Subgroups of U_3 are $H_1 = \{1\}$ and $H_2 = \{1,2\} = U_3$. The unit subgraphs of U_3 are:

Fig. 13: Unit Graph of H_1

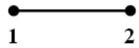

Fig. 14: Unit Graph of H_2

3.2.3. Unit Subgraphs of U_5

The multiplicative group of integers modulo 5 is $U_5 = \{1,2,3,4\}$. Subgroups of U_5 are $H_1 = \{1\}$, $H_2 = \{1,4\}$ and $H_3 = \{1,2,3,4\} = U_5$. The unit subgraphs of U_5 are:

Fig. 15: Unit Graph of H_1

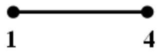

Fig. 16: Unit Graph of H_2

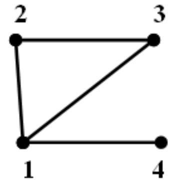

Fig. 17: Unit Graph of H_3

3.2.4. Unit Subgraphs of U_9

The multiplicative group of integers modulo 9 is $U_9 = \{1,2,4,5,7,8\}$. Subgroups of U_9 are $H_1 = \{1\}, H_2 = \{1,8\}$, $H_3 = \{1,4,7\}$ and $H_4 = \{1,2,4,5,7,8\} = U_9$. The unit subgraphs of U_9 are:

Fig. 18: Unit Graph of H_1

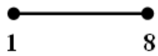

Fig. 19: Unit Graph of H_2

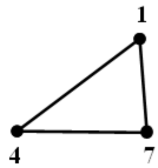

Fig. 20: Unit Graph of H_3

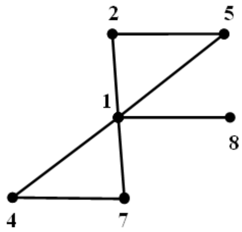

Fig. 21: Unit Graph of H_4

3.2.5. Unit Subgraphs of U_{13}

The multiplicative group of integers modulo 13 is $U_{13} = \{1,2,3,4,5,6,7,8,9,10,11,12\}$. Subgroups of U_{13} are $H_1 = \{1\}, H_2 = \{1,12\}$, $H_3 = \{1,3,9\}$ and $H_4 = \{1,2,3,4,5,6,7,8,9,10,11,12\} = U_{13}$ etc. The unit subgraphs of U_{13} are:

Fig. 22: Unit Graph of H_1

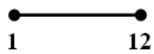

Fig. 23: Unit Graph of H_2

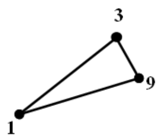

Fig. 24: Unit Graph of H_3

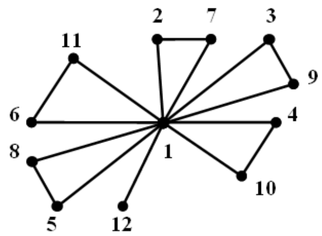

Fig. 25: Unit Graph of H_4

3.2.6. Unit Subgraphs of U_{16}

The multiplicative group of integers modulo 16 is $U_{16} = \{1,3,5,7,9,11,13,15\}$. Subgroups of U_{16} are $H_1 = \{1\}$, $H_2 = \{1,7\}$, $H_3 = \{1,9\}$, $H_4 = \{1,15\}$, $H_5 = \{1,5,9,13\}$ and $H_6 = \{1,3,5,7,9,11,13,15\} = U_{16}$ etc. The unit subgraphs of U_{16} are:

Fig. 26: Unit Graph of H_1

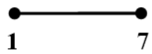

Fig. 27: Unit Graph of H_2

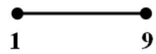

Fig. 28: Unit Graph of H_3

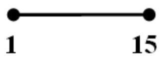

Fig. 29: Unit Graph of H_4

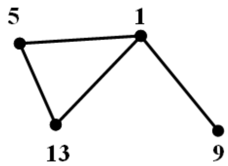

Fig. 30: Unit Graph of H_5

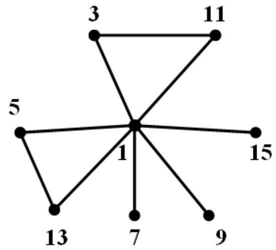

Fig. 31: Unit Graph of H_6

3.2.7. Unit Subgraph of Unit Group U_{17}

The multiplicative group of integers modulo 17 is $U_{17} = \{1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16\}$.
Subgroups of U_{17} are $H_1 = \{1\}$, $H_2 = \{1,16\}$, $H_3 = \{1,2,4,8,9,13,15,16\}$ and $H_4 = \{1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16\} = U_{17}$ etc. The unit subgraphs of U_{17} are:

Fig. 32: Unit Graph of H_1

Fig. 33: Unit Graph of H_2

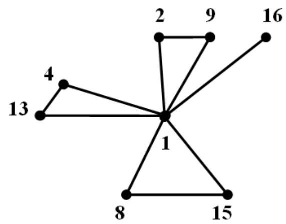

Fig. 34: Unit Graph of H_3

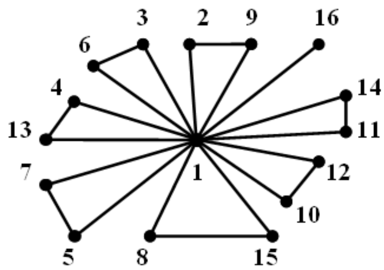

Fig. 35: Unit Graph of H_4

3.2.8. Unit Subgraphs of Unit Group U_{19}

The multiplicative group of integers modulo 19 is $U_{19} = \{1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18\}$.
Subgroups of U_{19} are $H_1 = \{1\}$, $H_2 = \{1,18\}$, $H_3 = \{1,7,11\}$ and $H_4 = \{1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18\} = U_{19}$ etc. The unit subgraphs of U_{19} are:

Fig. 36: Unit Graph of H_1

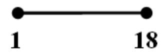

Fig. 37: Unit Graph of H_2

Fig. 38: Unit Graph of H_3

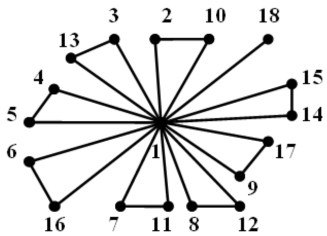

Fig. 39: Unit Graph of H_4

3.2.9. Unit Subgraphs of Unit Group U_{21}

The multiplicative group of integers modulo 21 is $U_{21} = \{1,2,4,5,8,10,11,13,16,17,19,20\}$. Subgroups of U_{21} are $H_1 = \{1\}$, $H_2 = \{1,8\}$, $H_3 = \{1,13\}$, $H_4 = \{1,20\}$, $H_5 = \{1,8,13,20\}$ and $H_4 = \{1,2,4,5,8,10,11,13,16,17,19,20\} = U_{21}$ etc. The unit subgraphs of U_{21} are:

Fig. 40: Unit Graph of H_1

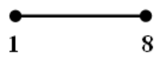

Fig. 41: Unit Graph of H_2

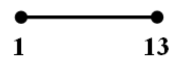

Fig. 42: Unit Graph of H_3

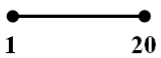

Fig. 43: Unit Graph of H_4

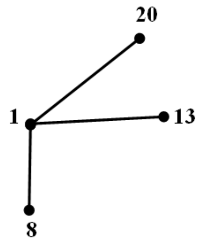

Fig. 44: Unit Graph of H_5

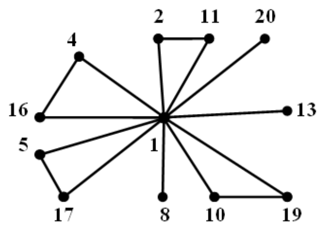

Fig. 45: Unit Graph of H_6

3.2.10. Unit Subgraphs of Unit Group U_{22}

The multiplicative group of integers modulo 22 is $U_{22} = \{1,3,5,7,9,13,15,17,19,21\}$. Subgroups of U_{22} are $H_1 = \{1\}$, $H_2 = \{1,21\}$, $H_3 = \{1,3,5,9,15\}$ and $H_4 = \{1,3,5,7,9,13,15,17,19,21\} = U_{22}$ etc. The unit subgraphs of U_{22} are:

Fig. 46: Unit Graph of H_1

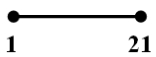

Fig. 47: Unit Graph of H_2

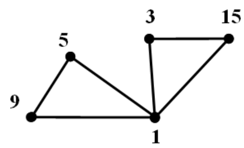

Fig. 48: Unit Graph of H_3

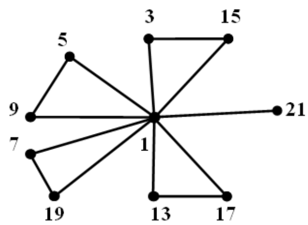

Fig. 49: Unit Subgraph of H_4

3.3. Unit Subgraphs of K_4

The Klein's four group K_4 is given by $K_4 = \{e, a, b, c\}$, where $a^2 = b^2 = c^2 = e$ and $ab = ba = c$, $bc = cb = a$ and $ca = ac = b$. Subgroups of K_4 are $H_1 = \{e\}, H_2 = \{e, a\}, H_3 = \{e, b\}, H_4 = \{e, c\}$ and $H_5 = \{e, a, b, c\} = K_4$. The unit subgraphs of K_4 are:

Fig. 50: Unit Graph of H_1

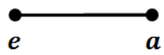

Fig. 51: Unit Graph of H_2

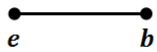

Fig. 52: Unit Graph of H_3

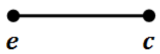

Fig. 53: Unit Graph of H_4

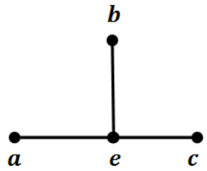

Fig. 54: Unit Graph of H_5

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