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An Extension of Some Common Fixed Point Results for Contractive Mappings in Cone Metric Spaces

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Abstract: *The purpose of this paper is to obtain existence of common fixed point theorem for some contractive mapping in the setting of cone metric spaces. Our results generalize and extends, unify some well known results of [1].*

Key Words: *Cone metric spaces, contractive mappings, common fixed point, fixed point, non- normal cone.*

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I. Introduction

Fixed point theory is one of the most dynamic research subjects in non-linear analysis. The theory itself is a beautiful mixture of analysis, topology and geometry. Over the last years or so the theory of fixed points has been revealed as a very powerful and important tool in the study of non-linear phenomena. In this area, the first important and significant result was proved by Banach [27] in 1922 for a contraction mapping in a complete metric space. The well known Banach contraction theorems may be stated as follows:

"Every contraction mapping of a complete metric space in X into itself has a unique fixed point."

Since then, this principle has been extended and generalized in several ways either by using the contractive condition or by imposing some additional conditions on an ambient space. This principle is one of the cornerstones in the development of fixed point theory. From inspiration of this work, several mathematicians heavily studied this field.

Recently, the real number's as the co-domain of a metric, by an ordered Banach space obtain a generalized metric space, called a cone metric space, was introduced by Huang and Zhang [4]. The described the convergence in cone metric space, introduced their completeness, and proved some fixed point theorem for contractive mapping on cone metric space. The initial work of Huang and Zhang[4] inspired many authors to prove fixed point theorems as well as common fixed point theorems for two or more mappings on cone metric space see for instance[13,14]. After wards, many authors have generalized the results of [4] and studied the existence of common fixed point of a pair of self mapping in the frame work of normal cone metric space, see for instance [1],[2], [3], [5], [6] [7] [8], [9] to [16]. Recently, Morals and Rojas [17], [18] have extended the concept T-contraction mappings to cone metric space by proving fixed point theorems for T- Kannan – Zamfirescu. T-weak contraction mappings. S. Moradi in [19] introduced the T- Kannan contractive mapping which extends the well known Kannan fixed point theorem given in [20, 21, and 22]. The results [3] and [19] very recently generalized by [23], [24] and [26].

Many authors have established and extended different type s of contractive mappings in cone metric spaces see for instances [1], [4], [28], [29], [30], [31], [32], [33], [34], [35], [36],[38] and the author [37] have also proved fixed point in cone metric space for generalized contractive mappings. In sequel, S.K.Tiwari and R.P. Dubey [1] obtain unique fixed point results in cone metric spaces which are generalized results are [4], [37]. In this paper is to extend and improve common fixed point theorem for this mappings in cone metric spaces with non normal cone conditions. Our results extend and generalized the respective theorem 2.1 of [1].

II. Preliminary Notes

First, we recall some standard notations and definitions in cone metric spaces with some of their properties [4].

Definition: 2.1 [4] Let E be a real Banach space and P be a subset of E. P is called a cone if and only if :

(i) P is closed, non-empty and $P \neq \{0\}$,

(ii) $ax + by \in P$ for all $x, y \in P$ and non-negative real number a, b;

(iii) $x \in P$ and $-x \in P \Longrightarrow x = 0 \Leftrightarrow P \cap (-P) = \{0\}$

Given a cone $P \subset E$, we define a partial ordering $\leq$ on E with respect to P by $x \leq y$ if and only if $y-x \in P$. We shall write $x \ll y$ if $y - x \in intP$, int P denotes the interior of P. The cone P is called normal if there is a number $K > 0$ such that for all $x, y \in E$,

$$0 \leq x \leq y \text{ implies } \| x \| \leq k \| y \|.$$

The least positive number k satisfying the above is called the normal constant of P.

Definition: 2.2[4] Let x be a non-empty set. Suppose the mapping $d: X \times X \longrightarrow E$ satisfies

(i) $0 < d(x,y) \text{ for all } x,y \in X \text{ and } (x,y) = 0 \text{ iff } x = y$

(ii) $d(x,y) = d(y,x) \text{ for all } x,y \in X$

(iii) $d(x,y) \leq d(x,z) + d(z,y) \text{ for all } x,y,z \in X$

Then d is called a cone metric on X, and (X,d) is called a cone metric space [27]. The concept of cone metric space is more general than that of a metric space.

Example 2.3[4] Let $E = R^2, P = \{(x,y) \in E : x, y \geq 0\}, X = R$ and $d: X \times X \to E$ defined by $d(x,y) = (|x - y|, \alpha|x - y|)$,where α ≥ 0 is a constant. Then (X, d) is a cone metric space.

Definition: 2.4 [4] Let(X, d) be a cone metric space, x∈ X and $\{x_n\}_{n\geq1}$a sequence in X. then,

(i) $\{x_n\}_{n\geq1}$ Converges to x whenever for every$c \in E$ with $0 \ll c$, there is a natural number N such that$d(x_n, x) \ll c$ for all n≥N. We denote this by $lim_{n\to\infty} x_n = x \text{ or } x_n \to x, (n \to \infty)$

(ii) $\{x_n\}_{n\geq1}$is said to be a Cauchy sequence if for every c ∈ E with 0 << c, there is a natural number N such that $d(x_n, x_m) \ll c$ for all $n.m \geq N$.

(iii) (X,d) is called a complete cone metric space if every Cauchy sequence in X is convergent

Lemma: 2.5 [30] Let E be a topological vector space. If $c_n \in E$ and $c_n \to 0$, then for each $c \in int(P)$ there exist N such that$c_n \ll c$ for all $n > N$

III. Main Results.

In this section we shall prove unique common fixed point results for contractive mappings in complete cone metric spaces by using non- normal cone.

Theorem3.1: Let (X, d) be a complete cone metric space and suppose the mapping $T_1, T_2: X \to X$ satisfy the contractive condition,

$$d(T_1x, T_2y) \leq a_1 d(x,y) + a_2 d(T_1x, x) + a_3 d(T_2y, y) + a_4 d(T_1x, y) + a_5 d(T_2y, x)$$

for all $x, y \in X$,where $a_i, i = 1,2,3,4,5$ are all non negative constants such that $a_1 + a_2 + a_3 + a_4 + a_5 < 1$.Then T_1 and T_2 have a unique common fixed point in X and for any $x \in X$, iterative sequence $\{T_1^{2n+1}x_0\}$and $\{T_2^{2n+2}x_0\}$ converge to the common fixed point. Moreover, any fixed point of $T_1$is the fixed point of T_2, and conversely.

Proof: For each $x_0 \in X$ and $n \geq 1$, set $x_1 = T_1x_0$ and

$$x_{2n+1} = T_1x_{2n} = T_1^{2n+1}x_0 \text{ , for all} n \in N., n = 0,1,2 \dots \dots\dots\dots$$

Similarly,

$$x_{2n+2} = T_2x_{2n} = T_2^{2n+2}x_0, \text{ for all } n \in N. n = 0,1,2 \dots\dots\dots\dots\dots$$

Let, $x = x_{2n}$ and $y = x_{2n-1}$ in (3.1)

Then we have

$$\begin{aligned}
d(x_{2n+1}, x_{2n}) &= d(T_1x_{2n}, T_2x_{2n-1}) \\
&\leq a_1 d(x_{2n}, x_{2n-1}) + a_2 d(T_1x_{2n}, x_{2n}) + a_3 d(T_2x_{2n-1}, x_{2n-1}) + a_4 d(T_1x_{2n}, x_{2n-1}) \\
&\qquad + a_5 d(T_2x_{2n-1}, x_{2n}) \\
&\leq a_1 d(x_{2n}, x_{2n-1}) + a_2 d(x_{2n+1}, x_{2n}) + a_3 d(x_{2n}, x_{2n-1}) \\
&\quad + a_4 d(x_{2n+1}, x_{2n-1}) + a_5 d(x_{2n}, x_{2n}) \\
&\leq a_1 d(x_{2n}, x_{2n-1}) + a_2 d(x_{2n+1}, x_{2n}) + a_3 d(x_{2n}, x_{2n-1}) + a_4 d(x_{2n+1}, x_{2n-1}) \\
&\leq a_1 d(x_{2n}, x_{2n-1}) + a_2 d(x_{2n+1}, x_{2n}) + a_3 d(x_{2n}, x_{2n-1}) + a_4[d(x_{2n}, x_{2n-1}) + d(x_{2n}, x_{2n+1})] \\
&\leq (a_2 + a_4) d(x_{2n+1}, x_{2n}) + (a_1 + a_3 + a_4)\, d(x_{2n}, x_{2n-1})
\end{aligned}$$

$$(1 - a_2 - a_4) d(x_{2n+1}, x_{2n}) \leq (a_1 + a_3 + a_4)\, d(x_{2n}, x_{2n-1})$$

So,

$$d(x_{2n+1}, x_{2n}) \leq \frac{a_1 + a_3 + a_4}{1 - a_2 - a_4} d(x_{2n}, x_{2n-1}).$$

$$= h d(x_{2n}, x_{2n-1}) \text{ , where, } h = \frac{a_1 + a_3 + a_4}{1 - a_2 - a_4}$$

Hence,

$$d(x_{2n+1}, x_{2n}) \leq h d(x_{2n}, x_{2n-1}) \leq h^2 d(x_{2n-1}, x_{2n-2}) \leq h^3 d(x_{2n-2}, x_{2n-3}) \leq \cdots \dots\dots\dots\dots\dots \leq h^n d(x_1, x_0)$$

We now show that $\{x_n\}$ is a Cauchy sequence in X.

for $> m$, we have,

$$d(x_{2n}, x_{2m}) \leq d(x_{2n}, x_{2n+1}) + d(x_{2n+1}, x_{2n+2}) + \cdots \dots\dots.. + d(x_{2m-1}, x_{2m})$$

$$\leq h^{2n}d(x_1,x_0) + h^{2n+1}d(x_1,x_0) + \cdots \ldots \ldots \ldots + h^{2m-1}d(x_1,x_0)$$
$$\leq \frac{h^{2m}}{1-h}d(x_1,x_0) \to 0$$

Hence,
$lim_{n\to\infty}d(x_{2n},x_{2m}) = 0$ by (ii). Applying lemma 2.5, $\{x_{2n}\}$ is a Cauchy sequence in X

Let 0 << c be given, choose a positive integer N_1, such that $\frac{h^{2m}}{1-h}d(x_1,x_0) \ll c$, for all $m \geq N_1$.Thus $d(x_{2n},x_{2m}) \ll c$, for $n > m$. Therefore $\{x_{2n}\}$ is a Cauchy sequence in (X,d). Since (X,d) be a complete cone metric space, there exist $x^* \in X$ such that $x_{2n} \to x^*$. Now choose a positive integer N_2 such that $d(x_{2n-1},x^*) \ll \frac{\sigma}{6}c$,where $1-(a_2+a_4)$, for all $n \geq N_2$. Hence, we have

$$d(T_1x^*,x^*) \leq d(T_1x^*,x_{2n}) + d(x_{2n},x^*)$$
$$= d(T_1x^*,T_2x_{2n-1}) + d(x_{2n},x^*)$$
$$\leq a_1d(x^*,x_{2n-1}) + a_2d(T_1x^*,x^*) + a_3d(T_2\,x_{2n-1},x_{2n-1}) + a_4d(T_1x^*,x_{2n-1})$$
$$+ a_5d(T_2\,x_{2n-1},x^*) + d(x_{2n},x^*)$$
$$\leq a_1d(x^*,x_{2n-1}) + a_2d(T_1x^*,x^*) + a_3d(x_{2n},x_{2n-1}) + a_4d(T_1x^*,x_{2n-1})$$
$$+ a_5d(x_{2n},x^*) + d(x_{2n},x^*)$$
$$\leq a_1d(x^*,x_{2n-1}) + a_2d(T_1x^*,x^*) + a_3[d(x^*,x_{2n-1}) + d(x_{2n},x^*)]$$
$$+ a_4[d(x^*,x_{2n-1}) + d(T_1x^*,x^*)] + a_5d(x_{2n},x^*) + d(x_{2n},x^*)$$
$$d(T_1x^*,x^*) \leq \frac{(a_1+a_3+a_4)}{1-(a_2+a_4)}d(x^*,x_{2n-1}) + [1+\frac{a_3+a_5}{1-(a_2+a_4)}]\,d(x_{2n},x^*)$$
$$\leq \frac{1}{\sigma}[\frac{3\sigma}{6}c + \frac{3\sigma}{6}c]$$
$$= c, \text{ for all } n \geq N_2.$$

Thus, $(T_1x^*,x^*) \ll \frac{c}{m}$, for all $m \geq 1$, so, $\frac{c}{m} - d(T_1x^*,x^*) \in P$, for all $m \geq 1$

Since, $\frac{c}{m} \to 0$ (as $m \to \infty$) and P is closed, $-d(T_1x^*,x^*) \in P$.But $d(T_1x^*,x^*) \in P$.

Therefore $d(T_1x^*,x^*) \in P = 0$ and $T_1x^* = x^*$. So, x^* is a fixed point of T_1.

Now we will show that x^* is also fixed point of T_2:

$$d(x^*,T_2x^*) \leq d(T_1x^*,T_2x^*)$$
$$\leq a_1d(x^*,x^*) + a_2\,d(T_1x^*,x^*) + a_3\,d(T_2x^*,x^*) + a_4\,d(T_1x^*,x^*) + a_5\,d(T_2x^*,x^*)$$
$$\leq (a_3+a_5)\,d(T_2x^*,x^*)$$

Which, using the definition of partial ordering on E and properties of cone P, gives that $d(x^*,T_2x^*) = 0$, and $x^* = T_2x^*$. Conversely, any fixed point of $T_1$is the fixed point of T_2.That means, $T_1x^* = x^* = T_2x^*$. Thus x^* is common fixed point of T_1 and T_2.

To prove uniqueness, let us suppose that, if y^* is another fixed point of and T_2.

Then $d(x^*,y^*) \leq d(T_1x^*,T_2y^*)$
$$\leq a_1d(x^*,y^*) + a_2\,d(T_1x^*,x^*) + a_3d(T_2y^*,y^*) + a_4\,d(T_1x^*,y^*) + a_5\,d(T_2y^*,x^*)$$
$$\leq (a_1+a_4+a_5)d(x^*,y^*)$$

Hence $d(x^*,y^*)$ =0 and so, $x^* = y^*$. Therefore x^* is a unique common fixed point of T_1 and T_2 . This completes the proof.

Example 3.1: Let X= {1, 2, 3}.Let d be the cone metric for Xdetermined by

$$d(1,2) = 1,\ d(2,3) = \frac{4}{7},\ d(1,3) = \frac{5}{7}.$$

Let $T_1,T_2: X \to X$ be the function on (X,d) such that

$$T_1(1) = T_1(2) = T_1(3) = 1;$$
$$T_2(1) = T_2(3) = 1,\ T_2(2) = 3.$$

Let $a_1 = 0. a_2 = 0, a_3 = 0, a_4 = \frac{5}{7}, a_5 = 0$. Then the conditions of theorem 3.1 are satisfied. However, no negative real numbers a_1,a_2,a_3,a_5 can be chosen such that

$$a_1 + a_2 + a_3 + a_5 < 1 \text{ and for } x,y \in X,$$

$d(T_1x,T_2y) \leq a_1d(x,y) + a_2d(T_1x,x) + a_3d(T_2y,y) + a_4d(T_1x,y) + a_5d(T_2y,x)$. For if there exist such a_1,a_2,a_3,a_5, then

$d(T_1(3),T_2(2)) \leq a_1d(3,2) + a_2d(T_1(3),3) + a_3d(T_2(2),2) + a_5\,d(T_2(2),3)$.So,

$$\frac{5}{7} \leq \frac{4a_1}{7} + \frac{5a_2}{7} + \frac{4a_5}{7} \leq \frac{5}{7}\,(a_1+a_2+a_5) \leq \frac{5}{7}, \text{ which is a contradiction.}$$

Theorem 3.2: Let (X,d)be a complete cone metric space and $T_1,T_2: X \to X$ be a self mappings satisfies the condition.

$$d(T_1x,T_2y) \leq a_1d(x,y) + a_2d(T_1x,x) + a_3d(T_2y,y) + a_4[d(T_1x,y) + d(T_2y,x)]$$

for all $x, y \in X$,where $a_i, i = 1,2,3,4,5$ are all non negative constants such that $a_1 + a_2 + a_3 + a_4 + a_5 < 1$.Then T_1 and T_2 have a unique common fixed point in X and for any $x \in X$, iterative sequence $\{T_1^{2n+1}x_0\}$and $\{T_2^{2n+2}x_0\}$ converge to the common fixed point. Moreover, any fixed point of $T_1$is the fixed point of T_2, and conversely.

Proof: For each $x_0 \in X$ and $n \geq 1$, set $x_1 = T_1 x_0$ and

$$x_{2n+1} = T_1 x_{2n} = T_1^{2n+1} x_0 \text{ , for all} n \in N., n = 0,1,2 \ldots \ldots\ldots$$

Similarly,

$$x_{2n+2} = T_2 x_{2n} = T_2^{2n+2} x_0, \text{ for all } n \in N. n = 0,1,2 \ldots \ldots \ldots$$

Let, $x = x_{2n}$ and $y = x_{2n-1}$ in (3.2)

Then we have,

$$\begin{aligned} d(x_{2n+1}, x_{2n}) &= d(T_1 x_{2n}, T_2 x_{2n-1}) \\ &\leq a_1 d(x_{2n}, x_{2n-1}) + a_2 d(T_1 x_{2n}, x_{2n}) + a_3 d(T_2 x_{2n-1}, x_{2n-1}) \\ &\quad + a_4[d(T_1 x_{2n}, x_{2n-1}) + d(T_2 x_{2n-1}, x_{2n})] \\ &\leq a_1 d(x_{2n}, x_{2n-1}) + a_2 d(x_{2n+1}, x_{2n}) + a_3 d(x_{2n}, x_{2n-1}) \\ &\quad + a_4[d(x_{2n+1}, x_{2n-1}) + d(x_{2n}, x_{2n})] \\ &\leq a_1 d(x_{2n}, x_{2n-1}) + a_2 d(x_{2n+1}, x_{2n}) + a_3 d(x_{2n}, x_{2n-1}) \\ &\quad + a_4 d(x_{2n+1}, x_{2n-1}) \\ &\leq a_1 d(x_{2n}, x_{2n-1}) + a_2 d(x_{2n+1}, x_{2n}) + a_3 d(x_{2n}, x_{2n-1}) \\ &\quad + a_4[d(x_{2n}, x_{2n-1}) + d(x_{2n}, x_{2n+1})] \\ &\leq (a_2 + a_4) d(x_{2n+1}, x_{2n}) + (a_1 + a_3 + a_4)\, d(x_{2n}, x_{2n-1}) \end{aligned}$$

$$(1 - a_2 - a_4) d(x_{2n+1}, x_{2n}) \leq (a_1 + a_3 + a_4)\, d(x_{2n}, x_{2n-1})$$

$$d(x_{2n+1}, x_{2n}) \leq \frac{a_1 + a_3 + a_4}{1 - a_2 - a_4} d(x_{2n}, x_{2n-1})$$

This implies $d(x_{2n+1}, x_{2n}) \leq h d(x_{2n}, x_{2n-1})$

$$= h d(x_{2n}, x_{2n-1}) \text{ , where, } h = \frac{a_1 + a_3 + a_4}{1 - a_2 - a_4} < 1$$

Similarly, we obtain,

$$d(x_{2n+2}, x_{2n+1}) \leq h d(x_{2n+1}, x_{2n})$$

Hence,

$$d(x_{2n+1}, x_{2n}) \leq h d(x_{2n}, x_{2n-1}) \leq h^2 d(x_{2n-1}, x_{2n-2}) \leq h^3 d(x_{2n-2}, x_{2n-3}) \leq \cdots \ldots \ldots \ldots \ldots \ldots \quad \leq h^n d(x_1, x_0)$$

We now show that $\{x_n\}$ is a Cauchy sequence in X.

$$\begin{aligned} d(x_{2n}, x_{2m}) &\leq d(x_{2n}, x_{2n+1}) + d(x_{2n+1}, x_{2n+2}) + \cdots \ldots \ldots .. + d(x_{2m-1}, x_{2m}) \\ &\leq h^{2n} d(x_1, x_0) + h^{2n+1} d(x_1, x_0) + \cdots \ldots \ldots \ldots + h^{2m-1} d(x_1, x_0) \\ &\leq \frac{h^{2m}}{1-h} d(x_1, x_0) \to 0 \end{aligned}$$

Hence,

$lim_{n \to \infty} d(x_{2n}, x_{2m}) = 0$. By applying lemma 2.5, $\{x_{2n}\}$ is a Cauchy sequence in X

Let 0 << c be given, choose a positive integer N_1, such that $\frac{h^{2m}}{1-h} d(x_1, x_0) \ll c$, for all $m \geq N_1$.Thus $d(x_{2n}, x_{2m}) \ll c$, for $n > m$. Therefore $\{x_{2n}\}$ is a Cauchy sequence in (X, d). Since (X, d) be a complete cone metric space, there exist $x^* \in X$ such that $x_{2n} \to x^*$. Now choose a positive integer N_2 such that $d(x_{2n-1,} x^*) \ll \frac{\sigma}{6} c$,where $1 - (a_2 + a_4)$, for all $n \geq N_2$. Hence, we have

$$\begin{aligned} d(T_1 x^*, x^*) &\leq d(T_1 x^*, x_{2n}) + d(x_{2n,} x^*) \\ &= d(T_1 x^*, T_2 x_{2n-1}) + d(x_{2n,} x^*) \\ &\leq a_1 d(x^*, x_{2n-1}) + a_2 d(T_1 x^*, x^*) + a_3 d(T_2\, x_{2n-1}, x_{2n-1}) \\ &\quad + a_4[d(T_1 x^*, x_{2n-1} + d(T_2\, x_{2n-1}, x^*)] + d(x_{2n,} x^*) \\ &\leq a_1 d(x^*, x_{2n-1}) + a_2 d(T_1 x^*, x^*) + a_3 d(\, x_{2n}, x_{2n-1}) \\ &\quad + a_4[d(T_1 x^*, x_{2n-1}) + d(x_{2n,} x^*)] + d(x_{2n,} x^*) \\ &\leq a_1 d(x^*, x_{2n-1}) + a_2 d(T_1 x^*, x^*) + a_3[d(x^*, x_{2n-1}) + d(x_{2n,} x^*)] \\ &\quad + a_4[d(x^*, x_{2n-1}) + d(T_1 x^*, x^*)] + d(x_{2n,} x^*) \end{aligned}$$

$$\begin{aligned} d(T_1 x^*, x^*) &\leq \frac{(a_1 + a_3 + a_4)}{1 - (a_2 + a_4)} d(x^*, x_{2n-1}) + [1 + \frac{a_3}{1 - (a_2 + a_4)}]\, d(x_{2n,} x^*) \\ &\leq \frac{1}{\sigma} [\frac{3\sigma}{6} c + \frac{3\sigma}{6} c] \\ &= c, \text{ for all } n \geq N_2. \end{aligned}$$

Thus, $(T_1x^*, x^*) \ll \frac{c}{m}$, for all $m \geq 1$, so, $\frac{c}{m} - d(T_1x^*, x^*) \in P$, for all $m \geq 1$Since, $\frac{c}{m} \to 0$ (as $m \to \infty$) and P is closed, $-d(T_1x^*, x^*) \in P$.But $d(T_1x^*, x^*) \in P$.Therefore $d(T_1x^*, x^*) \in P = 0$ and $T_1x^* = x^*$. So, x^* is a fixed point of T_1. Now we will show that x^* is also fixed point ofT_2:

$d(x^*, T_2x^*) \leq d(T_1x^*, T_2x^*)$

$\leq a_1d(x^*, x^*)+a_2\, d(T_1x^*, x^*)+a_3\, d(T_2x^*, x^*)+a_4[\, d(T_1x^*, x^*)+ d(T_2x^*, x^*)]$

$\leq (a_3 + a_4)\, d(T_2x^*, x^*)$

Which, using the definition of partial ordering on E and properties of cone P, gives that $d(x^*, T_2x^*) = 0$, and $x^* = T_2x^*$. Conversely, any fixed point of $T_1$is the fixed point of T_2.That means, $T_1x^* = x^* = T_2x^*$. Thus x^* is common fixed point of T_1 and T_2.

To prove uniqueness, let us suppose that, if y^* is another fixed point of T_1 and T_2.

$d(x^*, y^*) = d(T_1x^*, T_2y^*)$

$\leq a_1d(x^*, y^*) + a_2d(T_1x^*, x^*) + a_3d(T_2y^*, y^*) + a_4[d(T_1x^*, y^*) + d(T_2y^*, x^*)]$

Hence $d(x^*, y^*)$ =0 and so, $x^* = y^*$. Therefore x^* is a unique common fixed point of T_1 and T_2 . This completes the proof.

Theorem 3.3: Let (X, d)be a complete cone metric space and $T_1, T_2: X \to X$ be a self mappings satisfies the condition.

$$d(T_1x, T_2y) \leq a_1d(x, y) + a_2[d(T_1x, x) + d(T_2y, y)] + a_3[d(T_1x, y) + d(T_2y, x)]$$

for all $x, y \in X$,where $a_i, i = 1,2,3$ are all non negative constants such that $a_1 + 2a_2 + 2a_3 < 1$.Then T_1 and T_2 have a unique common fixed point in X and for any $x \in X$, iterative sequence $\{T_1^{2n+1}x_0\}$and $\{T_2^{2n+2}x_0\}$ converge to the common fixed point. Moreover, any fixed point of $T_1$is the fixed point of T_2, and conversely.

Proof: For each $x_0 \in X$ and $n \geq 1$, set $x_1 = T_1x_0$ and

$$x_{2n+1} = T_1x_{2n} = T_1^{2n+1}x_0 \text{ , for all} n \in N., n = 0,1,2 \dots \dots\dots$$

Similarly,

$$x_{2n+2} = T_2x_{2n} = T_2^{2n+2}x_0, \text{ for all } n \in N. n = 0,1,2 \dots \dots\dots$$

Let, $x = x_{2n}$ and $y = x_{2n-1}$ in (3.3)

Then we have,

$d(x_{2n+1}, x_{2n}) = d(T_1x_{2n}, T_2x_{2n-1})$

$\leq a_1d(x_{2n}, x_{2n-1}) + a_2[d(T_1x_{2n}, x_{2n}) + d(T_2x_{2n-1}, x_{2n-1})]$

$+a_3[d(T_1x_{2n}, x_{2n-1}) + d(T_2x_{2n-1}, x_{2n})]$

$\leq a_1d(x_{2n}, x_{2n-1}) + a_2[d(x_{2n+1}, x_{2n}) + d(x_{2n}, x_{2n-1})] +a_3[d(x_{2n+1}, x_{2n-1}) + d(x_{2n}, x_{2n})]$

$\leq a_1d(x_{2n}, x_{2n-1}) + a_2[d(x_{2n+1}, x_{2n}) + d(x_{2n}, x_{2n-1})] + a_3[d(x_{2n+1}, x_{2n-1})]$

$\leq a_1d(x_{2n}, x_{2n-1}) + a_2[d(x_{2n+1}, x_{2n}) + d(x_{2n}, x_{2n-1})] + a_3[d(x_{2n}, x_{2n-1}) + d(x_{2n}, x_{2n+1})]$

$\leq (a_2 + a_3)d(x_{2n+1}, x_{2n}) + (a_1 + a_2 + a_3)\, d(x_{2n}, x_{2n-1})$

$(1 - a_2 - a_3)d(x_{2n+1}, x_{2n}) \leq (a_1 + a_2 + a_3)\, d(x_{2n}, x_{2n-1})$

$d(x_{2n+1}, x_{2n}) \leq \frac{a_1+a_2+a_3}{1-a_2-a_3} d(x_{2n}, x_{2n-1})$

This implies, $d(x_{2n+1}, x_{2n}) \leq hd(x_{2n}, x_{2n-1})$

$= hd(x_{2n}, x_{2n-1})$, where, $h = \frac{a_1+a_2+a_3}{1-a_2-a_3} < 1$

Similarly, we obtain,

$$d(x_{2n+2}, x_{2n+1}) \leq hd(x_{2n+1}, x_{2n})$$

Hence,

$$d(x_{2n+1}, x_{2n}) \leq hd(x_{2n}, x_{2n-1}) \leq h^2d(x_{2n-1}, x_{2n-2}) \leq h^3d(x_{2n-2}, x_{2n-3}) \leq \cdots \dots \dots \dots \dots \dots \leq h^nd(x_1, x_0)$$

We now show that $\{x_n\}$ is a Cauchy sequence in X.

$d(x_{2n}, x_{2m}) \leq d(x_{2n}, x_{2n+1}) + d(x_{2n+1}, x_{2n+2}) + \cdots \dots \dots . . +d(x_{2m-1}, x_{2m})$

$\leq h^{2n}d(x_1, x_0) + h^{2n+1}d(x_1, x_0) + \cdots \dots \dots \dots + h^{2m-1}d(x_1, x_0)$

$\leq \frac{h^{2m}}{1-h} d(x_1, x_0) \to 0$

Hence,

$lim_{n\to\infty} d(x_{2n}, x_{2m}) = 0$. By applying lemma 2.5, $\{x_{2n}\}$ is a Cauchy sequence in X

Let 0 << c be given, choose a positive integer N_1, such that $\frac{h^{2m}}{1-h} d(x_1, x_0) \ll c$, for all $m \geq N_1$.Thus $d(x_{2n}, x_{2m}) \ll c$, for $n > m$. Therefore $\{x_{2n}\}$ is a Cauchy sequence in (X, d). Since (X, d) be a complete cone metric space, there exist $x^* \in X$ such that $x_{2n} \to x^*$. Now choose a positive integer N_2 such that $d(x_{2n-1,} x^*) \ll \frac{\sigma}{6} c$,where $1 - (a_2 + a_4)$, for all $n \geq N_2$. Hence, we have

$d(T_1x^*,x^*) \le d(T_1x^*,x_{2n}) + d(x_{2n,}x^*)$
$= d(T_1x^*,T_2x_{2n-1}) + d(x_{2n,}x^*)$
$\le a_1d(x^*,x_{2n-1}) + a_2[d(T_1x^*,x^*) + d(T_2\, x_{2n-1},x_{2n-1})]$
$+ a_3[d(T_1x^*,x_{2n-1} + d(T_2\, x_{2n-1},x^*)] + d(x_{2n,}x^*)$
$\le a_1d(x^*,x_{2n-1}) + a_2[d(T_1x^*,x^*) + d(\, x_{2n},x_{2n-1})]$
$+ a_3[d(T_1x^*,x_{2n-1}) + d(x_{2n,}x^*)] + d(x_{2n,}x^*)$
$\le a_1d(x^*,x_{2n-1}) + a_2[d(T_1x^*,x^*) + d(\, x_{2n},x_{2n-1})]$
$+ a_3[d(x^*,x_{2n-1}) + d(T_1x^*,x^*) + d(x_{2n,}x^*)] + d(x_{2n,}x^*)$
$\le \frac{1}{1-a_2-a_3}[\, a_1d(x^*,x_{2n-1}) + a_2d(\, x_{2n},x_{2n-1}) + a_3\{\, d(x^*,x_{2n-1}) + d(x_{2n,}x^*)\}$
$+ d(x_{2n,}x^*)]$
$\le \frac{1}{\sigma}[\, a_1d(x^*,x_{2n-1}) + a_2\{d(\, x^*,x_{2n-1}) + d(x_{2n,}x^*)\}$
$+ a_3\{\, d(x^*,x_{2n-1}) + d(x_{2n,}x^*)\} + d(x_{2n,}x^*)]$
$\le \frac{1}{\sigma}[(a_1+a_2+a_3)d(x^*,x_{2n-1}) + \{1+(a_2+a_3)d(x_{2n,}x^*)\}]$
$\le \frac{1}{\sigma}[\frac{3\sigma}{\sigma}c + \frac{3\sigma}{\sigma}c = c$ for all $n \ge N_2$.

Thus, $(T_1x^*,x^*) \ll \frac{c}{m}$, for all $m \ge 1$, so, $\frac{c}{m} - d(T_1x^*,x^*) \in P$, for all $m \ge 1$Since, $\frac{c}{m} \to 0$ (as $m \to \infty$) and P is closed, $-d(T_1x^*,x^*) \in P$.But $d(T_1x^*,x^*) \in P$.Therefore $d(T_1x^*,x^*) \in P = 0$ and $T_1x^* = x^*$. So, x^* is a fixed point of T_1.

Now we will show that x^* is also fixed point ofT_2:

$d(x^*,T_2x^*) \le d(T_1x^*,T_2x^*)$
$\le a_1d(x^*,x^*) + a_2[\, d(T_1x^*,x^*) + d(T_2x^*,x^*)] + a_3[\, d(T_1x^*,x^*) + d(T_2x^*,x^*)]$
$\le (a_3+a_4)\, d(T_2x^*,x^*)$.

Which, using the definition of partial ordering on E and properties of cone P, gives that $d(x^*,T_2x^*) = 0$, and $x^* = T_2x^*$. Conversely, any fixed point of $T_1$is the fixed point of T_2.That is $T_1x^* = x^* = T_2x^*$. Thus x^* is common fixed point of T_1 and T_2.

To prove uniqueness, let us suppose that, if y^* is another fixed point of T_1 and T_2.

$d(x^*,y^*) = d(T_1x^*,T_2y^*)$
$\le a_1d(x^*,y^*) + a_2[d(T_1x^*,x^*) + d(T_2y^*,y^*)] + a_3[d(T_1x^*,y^*) + d(T_2y^*,x^*)]$
$\le (a_1+2a_3)d(x^*,y^*)$.

Hence $d(x^*,y^*)$ =0 and so, $x^* = y^*$. Therefore x^* is a unique common fixed point of T_1 and T_2 . This completes the proof.

Remarks 3.4.

(1) If we put $a_1 = k$ & $a_2 = a_3 = a_4 = a_5 = 0$ in theorems 3.1, 3.2 and 3.3. Then we get the result of theorem 2.1 of Dubey et al.[38].

(2) If we put $a_1 = 0$, $a_2 = a_3 = k$ & $a_4 = a_5 = 0$ in theorems 3.1, 3.2 and also put $a_1\ \ 0, a_2 = k, a_3 = 0$ in theorem 3.3. Then we get the result of theorem 2.3 of Dubey et al. [38].

(3) If putting $a_1 = a_2 = a_3 = 0$ & $a_4 = a_5 = k$ in theorems 3.1 and $a_1 = a_2 = a_3 = 0$ and $a_4 = k$ in theorem 3.2 and also put $a_1 = a_2 = 0, a_3 = k$ in theorem 3.3, we get the result of theorem 2.4 of Dubey et al.[38].

Respectively in it,

Precisely, theorems 2.1, 2.3 and 2.4 are synthesizes and generalizes all the results of [38] for a contractive mappings in cone metric spaces by using normality of cone.

Theorem 3.5: Let (X,d)be a complete cone metric space and $T_1,T_2{:}\,X \to X$ be a self mappings satisfies the condition.

$$d(T_1x,T_2y) \le \alpha[d(x,y) + d(x,T_1x) + d(x,T_2y)] + \beta[d(y,T_1x) + d(y,T_2y) + d(T_1x,T_2y)]$$

for all $x,y \in X$,where $\alpha,\beta \ge 0$ are all non negative constants such that $3\,\alpha + 3\beta < 1$.Then T_1 and T_2 have a unique common fixed point in X and for any $x \in X$, iterative sequence $\{T_1^{2n+1}x_0\}$and $\{T_2^{2n+2}x_0\}$ converge to the common fixed point. Moreover, any fixed point of $T_1$is the fixed point of T_2, and conversely.

,**Proof:** For each $x_0 \in X$ and $n \ge 1$, set $x_1 = T_1x_0$ and

$$x_{2n+1} = T_1x_{2n} = T_1^{2n+1}x_0 \text{ , for all } n \in N., n = 0,1,2 \ldots \ldots\ldots\ldots$$

Similarly,

$$x_{2n+2} = T_2x_{2n} = T_2^{2n+2}x_0, \text{ for all } n \in N. n = 0,1,2 \ldots\ldots\ldots\ldots$$

Let, $x = x_{2n}$ and $y = x_{2n-1}$ in (3.3)

Then we have,

$$\begin{aligned} d(x_{2n+1}, x_{2n}) &= d(T_1x_{2n}, T_2x_{2n-1}) \\ &\le \alpha[d(x_{2n}, x_{2n-1}) + d(x_{2n}T_1x_{2n}) + d(x_{2n}, T_2x_{2n-1})] \\ &+\beta[d(x_{2n-1}, T_1x_{2n}) + d(x_{2n-1}, T_2x_{2n-1}) + d(T_1x_{2n}, T_2x_{2n-1})] \\ &\le \alpha[d(x_{2n}, x_{2n-1}) + d(x_{2n}, x_{2n+1}) + d(x_{2n}, x_{2n})] \\ &+\beta[d(x_{2n-1}, x_{2n+1}) + d(x_{2n-1}, x_{2n}) + d(x_{2n+1}, x_{2n})] \\ &\le \alpha[d(x_{2n}, x_{2n-1}) + d(x_{2n}, x_{2n+1})] \\ &+\beta[d(x_{2n}, x_{2n-1}) + d(x_{2n}, x_{2n+1}) + d(x_{2n-1}, x_{2n}) + d(x_{2n+1}, x_{2n})] \\ &\le (\alpha + 2\beta)\, d(x_{2n}, x_{2n+1}) + (\alpha + 2\beta) d(x_{2n+1}, x_{2n}) \end{aligned}$$

Therefore,

$d(x_{2n+1}, x_{2n}) \le \frac{\alpha+2\beta}{1-(\alpha+2\beta)} d(x_{2n+1}, x_{2n})$, where $h = \frac{\alpha+2\beta}{1-(\alpha+2\beta)}$

This implies, $d(x_{2n+1}, x_{2n}) \le hd(x_{2n}, x_{2n-1})$

Similarly, we obtain,

$$d(x_{2n+2}, x_{2n+1}) \le hd(x_{2n+1}, x_{2n})$$

Hence,

$$d(x_{2n+1}, x_{2n}) \le hd(x_{2n}, x_{2n-1}) \le h^2d(x_{2n-1}, x_{2n-2}) \le h^3d(x_{2n-2}, x_{2n-3}) \le \cdots \ldots \ldots \ldots \ldots \ldots \le h^nd(x_1, x_0)$$

We now show that $\{x_n\}$ is a Cauchy sequence in X.

$$\begin{aligned} d(x_{2n}, x_{2m}) &\le d(x_{2n}, x_{2n+1}) + d(x_{2n+1}, x_{2n+2}) + \cdots \ldots \ldots .. + d(x_{2m-1}, x_{2m}) \\ &\le h^{2n}d(x_1, x_0) + h^{2n+1}d(x_1, x_0) + \cdots \ldots \ldots \ldots + h^{2m-1}d(x_1, x_0) \\ &\le \frac{h^{2m}}{1-h} d(x_1, x_0) \to 0 \end{aligned}$$

Hence,

$lim_{n\to\infty} d(x_{2n}, x_{2m}) = 0$. By applying lemma 2.5, $\{x_{2n}\}$ is a Cauchy sequence in X

Let 0 << c be given, choose a positive integer N_1, such that $\frac{h^{2m}}{1-h} d(x_1, x_0) \ll c$, for all $m \ge N_1$.Thus $d(x_{2n}, x_{2m}) \ll c$, for $n > m$. Therefore $\{x_{2n}\}$ is a Cauchy sequence in (X, d). Since (X, d) be a complete cone metric space, there exist $x^* \in X$ such that $x_{2n} \to x^*$. Now choose a positive integer N_2 such that $d(x_{2n-1,} x^*) \ll \frac{\sigma}{6} c$,where $1 - (\alpha + 2\beta)$for all $n \ge N_2$. Hence, we have

$$\begin{aligned} d(T_1x^*, x^*) &\le d(T_1x^*, x_{2n}) + d(x_{2n,} x^*) \\ &= d(T_1x^*, T_2x_{2n-1}) + d(x_{2n,} x^*) \\ &\le \alpha[d(x^*, x_{2n-1}) + d(x^*, T_1x^*) + d(x^*, T_2x_{2n-1})] \\ &+ \beta[d(x_{2n-1}, T_1x^*) + d(x_{2n-1}, T_2\, x_{2n-1}) + d(T_1x^*, T_2x_{2n-1})] + d(x_{2n,} x^*) \\ &\le \alpha[d(x^*, x_{2n-1}) + d(x^*, T_1x^*) + d(x^*, x_{2n})] \\ &+ \beta[d(x_{2n-1}, T_1x^*) + d(x_{2n-1}, x_{2n}) + d(x^*, x_{2n})] + d(x_{2n,} x^*) \\ &\le \alpha[d(x^*, x_{2n-1}) + d(x^*, T_1x^*) + d(x^*, x_{2n})] \\ &+ \beta[d(x_{2n-1}, x^*) + d(x^*, T_1x^*) + d(x^*, x_{2n}) + d(x^*, x_{2n})] + d(x_{2n,} x^*) \\ &\le \frac{1}{1-(\alpha+\beta)} [(\alpha + \beta)d(x^*, x_{2n-1}) + \{1+(\alpha + 2\beta)\, d(x_{2n,} x^*)] \\ &\le \frac{1}{\sigma} [\frac{3\sigma}{\sigma} c + \frac{3\sigma}{\sigma} c = c \text{ for all } n \ge N_2. \end{aligned}$$

Thus, $(T_1x^*, x^*) \ll \frac{c}{m}$, for all $m \ge 1$. So $\frac{c}{m} - d(T_1x^*, x^*) \in P$, for all $m \ge 1$.Since, $\frac{c}{m} \to 0$ (as$m \to \infty$) and P is closed, $-d(T_1x^*, x^*) \in P$.But $d(T_1x^*, x^*) \in P$.Therefore $d(T_1x^*, x^*) \in P = 0$ and $T_1x^* = x^*$. So, x^* is a fixed point of T_1.

Now we will show that x^* is also fixed point of T_2:

$$\begin{aligned} d(x^*, T_2x^*) &\le d(T_1x^*, T_2x^*) \\ &\le \alpha[d(x^*, x^*) + d(x^*, T_1x^*) + d(x^*, T_2x^*)] + \beta[d(x^*, T_1x^*) + d(x^*, T_2x^*) + d(T_1x^*, T_2x^*)] \\ &\le (\alpha + 2\beta)\, d(T_2x^*, x^*). \end{aligned}$$

Which, using the definition of partial ordering on E and properties of cone P, gives that $d(x^*, T_2x^*) = 0$, and $x^* = T_2x^*$. Conversely, any fixed point of $T_1$is the fixed point of T_2.That is $T_1x^* = x^* = T_2x^*$. Thus x^* is common fixed point of T_1 and T_2.

To prove uniqueness, let us suppose that, if y^* is another fixed point of T_1 and T_2.

$$\begin{aligned} d(x^*, y^*) &= d(T_1x^*, T_2y^*) \\ &\le \alpha[d(x^*, y^*) + d(x^*, T_1x^*) + d(x^*T_2y^*,)] + \beta[d(y^*, T_1x^*) + d(y^*, T_2y^*) + d(T_1x^*, T_2y^*)] \\ &\le 2(\alpha + \beta)d(x^*, y^*) \end{aligned}$$

Hence $d(x^*, y^*)$ =0 and so, $x^* = y^*$. Therefore x^* is a unique common fixed point of T_1 and T_2 . This completes

Corollary 3.6: Let (X,d)be a complete cone metric space and $T_1, T_2: X \to X$ be a self mappings satisfies the condition.

$$d(T_1 x, T_2 y) \leq \alpha[d(x,y) + d(x, T_1 x) + d(x, T_2 y)]$$

for all $x, y \in X$,where $\alpha \geq 0$ are all non negative constants such that $\alpha \in [0, \frac{1}{3}]$.Then T_1 and T_2 have a unique common fixed point in X and for any $x \in X$, iterative sequence $\{T_1^{2n+1} x_0\}$and $\{T_2^{2n+2} x_0\}$ converge to the common fixed point. Moreover, any fixed point of $T_1$is the fixed point of T_2, and conversely.

Proof: the proof of corollary immediately follows by putting $\beta = 0$ in the previous theorem.

IV. Conclusion

In this article we have proved that the existence and uniqueness of common fixed point theorem for contraction in cone metric spaces. The results of this paper generalize and extend the results of, S. K. Tiwari & R. P. Dubey, [1] on contractive mappings in cone metric spaces.

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