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# An Inverse Transient Thermoelastic Behavior of Circular Plate by Using Marchi-Fasulo and Laplace Integral Transform

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***Abstract:*** *This article concerned with study of exact solution of deflection of inverse transient thermoelastic thin circular plate occupying the space* $D:\left\{(x,y,z)\,|\,0\le r=\sqrt{x^2+y^2}\le a,-h\le z\le h\right\}$ *with the stated boundary conditions. The Marchi-Fasulo integral transform, Laplace integral transform are used and graphs are plotted using Microsoft office excel 2010.*

***Keywords*** *–Inverse transient thermoelastic problem, Laplace Transform, Marchi-Fasulo Transform, Thin circular plate, Thermoelastic deflection.*

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## I. Introduction

The inverse problems of thermoelasticity consist of determination of temperature distribution and thermal deflection of solids when the conditions of temperature and deflection are known at the some points of the solid under consideration. Grysa and Cialkowski [1], Grysa and Koalowski [2] studied one-dimensional transient thermoelastic problems and derived the heating temperature and heat flux on the surface of an isotropic infinite slab. Khobragade et al.[3] and [4] discuss an inverse steady state and transient thermoelastic problem of thin circular plate and annular disc in Marchi-Fasulo transform domain. Deshmukh et al. [5] investigated inverse heat conduction problem of semi-infinite, clamped thin circular plate and their thermal deflection by quasi-static approach. Ghonge and Ghadle [6]-[10] derived the exact solution to deflection of thermoelastic circular plate by using Marchi-Fasulo, Marchi-Zgrablich and Laplace integral transform.

In this work we modify the problem of Ghonge and Ghadle [6], and the results for temperature distribution, heat flux and quasi-static thermal deflection on outer surface of circular plate are discuss. The inverse transient heat conduction equation is solved by using Marchi-Fasulo and Laplace integral transform and the results for temperature distribution, unknown heat flux and thermal deflection function are obtained in terms of infinite series of Bessel's function and it is solved for special case by using MathCAD 2007 software and illustrated graphically by using Microsoft office excel 2010.

## II. Formulation of The Problem

Consider a thin circular plate of thickness 2h occupying the space $D:\left\{(x,y,z)\,|\,0\le r=\sqrt{x^2+y^2}\le a,-h\le z\le h\right\}$. Suppose the plate is subjected to arbitrary known interior temperature $f(z,t)$ within the $0\le r\le a$ region with third kind condition which assume to be zero at upper surface $z=h$ and lower surface $z=-h$. Under this more realistic prescribed conditions, the unknown temperature on lower surface and quasi-static thermal deflection due to unknown temperature $g(z,t)$ are required to determine. The differential equation satisfying the deflection function as in Noda et al. [11] is given as

$$\nabla^4 w=\frac{-1}{(1-\nu)D}\nabla^2 M_T \tag{1}$$

Where, the operator $\nabla^2$ is defined by

$$\nabla^2=\frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r} \tag{2}$$

$M_T$ is the thermal moment of the plate defined as

$$M_T=\alpha E\int_{-h}^{h} zT(r,z,t)dz \tag{3}$$

and $D$ is the flexural rigidity of the plate denoted as

$$D = \frac{Eh^3}{12(1-\nu^2)} \tag{4}$$

$\alpha, E$ and $\nu$ are the coefficients of the linear thermal expansion, the Young's modulus and the Poisson's ration of the plate material respectively.
Since the edge of the circular plate is fixed and clamped;

$$w = \frac{\partial w}{\partial r} = 0 \qquad at \;\; r = a \tag{5}$$

The temperature of the circular plate satisfying the heat conduction equation as in Ozisik [12] is as

$$\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial z^2} = \frac{1}{k}\frac{\partial T}{\partial t} \quad in \; 0 \le r \le a, -h \le z \le h \tag{6}$$

with initial condition

$$T(r,z,t) = 0 \qquad 0 \le r \le a, -h \le z \le h \tag{7}$$

the boundary condition's

$$\frac{\partial T}{\partial r} = g(z,t) \qquad (unknown)\, r = a, -h \le z \le h \tag{8}$$

$$\left[ T(r,z,t) + k_1 \frac{\partial T(r,z,t)}{\partial z} \right]_{z=h} = 0 \tag{9}$$

$$\left[ T(r,z,t) + k_2 \frac{\partial T(r,z,t)}{\partial z} \right]_{z=-h} = 0 \tag{10}$$

and interior condition

$$\left[ T(r,z,t) + c \frac{\partial T(r,z,t)}{\partial r} \right]_{r=\xi} = f(z,t) \;\; (known) \tag{11}$$

where $k_1$ and $k_2$ are the radiation constants on the two plane surfaces, $k$ is the thermal diffusivity of the material of the circular plate. The equations (1) to (11) constitute the mathematical formulation of the inverse transient thermoelastic deflection problem of circular plate.

## III. Solution of The Problem

### 3.1 Determination of Temperature Function

First applying finite Marchi-Fasulo transform as defined in [3] to the equations (6), (7), (11) and using (8), (9), then applying Laplace transform as defined in Sneddon [13] to the equations in Marchi-Fasulo transform domain and then using inversion of Laplace transform as defined in Sneddon [13] and Marchi-Fasulo transform as defined in [3] respectively, one obtain the expression for temperature function $T(r,z,t)$ as

$$T(r,z,t) = \frac{2k}{\xi} \sum_{n=1}^{\infty} \frac{P_n(z)}{\lambda_n} \sum_{m=1}^{\infty} \frac{\lambda_m J_0(\lambda_m r)}{J_1(\lambda_m \xi) + c\lambda_m J_1^{'}(\lambda_m \xi)} \int_0^t \overline{f}\left(n,t'\right) e^{-\left(\lambda_m^2 + a_n^2\right)(t-t')} dt' \tag{12}$$

where m, n are positive integers, $\lambda_m$ are the positive roots of the transcendental equation

$$J_0(\lambda_m a) = 0 \tag{13}$$

$\lambda_n = \int_{-h}^{h} P_n^2(z)dz$ and $\overline{f}(n,t) = \int_{-h}^{h} f(z,t)P_n(z)dz$

$P_n(z) = Q_n \cos(a_n z) - W_n \sin(a_n z)$

$Q_n = a_n\left(\alpha_1 + \alpha_2\right)\cos(a_n h) + \left(\beta_1 - \beta_2\right)\sin(a_n h)$

$W_n = \left(\beta_1 + \beta_2\right)\cos(a_n h) + a_n\left(\alpha_2 - \alpha_1\right)\sin(a_n h)$

equation (12) is the desired solution of equation (6) with $\beta_1 = \beta_2 = 1, \alpha_1 = k_1$ and $\alpha_2 = k_2$ .

### 3.2 Determination of Unknown Temperature Function

Using (12)in (8) once obtain the unknown temperature function $g(z,t)$ as

$$g(z,t) = \frac{2k}{\xi} \sum_{n=1}^{\infty} \frac{P_n(z)}{\lambda_n} \sum_{m=1}^{\infty} \frac{\lambda_m^2 J_0^{'}(\lambda_m r)}{J_1(\lambda_m \xi) + c\lambda_m J_1^{'}(\lambda_m \xi)} \int_0^t \overline{f}\left(n,t'\right) e^{-\left(\lambda_m^2 + a_n^2\right)(t-t')} dt' \tag{14}$$

equation (14) is the desired solution of equation (8) with $\beta_1 = \beta_2 = 1, \alpha_1 = k_1$ and $\alpha_2 = k_2$ .

### 3.3 Determination of Quasi-Static Thermal Deflection Function

Using (12) in equation (3) , we obtain

$$M_T = \frac{4\alpha Ek}{\xi}\sum_{n=1}^{\infty}\frac{W_n}{a_n\lambda_n}\left(h\cos(a_n h)-\frac{\sin(a_n h)}{a_n}\right)\sum_{m=1}^{\infty}\frac{\lambda_m J_0(\lambda_m r)}{J_1(\lambda_m\xi)+c\lambda_m J_1^{'}(\lambda_m\xi)}\int_0^t \overline{f}(n,t')e^{-(\lambda_m^2+a_n^2)(t-t')}dt' \qquad (15)$$

Assume the solution of (1) satisfying the (5) as

$$w(r,t)=\sum_{n=1}^{\infty}C_n(t)\left[J_0(\lambda_m r)-J_0(\lambda_m a)\right] \qquad (16)$$

Using the (15), (16) and the result

$$\left[\frac{d^2}{dr^2}+\frac{1}{r}\frac{d}{dr}\right]J_0(\lambda_m r)=-\lambda_m^2 J_0(\lambda_m r),$$

in (1), once we obtain the expression for $C_n(t)$ as

$$\begin{aligned} C_n(t) &= \frac{4\alpha Ek}{(1-\nu)D\xi}\sum_{n=1}^{\infty}\frac{W_n}{a_n\lambda_n}\left(h\cos(a_n h)-\frac{\sin(a_n h)}{a_n}\right) \\ &\quad\times\sum_{m=1}^{\infty}\frac{1}{\lambda_m\left[J_1(\lambda_m\xi)+c\lambda_m J_1^{'}(\lambda_m\xi)\right]}\int_0^t \overline{f}(n,t')e^{-(\lambda_m^2+a_n^2)(t-t')}dt' \end{aligned} \qquad (17)$$

Substituting the equation (17) in the equation (16), once obtain the expression for thermal deflection function as

$$w(r,t)=\frac{2\alpha E}{D(1-\nu)}\sum_{n=1}^{\infty}\frac{\overline{f}(n)W_n\left[a_n h\cos(a_n h)-\sin(a_n h)\right]\left[I_0(a_n r)-I_0(a_n a)\right]}{\lambda_n a_n^4\left[I_0(a_n\xi)+ca_n I_0^{'}(a_n\xi)\right]} \qquad (18)$$

## IV. Special Case and Numerical Results

For formulation of special case of a circular plate.
Setting

$$f(z,t)=\left(1-e^{-t}\right)\left(z-h\right)^2\left(z+h\right)^2\xi \qquad (19)$$

Applying finite Marchi-Fasulo transform as define in [3] to the equations (19), one obtain,

$$\overline{f}(n,t)=4\left(k_1+k_2\right)\left(1-e^{-t}\right)\xi\left[\frac{a_n h\cos^2(a_n h)-\cos(a_n h)\sin(a_n h)}{a_n^2}\right] \qquad (20)$$

Now using (20), we obtain the necessary integral bellow

$$\begin{aligned} \int_0^t \overline{f}(n,t')e^{-(\lambda_m^2+a_n^2)(t-t')}dt' &= 4\left(k_1+k_2\right)\left(1-e^{-t}\right)\xi\left[\frac{a_n h\cos^2(a_n h)-\cos(a_n h)\sin(a_n h)}{a_n^2}\right] \\ &\quad\times\left[\frac{1-e^{-(\lambda_m^2+a_n^2)t}}{\lambda_m^2+a_n^2}+\frac{e^{-t}-e^{-(\lambda_m^2+a_n^2)t}}{1-\lambda_m^2-a_n^2}\right] \end{aligned} \qquad (21)$$

For numerical result we set $k_1=k_2=1,\ a=1m,\ \xi=0.5m,\ c=1, h=0.1m$ in equations (12),(14), (18) and once making use of integral in (21) and obtain the numerical results for temperature, heat flux and thermal deflection using MathCAD 2007 and depicted graphically using Microsoft office Excel 2010 as in Fig.1- Fig.3

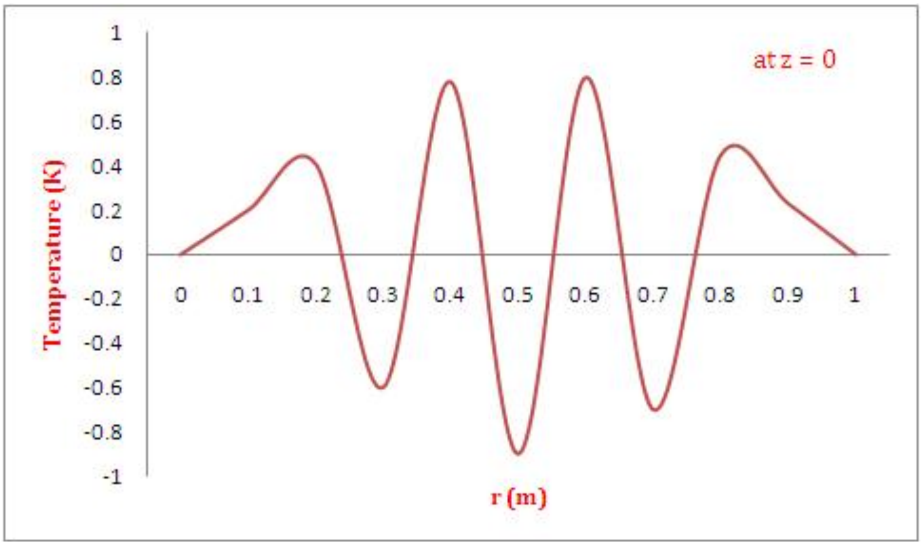


**Fig.1** Temperature distribution in circular plate

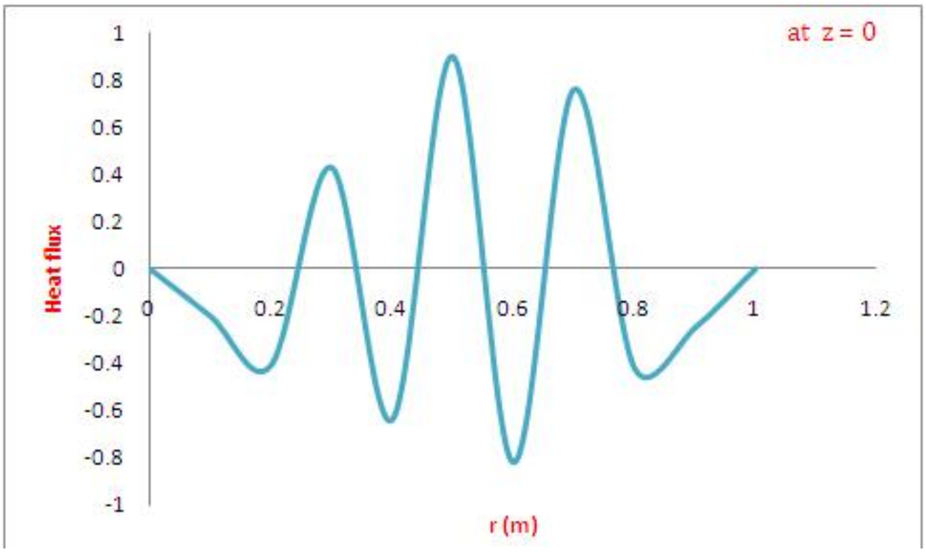


**Fig. 2** Unknown heat flux distribution in circular plate

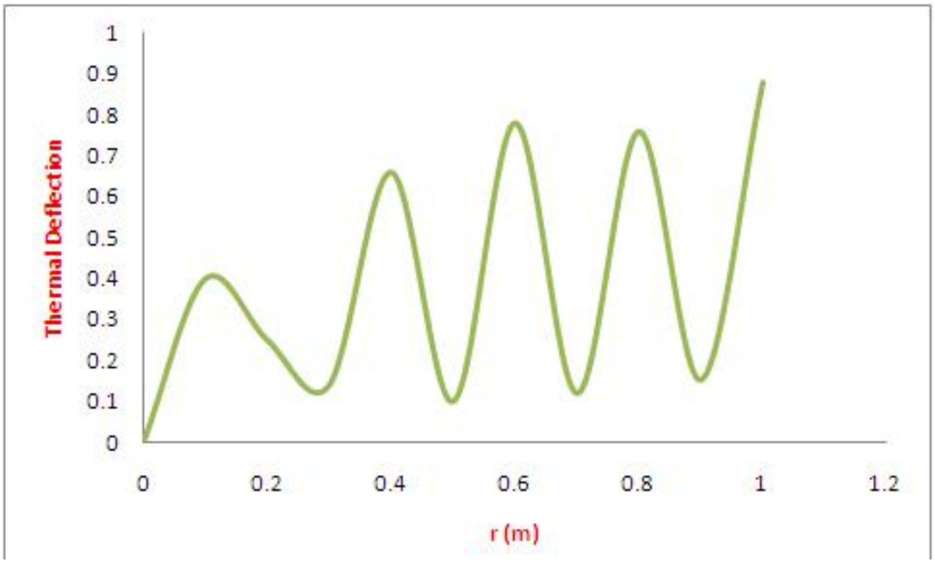


**Fig. 3** Quasi-static thermal deflection in circular plate

## V. Conclusions

In this work we have applied Marchi-Fasulo and Laplace integral transform to find the analytical solution of inverse transient heat conduction equations. The results are obtained in terms of Bessel's function in the form of Marchi-Fasulo transform series. The series solution converges provided we take sufficient number of terms in the series. Since the thickness of the plate is very small, the series solution given here will be definitely convergent.

Any particular case can be derived by assigning suitable values to the parameters and function in the series expressions. The temperature and quasi -static thermal deflection can be applied to the design of useful structures or machines in engineering applications. These types of inverse problems are very important in view of its relevance to various industrial machines subjected to heating such as the main shaft of lathe, turbines and the role of the rolling mill.

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