

Details of Constructed Laguerre Wavelet Transform with the Mathematical Framework

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Abstract:In this paper, the details of the construct Laguerre wavelet transform were identified with the important mathematical aspects. Several theorems were constructed that demonstrate the validity of the built in wavelet to deal with the details of the analysis based on the principle of multi resolution analyze. Where the discrete wavelet is built from the continuous wavelet and the decision tree is built The intermittent wave conversion packet to be used for image processing and the signal and examples shown will indicate the efficiency of the proposed theory where the wave is programmed to be ready for image and signal processing using the Matlab

Keywords -Approximation coefficients,detail coefficientsdiscrete Laguerre wavelettransform ,Laguerre wavelets transform, MATLAB program.

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I. INTRODUCTION

The theory of integrated transformation. The inverse of analysis is the integration of the signal multiplied by the basic analysis functions but the reconstruction process starts from, [9].

If $g=T(f)$ is the transform of f .

The question here, is whether it is possible to reconstruct f knowing g ?

In the appropriate circumstances depends on the function used with the features that qualify for that process then the answer is yes.[1-5]

It is possible to reconstruct or form desecrate f conversion, which requires many advantage in numerical terms. So on to reduce the number of separate information for reconstruction, leading to the concept of wavelet base after the introduction of the converted wave used this applicable to discrete Laguerre wavelet transform starting from the concept of multi resolution analysis of the norm space of the finite energy signals, [8].

The previous results period setting for signal analysis in the form of a succession of approximation with low of resolution supplemented by a series of details, [5-7].

In this paper it is possible to determine the best decomposition of a specific signal, which relates to the entropy criterion or standard by displaying the bases of the waveforms where the orthogonal rules are subject to orthonormal at the same time in order to improve the frequency accuracy of the wavelet analysis then suggest a richer associated with a large group of decomposition .

II. LAGUERRE WAVELETS TRANSFORM (LWT)

Laguerre wavelet is denoted by $(Lag)_{wav}$, is the type of wavelets and has used for solving differential equation, integral equation , variation problems and different sciences and engineering problems as well as fractional differential equation.That is why it is an important part. Laguerre wavelet $\rho_{n,m}(t) = \rho_{t,n,m,k}$ have

four arguments; $k = 1, 2, \dots$, $n = 1, 2, \dots, 2^{k-1}$, m is order for Laguerre polynomials and t is normalized time. If we dilation by parameter $s = 2^{-(k+1)}$ and translation by parameter $r = 2^{-(k+1)}(2n - 1)$ and use

$$\text{transform } x \text{ in } \rho_{s,r}(t) = |s|^{-\frac{1}{2}} \rho\left(\frac{t-r}{s}\right) \quad s, r \in R, \quad s \neq 0. \quad (1)$$

$x = 2^{-(k+1)}(2^k t)$, then we will get the following equation

$$\rho_{n,m}(t) = \begin{cases} 2^{k+1/2} \tilde{L}_m(2^k t - 2n + 1) & \frac{n-1}{2^{k-1}} \leq t \leq \frac{n}{2^{k-1}} \\ 0 & \text{otherwise} \end{cases} \quad (2)$$

Where $\tilde{L}_m = \frac{1}{m!} L_m$ for $k=2$, a function approximation $f(t) \in L^2[0,1]$ may be expanded as

$$f(t) = \sum_{n=1}^{\infty} \sum_{m=0}^{\infty} A_{n,m} \rho_{n,m}(t) \quad (3)$$

where

$$A_{n,m} = \langle f(t), \rho_{n,m}(t) \rangle \quad (4)$$

$t \in R$ and the frequency denoted by $\xi \in R$ the user space is square integrable functions (called signals) is denoted $L^2(R)$ then $\int_R |s(t)|^2 dt$ the square norm is called energy of the signal s . the quantities are often neglected.

Laguerre transform L_n in [3] we assume the inverse \bar{L}_n

$$1- \text{ The analysis step is } A(\xi) = (L_m f)(\xi) = \frac{1}{(m!)^2} \int_R f(t) L_m(t) e^{-\xi t} dt \quad \xi \in R$$

$$2- \text{ The synthesis step is } f(t) = (\bar{L}_m A)(t) = \int_R A(\xi) e^{+\xi t} dt \quad t \in R$$

The synthesis formula is formal moreover from the above steps we can say that Laguerre transform extends to finite energy functions and convergence in the synthesis formula is in $L^2(R)$ then expressed the synthesis mode is official, with f .

f is a function to be without additional assumptions and signal processing ($A(\xi)$) is often called the series of f , leading to $|A(\xi)|$ is the energy series.

The properties of signals

- 1- It is linearly
- 2- It is continuous
- 3- Above properties admits inverse transform \bar{L}_m keeps the angles and energies

Laguerre transform L_m is continuous linearly independent application of $L^2(R)$ on $L^2(R)$ then \bar{L}_m is thus

$$f = \bar{L}_m L_m f = L_m \bar{L}_m f$$

The inner product of all continuous real valued functions on norm space

$$\langle f, g \rangle_{L^2} = \langle L_m f, L_m g \rangle = \langle \bar{L}_m f, \bar{L}_m g \rangle$$

$$\text{In the norm form } \|f\|_{L^2} = \|L_m f\|_{L^2} = \|\bar{L}_m f\|_{L^2}$$

The functions of $t = e^{-\xi t}$ is atoms of this transform, if we test the analysis formula some of the abuses of the Laguerre transform appear. The following some negatives of L_m , the temporal parts of the function f vanish in A sure of that if take f is not continuous that mean impossible to test it by using A Lack of causality of Laguerre transform. If calculation of A this request f be defined in R , gradual calculation of the transform thus the analysis is impossible of real time, sure know the future of signal and we cannot even approximately know the spectrum A of signal f . Principle the doubt for Heisenberg, .

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4-

2.1. Details of analysis LWT

The general aspects of constructed LWT from the following steps Laguerre wavelet $\rho \in L^1 \cap L^2$ is a regular function and well, it proves the frequency domain with admissibility condition $\int_R \frac{|\rho(\xi)|^2}{|\xi|} d\xi \in [0, \infty)$ where

ρ is signals the Laguerre transform from $\rho_{n,m}$, this state includes in special case, the basic requirement the Laguerre wavelet combines to zero, fixed condition of Laguerre wavelet m vanishing moments proved by

$$\int_R t^r \rho(t) dt = 0 \quad \text{for } r = 0, 1, 2, \dots, m \quad (5)$$

the admissibility condition is enough and simpler to verify for Laguerre wavelet as long as

$$\rho \in L^1 \cap L^2 \quad t \in L^1 \quad \text{and} \quad \int_R \rho(t) dt = 0 \quad (6)$$

In order to move away from the result of zero, it is possible to believe that temporal factor plays a large role in the possibility of wave oscillation and translation because of damping. This means that the oscillation of apportion of waves can be measured equal to the number of evaporation moment allowing assessment of their localization by the interval. The resulting values vary greatly from zero.

From [3] the function f defined in R in finite interval we can defined its continuous wavelet transform by the function

$$A_f(s, r) = \int_R f(t) \rho_{s,r}(t) dt \quad s \in R^+, r \in R \quad (7)$$

The calculation of the coefficients of Laguerre wavelet is equal to the analysis of f with the Laguerre wavelet ρ , which is called continuous functions and whose significance is shown in the measurement of the function fluctuations at scale s. this is to eliminate the trend in a rang containing the developments in $A_f(s, r)$

. The comparison to the transaction values of the value s even if we assume that zero is out of the interval $\left[\frac{n-1}{2^{k-1}}, \frac{n}{2^{k-1}} \right]$ so zero is outside $\left[\frac{(n-1)s}{2^{k-1}} + r, \frac{ns}{2^{k-1}} + r \right]$, we obtained a family value which depends on the value f in the neighborhood with the length corresponding to the s.

In qualitative terms, large values of $A_f(s, r)$ provides information on local irregularity f around position r and scale s see [3].

In particular, in the orthogonal with use scaling function denoted by \mathcal{G} is associated with ρ , dilated and translate it as ρ . The \mathcal{G} function is for local approximations what the ρ function is for fluctuations.

The following two basic atoms of wavelets called the Laguerre wavelet re call (2).

$$\begin{cases} \rho_{n,m}(t) = 2^{\binom{k+1}{2}} \rho(2^k t - n + 1) \quad \text{for } (n, m) \in Z^2 \\ \mathcal{G}_{n,m}(t) = 2^{\binom{k+1}{2}} \mathcal{G}(2^k t - n + 1) \quad \text{for } (n, m) \in Z^2 \end{cases} \quad (8)$$

The wavelet coefficients of a signal S.

$$\begin{cases} \lambda_{n,m} = \int_R S(t) \rho_{n,m}(t) dt \\ \gamma_{n,m} = \int_R S(t) \mathcal{G}_{n,m}(t) dt \end{cases} \quad (9)$$

under certain conditions we used these coefficients are to reconstruct the signal

$$S(t) = \sum_{n \in Z} \sum_{m \in Z} \lambda_{n,m} \rho_{n,m}(t) \quad (10)$$

The family $\{\rho_{n,m}\}_{(n,m) \in Z^2}$ is an orthonormal basis of $L^2(R)$ in the concept of multi resolution analysis

(MRA) with finite energy is a sequence $\{V_n\}_{n \in Z}$

... $\subset V_2 \subset V_1 \subset V_0 \subset V_{-1} \subset V_{-2} \subset \dots$ of $L^2(\mathbb{R})$ these spaces its central space V_0 by contraction for $n < 0$.
 Let $f(t) \in V_n \Leftrightarrow f(2t) \in V_{n-1}$ for $n \in \mathbb{Z}$ the function \mathcal{G} of V_0 which generates V_0 by integer translations

$$V_0 = \left\{ f \in L^2(\mathbb{R}) \mid f(t) = \sum_{n \in \mathbb{Z}} A_{n,m} \mathcal{G}(2^k t - 2n + 1) \in l^2(\mathbb{Z}) \right.$$

where the \mathcal{G} function is the scaling function of Laguerre wavelet multi resolution analysis of Laguerre wavelet $(MRA)_{Lag}$ the subspace $\{V_n\}_{n \in \mathbb{Z}}$ of $(MRA)_{Lag}$ is approximation spaces with detail spaces noted as $\{W_n\}_{n \in \mathbb{Z}}$, W_n space is the orthogonal of V_n in V_{n-1} , $V_{n-1} = V_n \oplus W_n$, $n \in \mathbb{Z}$ the approximation space of level n-1 is decomposed in to approximation at level n, the scalar function \mathcal{G} and the family $\{\mathcal{G}_{n,m}\}_{m \in \mathbb{Z}}$ generate V_n in the same time ρ , $\{\rho_{n,m}\}_{m \in \mathbb{Z}}$ generated W_n , and $L^2(\mathbb{R}) = \oplus W_n$, $n \in \mathbb{Z}$ then S is the sum of the details and $\{\rho_{n,m}\}_{(n,m) \in \mathbb{Z}^2}$ is orthonormal wavelet basis of $L^2(\mathbb{R})$. Thus $\lambda_{n,m}$ is the coefficient orthogonal projection of S on to W_n .

The coefficients of Laguerre wavelet is approximation $\gamma_{n,m} = \int_R S(t) \mathcal{G}_{n,m}(t) dt$ is an approximation of

$$A_n(t) = \sum_{m \in \mathbb{Z}} \gamma_{n,m} \mathcal{G}_{n,m}(t) \quad (11)$$

$$D_n(t) = \sum_{m \in \mathbb{Z}} \lambda_{n,m} \rho_{n,m}(t) \quad (12)$$

The function of calculation the coefficients of Laguerre wavelet $\lambda_{n,m} = \int_R S(t) \rho_{n,m}(t) dt$ is the details of

The following four important definitions

- 1- Detail coefficients $\lambda_{n,m} = \int_R S(t) \rho_{n,m}(t) dt$ is Laguerre wavelet coefficients
- 2- The detail signals $D_n(t) = \sum_{m \in \mathbb{Z}} \lambda_{n,m} \rho_{n,m}(t)$
- 3- Approximation coefficients $\gamma_{n,m} = \int_R S(t) \mathcal{G}_{n,m}(t) dt$
- 4- Approximation signals $A_n(t) = \sum_{m \in \mathbb{Z}} \gamma_{n,m} \mathcal{G}_{n,m}(t)$

The coefficients $\lambda_{n,m}, \gamma_{n,m}$ $n, m \in \mathbb{Z}$ in the level n, the orthogonal Laguerre wavelet projection on to spaces W_n, V_n of S the signal of the coefficients $\{\gamma_{0,m}\}_{m \in \mathbb{Z}} \in V_0$, $\{\lambda_{n,m}\}_{m \in \mathbb{Z}}$ are the coefficients of Sin the two spaces W_n, V_n when D_n and A_n belong to W_n, V_n in V_0 .

III. Decision Tree Of Signal From Lwt

The signal S is found its root started analysis signal from the approximation $A_0 = S$ in level n=0 second step will get the detail in level 1.

$$D_1 = A_0 - A_1 = s - A_1$$

$$S = D_1 + A_1$$

And in level 2

$$D_2 = A_1 - A_2$$

$$= S - D_1 - A_2$$

$$S = D_2 + D_1 + A_2$$

If we return in continuous time starting from [3] $S = f_{n,m}$ and $\gamma_{n,m} = A_{n,m}$

$$S = \sum_{n=1}^{2^{k-1}} \sum_{m=0}^{M-1} \lambda_{n,m} \rho_{n,m}(t) \quad (13)$$

The above equation means we can reconstruct the signal from its coefficients $\lambda_{n,m}$, with use it in define the detail at level n differently.

Let take n with using m we can find the detail D_n , in the general this sum defines will be point to as the approximation at level N of signal S and

$$S = A_N + \sum_{n \leq N} D_n \quad (14)$$

The equation (14) means $A_{N-1} = A_N + D_N$, the family $\{\rho_{n,m}\}_{n,m \in \mathbb{Z}}$ is orthogonal, the following some results are taken from above

- 1- A_N is the approximation in level N and orthogonal to $D_N, D_{N-1}, D_{N-2}, \dots$
- 2- S is the sum of two orthogonal signals A_N and $\sum_{n \leq N} D_n$ the quality Q_N of the approximation of S by

A_N is equal to

$$Q_N = \frac{\|A_N\|^2}{\|S\|^2}, Q_{N-1} = Q_N + \frac{\|D_N\|^2}{\|S\|^2} \quad (15)$$

IV. DISCRETE LAGUERRE WAVELET TRANSFORM PACHETS (Dlwtp)

Wavelet packets are a generalization of orthogonal wavelets allow a smooth analysis by breaking up detail spaces, which are never decomposed in the case of wavelets.

Started from two relations by Multi Resolution Analysis with $\mathcal{G} \in V_0$, $\mathcal{G}_{0,m} = \mathcal{G}(t-m)_{m \in \mathbb{Z}}$ is an orthonormal base of V_0 then $\exists! s = \{s_m\}_{m \in \mathbb{Z}}, s \in L^2(\mathbb{Z})$ such that

$$\frac{1}{2} \mathcal{G}\left(\frac{1}{2}\right) = \sum_{m \in \mathbb{Z}} s_m \mathcal{G}(t-m) \in L^2 \quad (16)$$

and the second relation $\rho \in W_0$, $\rho_{0,m} = \rho(t-m)_{m \in \mathbb{Z}}$ is an orthonormal base of W_0 then

$\exists! r = \{r_m\}_{m \in \mathbb{Z}}, r \in L^2(\mathbb{Z}) \in L^2$ such that

$$\frac{1}{2} \rho\left(\frac{1}{2}\right) = \sum_{m \in \mathbb{Z}} r_m \rho(t-m) \in L^2 \quad (17)$$

in the interval $[0,1] \in L^2(0,1) \rightarrow \{\rho_{n,m}\}_{n,m \in \mathbb{Z}}$ is an orthonormal base of $L^2(\mathbb{R})$

that mean the DLWTP are generated by recurrence, the following steps shows the stages of construction DLWTP.

The two filters g_m, h_m with length 2M associated with orthogonal Laguerre wavelet compact support ρ and scaling function \mathcal{G} MRA of $L^2(\mathbb{R})$ this point obtained the basis of sequences $\{s_m\}_{m \in \mathbb{Z}}$ and $\{r_m\}_{m \in \mathbb{Z}}$ in l^2 norm equal 1. Define the sequence of functions $(L_n)_{n \in \mathbb{N}}$ starting $L_0 = \mathcal{G}$ the scaling function

$$\begin{cases} L_{2m}(t) = \sqrt{2} \sum_{m=0} h_m L_m(2t-n) \\ L_{2m+1}(t) = \sqrt{2} \sum_{m \neq 0} g_m L_m(2t-n) \end{cases} \quad (18)$$

$L_0 = \mathcal{G}$ the scaling function m=0 and $L_1 = \rho$ the wave of Laguerre wavelet in (19)

$$h_0 = h_1 = \frac{1}{\sqrt{2}}, g_0 = -g_1 = \frac{1}{\sqrt{2}} \quad (19)$$

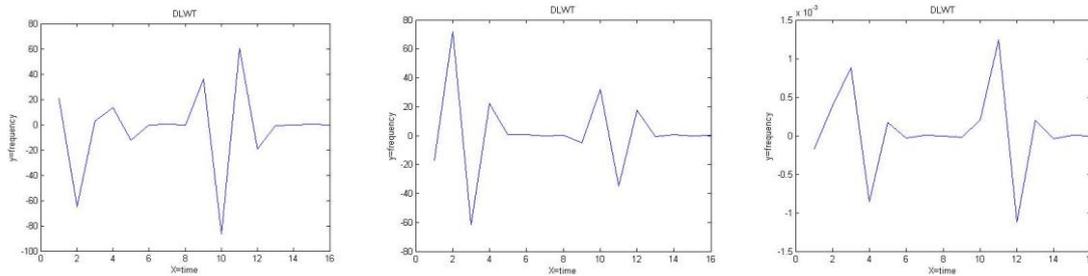
Let L_0 is the Laguerre scaling function and L_1 is the Laguerre wavelet in $[0,1)$

$$\begin{aligned} L_{2m}(t) &= L_m(2t) + L_m(2t-1) \\ L_{2m+1}(t) &= L_m(2t) - L_m(2t-1) \end{aligned} \quad (20)$$

- 1- We obtain L_{2m} by two copies of L_m contracted in $[0, 0.5)$ for $L_m(2t)$ and $[0.5, 1)$ for $L_m(2t-1)$ we obtain L_{2m+1} .
- 2- Subtraction the same versions $m=0,1,\dots,8$ we obtained the functions $\rho_{n,m}(t)$ the basis of DLWTP is regular and orthogonal wavelet.

The result of smoothed relation for regular function in the interval $[0, 2M-1]$ Fig.8.4 the DLWT in (16×16)

then the set $\{L_m(2^k - 2n + 1)\}_{n,m \in \mathbb{Z}}$ of $L^2(\mathbb{R})$ for examples



$$\rho_{1,0}(t) \rho_{1,1}(t) \rho_{1,2}$$

4.1. Atoms of DLWTP:

From deletion and translation of Scalar of Laguerre function and wavelet of Laguerre function \mathcal{G}, ρ consequently and the atoms of LWT are constructed.

For DLWTP we go to a head in like mode on the basis of the functions $(L_m; m \in \mathbb{N})$ we can index almost for arguments or signals

$$(L_m)_{j,n}(t) = 2^{-\frac{j}{2}} L_m(2^j t - n) \quad \text{for } m \in \mathbb{N}, (j,n) \in \mathbb{Z}^2 \quad (21)$$

If we fixed j the useful values n for $0 \leq n \leq 2^j - 1$, because these atoms are not all useful, in case k is the location parameter j is the scale parameter.

So we can as k the following question about the interpretation of m , what means of j ?

The concept for fixed values of j and n , $(L_m)_{j,n}$ analyzes, the signal around position will have fluctuations,

this position is $k 2^j$ the scale 2^j and all the frequencies corresponding to the various useful values of the last parameter m .

If we attentively testing discrete Laguerre wavelet transform packets linked with ρ offered already, the normal seek or require of the function $(L_m; m = 0, 1, \dots, 16)$ doesn't completely with the number of vibrations above L_m times, which makes us believe that the re arrangement of the wave is useful for the analysis of the signal.

The set of functions $P_{j,n} = (L_m)_{j,n}(t); m \in \mathbb{Z}$ include the (j,n) Laguerre Wavelet Packet. It is natural that the organization of DLWTP is in the form of a tree if we have a level of analysis equal to 2, we will get with respect of the possible values of the integers from 0 to $2^j - 1$, moreover of depth in the tree which signal the position in the tree or in the other words, the registration $P_{j,n}$ used is regular registration of tree nodes labeling.

By functions of DLWTP $P_{j,n}$ as $\bar{P}_{n,j}$ will note the space generate $\forall j$

$\bar{P}_{j,0} = V_j$ and $\bar{P}_{j,1} = W_j$ the library of DLWTP bases contains the Laguerre wavelet bases, with $V_0 = \bar{P}_{0,0}$. So we have

- $P_{r,1}; r \geq 1$ is an orthogonal base of V_0

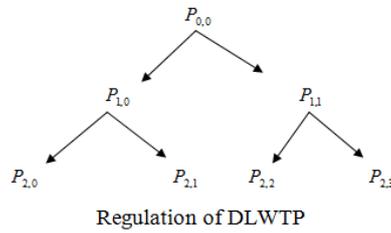
- $N \geq 0$ ($P_{N,0}, (P_{j,1}; j \in [1, N])$) is an orthogonal base of V_0

Levels

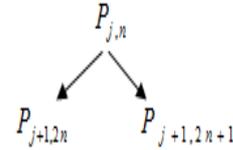
j=0

j=1

j=2



Regulation of DLWTP



Division and merge in the tree of DLWTP

The property that gives the meaning of the division is the orthogonal property of $(P_{j+1,2n}, P_{j+1,2n+1})$ where the merge is then reversed in a tree of DLWTP where we can put the following form for each node.

Finally with respect to the finite energy signal of V_0 for each discrete Laguerre wavelet packet basis shows a fixed method of exemplification that mean from the set of leaves of any binary tree agree to an orthogonal wavelet packet basis of the initial space, after that we can choose the decomposition best favorable to a signal.

If we applied equation (21) with levels 0,1,2 that mean take $j=0,1,2$ we have

$$\left\{ \begin{array}{l} j = 0 \\ \rho_{0,0}(t) = L_0(t) = 1 \subset V_0 \\ j = 1 \\ \rho_{1,0}(t) = \frac{1}{\sqrt{2}} L_0(2t) = \frac{1}{\sqrt{2}} \\ \rho_{1,1}(t) = \frac{1}{\sqrt{2}} L_1(2t-1) = \frac{1}{\sqrt{2}} (1-(2t-1)) \\ j = 2 \\ \rho_{2,0}(t) = \frac{1}{\sqrt{2}} L_0(4t) = \frac{1}{\sqrt{2}} \\ \rho_{2,1}(t) = \frac{1}{\sqrt{2}} L_1(4t-1) = \frac{1}{\sqrt{2}} (1-(4t-1)) \\ \rho_{2,2}(t) = \frac{1}{\sqrt{2}} L_2(4t-2) = \frac{1}{\sqrt{2}} ((4t-2)^2 - 4(4t-2) + 2) \\ \rho_{2,3}(t) = \frac{1}{\sqrt{2}} L_3(4t-3) = \frac{1}{\sqrt{2}} (6 - 18(4t-3) + 9(4t-3)^2 - (4t-3)^3) \\ n = 0,1,2, \dots, 2^j - 1 \quad j = 1,2,3, \dots \end{array} \right.$$

Then

$$\mathcal{G}_{j,n}(t) = \begin{cases} 1 & t \in [0,1) \\ 0 & \text{otherwise} \end{cases} \text{ the scale function of DLWT}$$

$$f(t) = A_0 + \sum_{j=0}^{\infty} \sum_{n=0}^{2^j-1} A_{j,n} \rho_{j,n}(t) \tag{22}$$

4.2. Four Filters Associated with DLWT

From wavelet toolbox by using Matlab program using the "wfilters" function
`[LO_D,HI_D,LO_R,HI_R] = wfilters('lgw');`

Will get four filters by using (8×8) matrix in $[0,1)$

$$\begin{array}{l} \text{LO_D} = 0.4527 \quad 0.3402 \quad 0.2459 \quad 0.1693 \quad 0.1087 \quad 0.0623 \quad 0.0288 \quad 0.0064 \\ \text{HI_D} = -0.0064 \quad 0.0288 \quad -0.0623 \quad 0.1087 \quad -0.1693 \quad 0.2459 \quad -0.3402 \quad 0.4527 \\ \text{LO_R} = 0.0064 \quad 0.0288 \quad 0.0623 \quad 0.1087 \quad 0.1693 \quad 0.2459 \quad 0.3402 \quad 0.4527 \\ \text{HI_R} = 0.4527 \quad -0.3402 \quad 0.2459 \quad -0.1693 \quad 0.1087 \quad -0.0623 \quad 0.0288 \quad -0.0064 \end{array}$$

The following figure illustrates the scaling and wavelet functions by using the following functions

```
wname = 'mywa';
wavefun(wname,'plot',7);
a = findobj(gcf,'Type','axes');
axis(a,'tight')
```

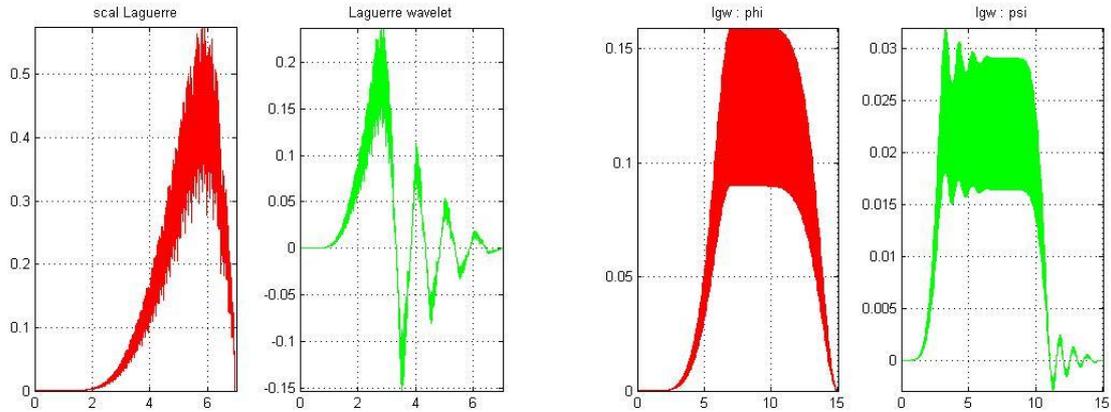


Fig.1 analysis DLWT (8×8) Fig.2 analysis DLWT (16×16)

Obtain the first 4 wavelet packets for the Discrete Laguerre wavelet with 4 vanishing moments.

By using

```
[wpws,x] = wfun('lgw',4,10);
formn = 1:size(wpws,1)
subplot(3,2,nn)
plot(x,wpws(nn,:)); axis tight;
title(['W',num2str(nn-1)]);
end
```

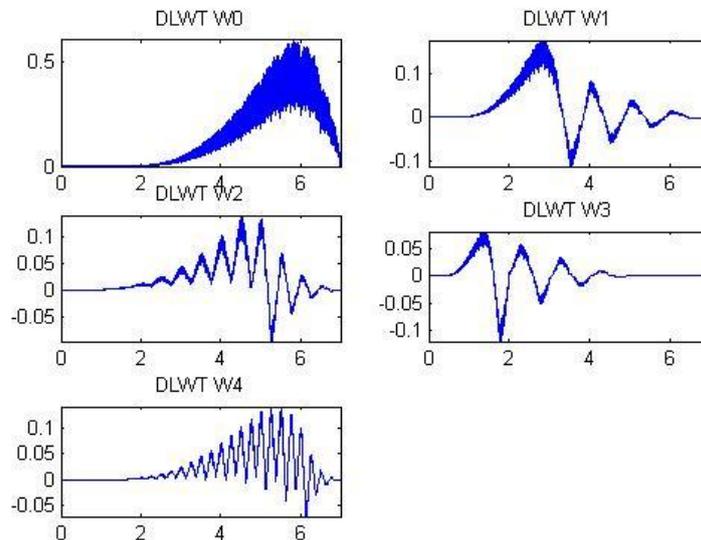


Fig.3 analysis DLWT level 4

When using the Wavelet and the Wavelet Packet Display tools from the "wavemenu" WT, the new wavelet is also available. Here is a screen shot of the Wavelet Display Tool.

The following figure illustrates all the DLWT analysis functions by using load signal in matlab program in level 3

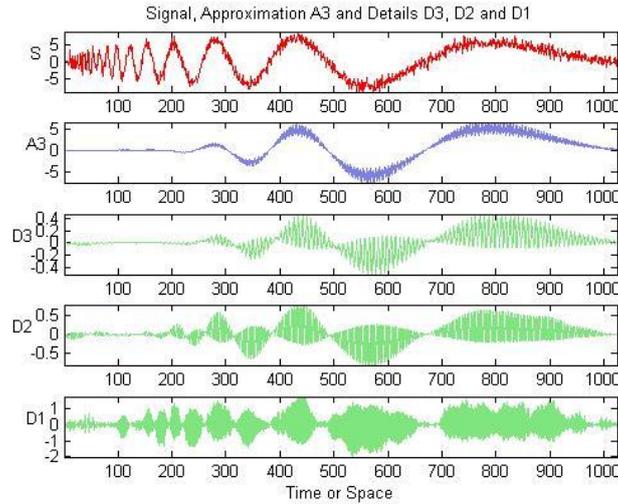


Fig.4 analysis DLWT with load signal

The signals from top to bottom in the figure above are S (the signal), A2 (Approximation of level 2), D2 (Detail of level 2) and D1 (Detail of level 1). If the wavelet is an orthogonal wavelet, the reconstruction must be perfect, which is, $S = A2 + D2 + D1$

This is the case as we can see below level 2 with norm $\left(\frac{1}{\sqrt{2}}\right)$ and without norm Respectively .

$\text{maxdiff} = \max(\text{abs}(x - (A2 + D1 + D2)))$

1. $\text{maxdiff} =$

1.1421

2. $\text{maxdiff} =$

0.8093

We can now use this new DLWT orthogonal wavelet to analyze a signal.

`loadnoisdopp; % Load a signal`

`x = noisdopp;`

`scales = 1:1:128;`

`coefs = cwt(x,scales,wname);`

`clf; wscalogram('image',coefs,'scales',scales,'ydata',x);`

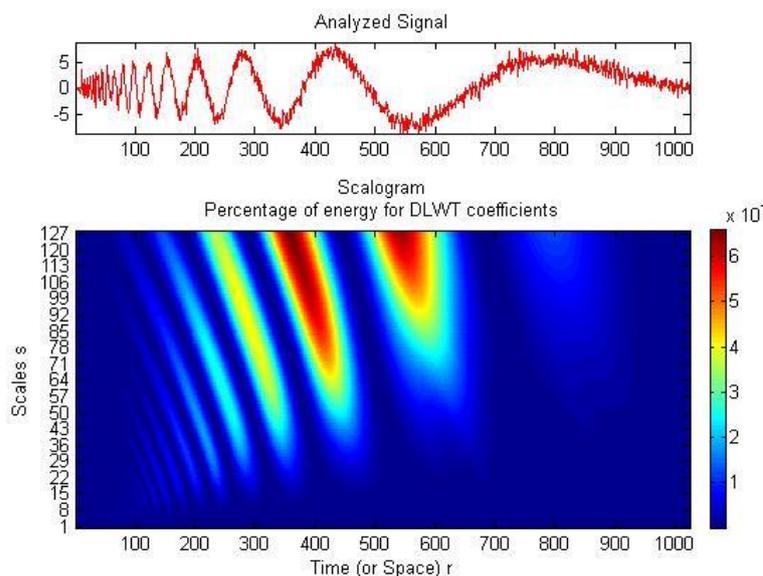


Fig.5 Doppler signal analysis by DLWT

DLWT Used to Analyze an Image

The DLWT can also be used to analyze an image by using

```
load mask; % Load an image
[cA,cH,cV,cD] = dlwt2(X,wname);
clf; imagesc(abs([cAcH ; cVcD]));
clearcAcHcVcD
```

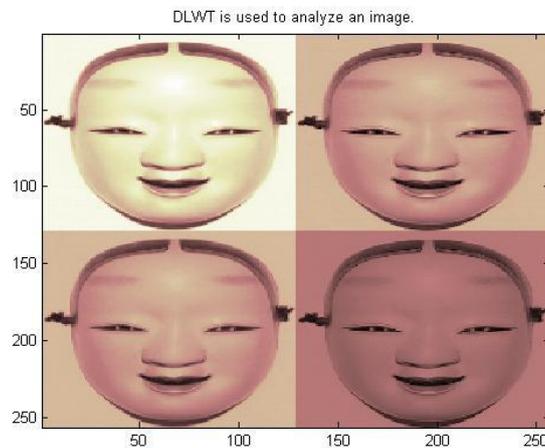


Fig.16 analysis DLWT with loadMask

V. CONCLUSION

In this work we identified how to obtain the wave forms of DLWT with its behavior with the organization of Laguerre wavelet packets and by calculate those packets that will obtain in the coming works, which show us efficiency of the DLWT used it in image processing, Moreover we proved possibility of add our wavelets in wavelet toolbox by using matlab program.

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