

# Maximization of Profit in a Product Mix Company Using Linear Programming

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**Abstract :** This paper illustrates the use of linear programming methods as applied in the manufacturing industry. We collected data from the records of KASMO industry limited, Osogbo, Nigeria, on four types of sales packages adopted for selling her medicated soap product which include one (1) tablet per pack, three (3) tablets per pack, twelve (12) tablets per pack and one hundred and twenty (120) tablets per pack. Information on selling price per pack and on the cost of five (5) basic raw materials used couple with the quantity of each of the raw materials held in stock per month for the production of medicated soap tablets were available in the records of the company. In accordance with the costs of raw materials, the maximum profit that would accrue to the company given the product mix was determine. The results showed that the company would attain optimum profit level monthly of about N271,296 if she concentrates mainly on the unit sales (one tablet per pack) of her medicated soap product, ignoring other forms of sales packages.

By this, let her sales turn-over per month would be about 18,900 soap tablets. The analysis was carried out with the help of TORA software.

**Key words:** Linear programming, simplex method, objective function, product mix, tora soft ware.

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## I. Introduction

We defined linear programming (LP) as a mathematical method used in operation research (OR) or limited resources to attain optimum goal it is also seen as a mathematical method used in operation research (OR) or business science to solve specific types of problems such as allocation, transportation and assignment problems that allows a choice or choices between alternative courses of action (Yanya, 2004) Linear programming is a term that covers a whole range of mathematical methods that is aimed at optimizing performance in terms of combinations of resources (Lucey, 1996). Linear programming being the most important, it is designed for models with linear objective and constraint function ALP models can be designed and solved to determine the best course of action as in a product mix subject to the available constraints.

In general, the objective function may be of maximization of profit (which is the focus of this paper) or minimization of costs or labour hours. Moreover, the model also consists of certain structural constraints which are sets of conditions that the optimal solution should justify. Examples of the structural constraints include the raw material constraints production time constraint and skilled labour constraints to mentions just a few. An optimum solution is a solution that fulfils both the constraints of the problem and the set objective to be met.

The term “Linear” as stated by Akingbade (1996), means proportionality, which implies that the elements in a situation are so connected that they appear as a straight line when plotted on a graph. While the “programming” shows the solution methods which can be carried out by an interactive process in which a researcher advances from one solution to a better solution until a final solution is reached which cannot be improved upon. This final solution is termed the optimal solution of the LP problem.

This paper shows the pragmatic use of LP methods in a manufacturing company in Nigeria- KASMO industry limited. The problem addressed here was to obtain the product mix (combination of sales package) to be adopted by the company for selling her medicated soap product at which the optimal profit level would be attained.

## II. Materials and methods.

The dataset used for this paper was collected as extracts from the records of KASMO industry limited, Osogbo, Osun state Nigeria on her main product (Medicated soap) and four different types of sales packages adopted for selling her soap product in 2010. These four(4) types of sales packages include sales in one (1) soap tablet (unit sales) sales in three (3) soap tablets (three (3) tablets per pack) sales in dozen soap tablets (twelve

(12) tablets per pack), and sales in the selling price per unit soap tablet as the number of unit per pack increases. This was designed to motivate wholesales purchase of the soap product by the users.

Data on the five basic raw materials used for the manufacturing of KASMO medicated soap were available in the records of the company. Caustic soda, palm kernel oil (PKO), colourant, disinfectant and perfume are the raw materials used in the production of the mediated soap. Information on the quantity of each raw material held in stock pro month probably due to space or financial constraint was obtained. Also information on the quantity mix of these basic five raw material and their costs for effective manufacturing of the soap product was equally available and obtained from the company's records. Finally, data were collected on the quoted selling prices of the four sales packages of soap adopted by the company. However, information respect to labour cost, sale and marketing expenses and other related overheads were not obtainable and as a matter of fact, their effects were neglected in the analysis. Therefore, the only cost element considered for soap manufacturing in this paper is the cost of raw materials.

In a nutshell, the main aim of this paper is to determine the quality of each of the four sales packages that will maximize the profit of the company given. The afore mentioned raw material constraints.

The analysis was carried out using Linear programming (LP) techniques or methods. The LP problem developed here is a mathematical program in which the objective function is linear in the unknown variables and the constraints have linear equation or linear inequality or both. The general form of a LP problem is stated in matrix form as shown below;

Objective function maximize  $Z = CX$  .....(1)

Linear constraints: subject to  $AX=b$  .....(2)

Non-negativity condition  $X > 0$ .....(3)

The C in (1) is a row vector of m-dimension representing the objective function coefficients X is a  $M \times 1$  column vector of the decision variables of the LP model, A is an  $MXK$  matrix of coefficients and b is an  $K \times 1$  column vector of values in the RHS of the constraint equations in (2).

Based on the analysis in this paper this number of decision variables is four (4) setting  $M=4$  in (1). The four (4) decision variables  $X_1, X_2, X_3$  and  $X_4$  in vector X in the objective function represent the four (4) types of sales packages of soaps adopted by the company with  $X_1$  represent sales in one (1) tablet (unit sales).

$X_2$  represents sales in three (3) tablets per pack,  $X_3$  represents sales in twelve (12) tablets per pack, while  $X_4$  represents sales of soaps in carton (120 tablets per pack).

Also, since the company uses five (5) different raw materials for the manufacturing of her medicated soaps, therefore, there are five (5) linear constraints for the LP model, setting  $K=5$  in (2). The whole analysis was performed using Tora software.

### III. Data Analysis

The data collected from KASMO industry limited (KIL) Osogbo, Nigeria on her main product line medicated soap were analysed to determine the best sales package that would yield maximum profit to the company. All the information provided in tables 1 through 4 was used to form the linear programming model of the maximize objective function (profits):

$$Z - 14.36X_1 + 33.08X_2 + 112.32X_3 + 1023.20X_4$$

Subject to (raw materials constraints).

$$\text{Caustic soda: } 0.89X_1 + 2.67X_2 + 10.68X_3 + 106.79X_4 + X_5 = 16820$$

$$\text{Palm Kernel oil: } 2.15X_1 + 6.44X_2 + 25.78X_3 + 257.78X_4 \leq 40600$$

$$\text{Colourant: } 0.0015X_1 + 0.0046X_2 + 0.018X_3 + 0.184X_4 \leq 29$$

$$\text{Perfume: } 0.0061X_1 + 0.018X_2 + 0.074X_3 + 0.737X_4 \leq 116$$

$$\text{Disinfectant: } 0.028X_1 + 0.083X_2 + 0.331X_3 + 3.314X_4 \leq 522$$

$$X_1, X_2, X_3, X_4 \geq 0$$

In order to represent the above LP model in canonical form, five (5) slack variables  $X_i$  ( $i=5,6,7,8,9$ ) were introduced into the model. This changed the inequalities signs in the constraint aspect of the model to equality signs. A slack variable will account for the unused quantity of raw material (if any) it represents at end of the production. As a result, the above linear programming (LP) model becomes that of

$$\text{Maximize } = 14.36X_1 + 33.08X_2 + 112.32X_3 + 1023.20X_4$$

Subject to (raw material constraints)

$$\text{Caustic soda: } 0.89X_1 + 2.67X_2 + 10.68X_3 + 106.79X_4 + X_5 = 16820$$

$$\text{Palm kernel oil: } 2.15X_1 + 6.44X_2 + 25.78X_3 + 257.78X_4 = 40600$$

$$\text{Colourant: } 0.0015X_1 + 0.0046X_2 + 0.018X_3 + 0.184X_4 = 29$$

$$\text{Perfume: } 0.0061X_1 + 0.018X_2 + 0.074X_3 + 0.737X_4 = 116$$

With non-negativity constraint that  $X_1, X_2, X_3, X_4, X_5, X_6, X_7, X_8, X_9 \geq 0$

For analysis proper the simplex method proposed by George B. Dantzig in 1947 as published in Dantzig (1963) was adopted to solve the above LP problem. The simplex method has been found to be more efficient and

convenient for computer software implementation in many instances (Yahya, 2004). The initial table of the above LP model as used by simplex algorithm is given by table 14m Appendix A. The Tora software was employed to fit the LP model. The result obtained from this analysis are presented in the next section.

#### **IV. Results**

Result from the analysis carried out on the LP model in previous section using simplex method estimated the value of the objective function to be 271, 2964. The contributions of the four decision variables  $X_1$ ,  $X_2$ ,  $X_3$  and  $X_4$  into the objective function are 18893, 0.0 and 0 respectively. This simply indicates that only  $X_1$  variable contributed meaningfully to improve the value of the objective function of the LP model.

It is observed from the result's output in the appendix that the optimal solutions programming model fitted here was attained at the first iteration. This simply shows the goodness of the structure of the data collected for analysis.

#### **V. Discuss and Conclusion**

The appropriateness of the linear programming methods for optimal resource allocation in industry has been demonstrated in this work. This is evident from the results obtained from the profit maximization type of the LP model fitted to the data collected on soap manufacturing from KASMO Industry Limited Osogbo, Nigeria.

From the result of the LP model in section 3 as reported in section 4, it is desirable for the KIL company to concentrate on the unit sales (1 soap tablet per pack) of her soap products. By this, total sales of about 18.893 tablets would be sold by the company per month. This would fetch the company an optimal profit of about N271,296.4 per month based on the costs of raw materials only. A simple division of the value of the objective function, N271,296.4 by 18,893 soap tablets sold based on 1 tablet per pack strategy (as shown in Table 3) yields a profit of N14.36 per soap tablet. This value agreed perfectly with the profit expected by the company on the sales of a soap tablet as contained in Table 4.

The result of the LP model fitted to the data collected from KIL are only based on the cost of raw materials used for soap production. Therefore, it is quite instructive to remark that if information on other elements of cost of production such as labour and overhead costs is available and incorporated into the LP model formulation and analysis, the results reported here might be remarkably different. Nonetheless, findings from this work could still serve as useful guides to the management of KIL in the formulation of production and marketing strategies for their soap product.

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Appendix A

TORA C:\Users\ARINZE MICHAEL\Documents\Kasmo.txt

LINEAR PROGRAMMING

Final Iteration All Iterations Write to Printer

Variable	Value	Obj Coeff	Obj Val Coeffs
x1: Tab1	18892.51	14.36	271296.42
x2: Tab2	0.00	33.08	0.00
x3: Tab3	0.00	112.32	0.00
x4: Tab4	0.00	1023.20	0.00

Constraint	RHS	Slack / Surplus
1 (-)	16820.00	6.61-
2 (-)	40600.00	16.06-
3 (-)	29.00	0.09-
4 (-)	116.00	0.00
5 (-)	522.00	0.19-

\*\*\* Sensitivity Analysis \*\*\*

Variable	Current Obj Coeff	Min Obj Coeff	Max Obj Coeff	Reduced Cost
x1: Tab1	14.36	11.03	infinity	0.00
x2: Tab2	33.08	infinity	43.06	9.98
x3: Tab3	112.32	infinity	172.25	59.93
x4: Tab4	1023.20	infinity	1722.52	699.32

Constraint	Current RHS	Min RHS	Max RHS	Dual Price
1 (-)	16820.00	16813.39	infinity	0.00
2 (-)	40600.00	40583.94	infinity	0.00
3 (-)	29.00	28.91	infinity	0.00
4 (-)	116.00	0.00	116.04	2338.76
5 (-)	522.00	521.81	infinity	0.00

View/Modify Input Data MAIN Menu Exit TORA

3 TORA C:\Users\ARINZE MICHAEL\Documents\Kasmo.txt

LINEAR PROGRAMMING

Final Iteration All Iterations Write to Printer

Variable	Value	Obj Coeff	Obj Val Coeffs
5 (-)	522.00	0.19-	

\*\*\* Sensitivity Analysis \*\*\*

Variable	Current Obj Coeff	Min Obj Coeff	Max Obj Coeff	Reduced Cost
x1: Tab1	14.36	11.03	infinity	0.00
x2: Tab2	33.08	infinity	43.06	9.98
x3: Tab3	112.32	infinity	172.25	59.93
x4: Tab4	1023.20	infinity	1722.52	699.32

Constraint	Current RHS	Min RHS	Max RHS	Dual Price
1 (-)	16820.00	16813.39	infinity	0.00
2 (-)	40600.00	40583.94	infinity	0.00
3 (-)	29.00	28.91	infinity	0.00
4 (-)	116.00	0.00	116.04	2338.76
5 (-)	522.00	521.81	infinity	0.00

View/Modify Input Data MAIN Menu Exit TORA

## Maximization Of Profit In A Product Mix Company Using Linear Programming

Linear Programming Solver: Worksheet: Simplex  
 Solver Parameters  
 Set Objective: \$D\$10 To: =SUM(\$C\$5:\$C\$7) (Max)  
 Variable Variable Cells: \$B\$5:\$B\$7  
 Subject to the Constraints:  
 \$D\$8:\$D\$10 <= \$E\$8:\$E\$10  
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 \$D\$545:\$D\$547 <= \$E\$545:\$E\$547  
 \$D\$548:\$D\$550 <= \$E\$548:\$E\$550  
 \$D\$551:\$D\$553 <= \$E\$551:\$E\$553  
 \$D\$554:\$D\$556 <= \$E\$554:\$E\$556  
 \$D\$557:\$D\$559 <= \$E\$557:\$E\$559  
 \$D\$560:\$D\$562 <= \$E\$560:\$E\$562  
 \$D\$563:\$D\$565 <= \$E\$563:\$E\$565  
 \$D\$566:\$D\$568 <= \$E\$566:\$E\$568  
 \$D\$569:\$D\$571 <= \$E\$569:\$E\$571  
 \$D\$572:\$D\$574 <= \$E\$572:\$E\$574  
 \$D\$575:\$D\$577 <= \$E\$575:\$E\$577  
 \$D\$578:\$D\$580 <= \$E\$578:\$E\$580  
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 \$D\$584:\$D\$586 <= \$E\$584:\$E\$586  
 \$D\$587:\$D\$589 <= \$E\$587:\$E\$589  
 \$D\$590:\$D\$592 <= \$E\$590:\$E\$592  
 \$D\$593:\$D\$595 <= \$E\$593:\$E\$595  
 \$D\$596:\$D\$598 <= \$E\$596:\$E\$598  
 \$D\$599:\$D\$601 <= \$E\$599:\$E\$601  
 \$D\$602:\$D\$604 <= \$E\$602:\$E\$604  
 \$D\$605:\$D\$607 <= \$E\$605:\$E\$607  
 \$D\$608:\$D\$610 <= \$E\$608:\$E\$610  
 \$D\$611:\$D\$613 <= \$E\$611:\$E\$613  
 \$D\$614:\$D\$616 <= \$E\$614:\$E\$616  
 \$D\$617:\$D\$619 <= \$E\$617:\$E\$619  
 \$D\$620:\$D\$622 <= \$E\$620:\$E\$622  
 \$D\$623:\$D\$625 <= \$E\$623:\$E\$625  
 \$D\$626:\$D\$628 <= \$E\$626:\$E\$628  
 \$D\$629:\$D\$631 <= \$E\$629:\$E\$631  
 \$D\$632:\$D\$634 <= \$E\$632:\$E\$634  
 \$D\$635:\$D\$637 <= \$E\$635:\$E\$637  
 \$D\$638:\$D\$640 <= \$E\$638:\$E\$640  
 \$D\$641:\$D\$643 <= \$E\$641:\$E\$643  
 \$D\$644:\$D\$646 <= \$E\$644:\$E\$646  
 \$D\$647:\$D\$649 <= \$E\$647:\$E\$649  
 \$D\$650:\$D\$652 <= \$E\$650:\$E\$652  
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 \$D\$659:\$D\$661 <= \$E\$659:\$E\$661  
 \$D\$662:\$D\$664 <= \$E\$662:\$E\$664  
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 \$D\$671:\$D\$673 <= \$E\$671:\$E\$673  
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 \$D\$677:\$D\$679 <= \$E\$677:\$E\$679  
 \$D\$680:\$D\$682 <= \$E\$680:\$E\$682  
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 \$D\$686:\$D\$688 <= \$E\$686:\$E\$688  
 \$D\$689:\$D\$691 <= \$E\$689:\$E\$691  
 \$D\$692:\$D\$694 <= \$E\$692:\$E\$694  
 \$D\$695:\$D\$697 <= \$E\$695:\$E\$697  
 \$D\$698:\$D\$700 <= \$E\$698:\$E\$700  
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 \$D\$710:\$D\$712 <= \$E\$710:\$E\$712  
 \$D\$713:\$D\$715 <= \$E\$713:\$E\$715  
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 \$D\$728:\$D\$730 <= \$E\$728:\$E\$730  
 \$D\$731:\$D\$733 <= \$E\$731:\$E\$733  
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 \$D\$752:\$D\$754 <= \$E\$752:\$E\$754  
 \$D\$755:\$D\$757 <= \$E\$755:\$E\$757  
 \$D\$758:\$D\$760 <= \$E\$758:\$E\$760  
 \$D\$761:\$D\$763 <= \$E\$761:\$E\$763  
 \$D\$764:\$D\$766 <= \$E\$764:\$E\$766  
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 \$D\$773:\$D\$775 <= \$E\$773:\$E\$775  
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 \$D\$779:\$D\$781 <= \$E\$779:\$E\$781  
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 \$D\$1061:\$D\$1063 <= \$E\$1061:\$E\$1063  
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 \$D\$1142:\$D\$1144 <= \$E\$1142:\$E\$1144  
 \$D\$1145:\$D\$1147 <= \$E\$1145:\$