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Spreading dynamics of COVID-19 SELIR model in complex social networks

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ABSTRACT: *In this paper, we propose a new SELIR (susceptible-exposed-latent-infected-recovered) COVID-19 spreading model in complex social networks. The global stability of the virus free equilibrium is proved in detail, and the basic reproduction number R_0 of the model is obtained. The existence of COVID-19 equilibrium and the dynamic behavior of the model are determined by the basic reproduction number R_0. The global attractivity of the COVID-19 prevailing equilibrium is proved by monotone iterative technique. Numerical simulation confirm the analysis results.*

KEYWORDS - *COVID-19; Virus spreading model; Equilibrium; Global stability*

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I. INTRODUCTION

The outbreak of COVID-19 epidemic has been developing for two years and is now spreading around the world. Because people have different understanding of COVID-19, and different preventive measures have been taken, the effect of epidemic prevention and control is also very different. According to official information, the initial symptoms of covid-19 infection are fever and dry cough, similar to the common influenza. However, COVID-19 can develop into pneumonia, dyspnea and even death[1]. Some patients infected with covid-19 virus have no obvious symptoms, and the incubation period of the virus is about 14 days, but asymptomatic infected people are also infectious[2,3]. Covid-19 virus is mainly transmitted through the air. When people are in close contact with virus carriers (within 2 meters), they may be infected with COVID-19[4]. At present, there is no specific drug for COVID-19，maintaining social distance and reducing the flow of people is considered to be the most effective epidemic prevention measures[5].

The official COVID-19 information is limited, it is not enough for us to understand the characteristics and transmission mechanism of COVID-19 virus. The study of COVID-19 virus model is of great significance to our understanding of COVID-19 virus, which has attracted many scholars' attention[6,7]. Mathematical models are powerful tools to understand real world phenomena, especially the dynamics of infectious diseases, mathematical models describing infectious diseases play an important role in both theory and practice[8-10]. As for the diffusion of general infectious diseases, there are some classical models, such as logistic model, SIS model, SIR model, SEIR model, etc[11,12]. The establishment of these models will help us to understand the transmission mechanism and characteristics of the disease, so as to put forward effective strategies for disease prediction, prevention and suppression.

In this paper, a new mathematical model of COVID-19 is established. The whole society is regarded as a complex network, and community is the basic unit of society. The community members are divided into five categories: S (susceptible), E(exposed), L(latent asymptomatic infected), I(symptomatic infected) and R(recovered). Each community is connected with each other. The purpose of the model is to study the transmission mechanism and characteristics of COVID-19, and put forward effective strategies for epidemic prevention and control.

The rest of the paper is organized as follows: In Sect. 2, we present a new SELIR model in social networks. In Sect. 3, it shows that COVID-19 free equilibrium exists and proves the COVID-19 free equilibrium is globally stable. In Sect. 4, the system has a COVID-19 prevailing equilibrium, and proves the COVID-19 prevailing equilibrium is globally asymptotically attractive. In Sect. 5, numerical simulations are given to illustrate the main results. Finally, the conclusions are given in Sect. 6.

II. MODEL FORMULATION

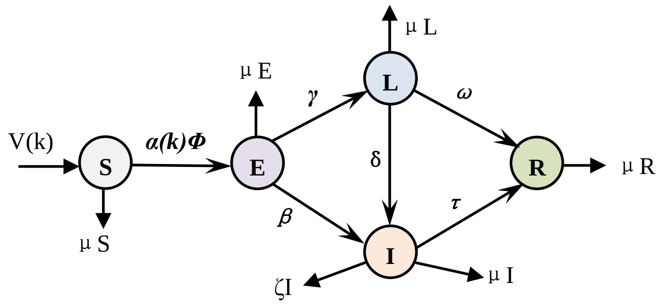

Figure 1 Transfer diagram for COVID-19 spreading model

In this paper, it is assumed that the whole population is an associated social network, and community is the basic unit of the society. The nodes in the network represent the individuals, and the edges connecting the nodes represent the connections between the individuals. The community members are divided into five categories: S (susceptible), E(exposed), L(latent), I(infected) and R(recovered). Each community is connected with each other. Each individual adopts one of the five states of S, E, L, I and R. S refers to people who have never contacted COVID-19 without immunity (susceptible); E refers to the close contacts of COVID-19 infected individuals (exposed); L refers to asymptomatic infection which is in latent state and may develop into symptomatic infection(latent); I refers to symptomatic infection(infected); R refers to the recovered and has immunity to COVID-19 (recovered). $S_k(t)$, $E_k(t)$, $L_k(t)$, $I_k(t)$ and $R_k(t)$ express the relative density of susceptible, exposed, latent, infected and recovered nodes of the community k at time t, respectively. The COVID-19 propagation model is shown in Fig. 1.

In the SELIR model, COVID-19 spread according to the following rules: The parameter $\alpha(k) > 0$ is the degree dependent rate, which indicates the accessibility of community k to COVID-19. E node represents the close contact of the infected individuals. Close contacts become asymptomatic infected individuals at rate γ, and become infected individuals with clinical symptoms at rate β. Latent individuals become into recovered individuals at rate ω, and become into infected individuals with clinical symptoms at rate δ. Infected individuals with clinical symptoms become into recovered individuals at rate τ, and the case fatality rate of infected individuals was ζ due to COVID-19.

The degree-dependent parameter $V(k)>0$ represents the number of newly immigrated individuals in community K per unit time, and each newly immigrated individual is susceptible, the reconnection of these nodes follows the above propagation rules. This type of rewiring preserves the network mean degree(the total number of links remains constant) but changes the mean degree of susceptible and infected nodes. The natural mortality rate in community K was μ. The parameters are all nonnegative. The model can be described by the following system of ordinary differential equations.

$$\begin{cases} \frac{dS_k(t)}{dt} = V(k) - \alpha(k)\Phi(t)S_k(t) - \mu S_k(t) \\ \frac{dE_k(t)}{dt} = \alpha(k)\Phi(t)S_k(t) - \beta E_k(t) - \gamma E_k(t) - \mu E_k(t) \\ \frac{dL_k(t)}{dt} = \gamma E_k(t) - \omega L_k(t) - \delta L_k(t) - \mu L_k(t) \\ \frac{dI_k(t)}{dt} = \beta E_k(t) + \delta L_k(t) - \tau I_k(t) - \zeta I_k(t) - \mu I_k(t) \\ \frac{dR_k(t)}{dt} = \omega L_k(t) + \tau I_k(t) - \mu R_k(t) \end{cases} \tag{2.1}$$

$\Phi(t)$ denotes the probability of susceptible individuals contacts with a virus carrier(infected individual or latent individual) at time t, which satisfies the relation:

$$\Phi(t) = \sum_{i=1}^{n} \frac{\varphi_1(i)}{i} P(i \mid k) \frac{I_i(t)}{N_i(t)} + \frac{\varphi_2(i)}{i} P(i \mid k) \frac{L_i(t)}{N_i(t)}$$

Here $P(i|k)$ is the conditional probability of a node with degree k connecting to a node with degree i. Because of the uncorrelated network [13], $P(i|k) = iP(i)/\langle k\rangle$. The probability of a randomly selected node with degree k is $P(k)$, thus $\sum_{k=1}^{n} P(k) = 1$, $\langle k\rangle = \sum_{k=1}^{n} kP(k)$ denotes the average degree, and $\varphi_1(i)$ indicates the probability of infection of a node with degree i caused by infected individuals; $\varphi_2(i)$ indicates the probability of infection of a node with degree i caused by latent individuals. 1/i denotes the probability of contacting an infected neighbor node with degree i at the present time step. The size of the population is constant.

$$S_k(t) + E_k(t) + L_k(t) + I_k(t) + R_k(t) = N_k(t) = \eta_K = V(k)/\mu \tag{2.2}$$

So we can obtain:

$$\Phi(t) = \frac{1}{\langle k\rangle}\sum_{k=1}^{n}\frac{\varphi_1(k)}{\eta_K}P(k)I_k(t) + \frac{1}{\langle k\rangle}\sum_{k=1}^{n}\frac{\varphi_2(k)}{\eta_K}P(k)L_k(t) \tag{2.3}$$

The initial conditions for system can be given as follows:

$$S_k(0) = \eta_K - E_k(0) - L_k(0) - I_k(0) - R_k(0) > 0$$
$$E_k(0) \geq 0 \quad L_k(0) \geq 0 \quad I_k(0) \geq 0 \quad R_k(0) \geq 0 \quad \Phi(0) > 0$$

The parameters of the system are all nonnegative.

III. THE BASIC REPRODUCTION NUMBER AND STABILITY ANALYSIS OF COVID-19 FREE EQUILIBRIUM

We define the solution vector of the system (2.1) as:

$$X_k(t) = [S_k(t), E_k(t), L_k(t), I_k(t), R_k(t)]^T \tag{3.1}$$

The basic reproduction number is defined as:

$$R_0 = \frac{\beta(\delta+\omega+\mu)+\gamma\delta}{(\tau+\zeta+\mu)(\beta+\gamma+\mu)(\delta+\omega+\mu)}\frac{\langle\alpha(k)\varphi_1(k)\rangle}{\langle k\rangle} + \frac{\gamma}{(\beta+\gamma+\mu)(\delta+\omega+\mu)}\frac{\langle\alpha(k)\varphi_2(k)\rangle}{\langle k\rangle} \tag{3.2}$$

Theorem1 If $R_0 < 1$, the COVID-19 free equilibrium $X_0 = [\eta_K, 0, 0, 0, 0]^T$ of system (2.1) is locally asymptotically stable, and it is unstable when $R_0 > 1$.

Proof:

Obviously, X_0 is a special solution of the equation(2.1), which is an equilibrium state. Next, we will prove that X_0 is locally asymptotically stable by Lyapunov's first method.

Jacobian matrix of the equation(2.1) at X_0 as follows:

$$J_{X=X_0} = \begin{bmatrix} A_{11} & A_{12} & \cdots & A_{1n} \\ A_{21} & A_{22} & \cdots & A_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ A_{n1} & A_{n2} & \cdots & A_{nn} \end{bmatrix}_{5n\times5n}$$

$$A_{kk} = \begin{bmatrix} -\mu & 0 & -\frac{\alpha(k)\varphi_2(k)P(k)}{\langle k\rangle} & -\frac{\alpha(k)\varphi_1(k)P(k)}{\langle k\rangle} & 0 \\ 0 & -(\beta+\gamma+\mu) & \frac{\alpha(k)\varphi_2(k)P(k)}{\langle k\rangle} & \frac{\alpha(k)\varphi_1(k)P(k)}{\langle k\rangle} & 0 \\ 0 & \gamma & -(\delta+\omega+\mu) & 0 & 0 \\ 0 & \beta & \delta & -(\tau+\zeta+\mu) & 0 \\ 0 & 0 & \omega & \tau & -\mu \end{bmatrix} \qquad k=1,2,3\ldots n$$

$$\underset{(i\neq k)}{A_{ik}} = \begin{bmatrix} 0 & 0 & -\frac{\alpha(i)\varphi_2(k)P(k)}{\langle k\rangle} & -\frac{\alpha(i)\varphi_1(k)P(k)}{\langle k\rangle} & 0 \\ 0 & 0 & \frac{\alpha(i)\varphi_2(k)P(k)}{\langle k\rangle} & \frac{\alpha(i)\varphi_1(k)P(k)}{\langle k\rangle} & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \qquad k=1,2,3,\ldots n;\ i=1,2,3\ldots n$$

The characteristic equation of Jacobian matrix as follows:

$$(x+\mu)^{2n}(x+\tau+\zeta+\mu)^{(n-1)}(x+\beta+\gamma+\mu)^{(n-1)}(x+\delta+\omega+\mu)^{(n-1)}\{(x+\tau+\zeta+\mu)(x+\beta+\gamma+\mu)(x+\delta+\omega+\mu) - \gamma(x+\tau+\zeta+\mu)\frac{\langle\alpha(k)\varphi_2(k)\rangle k}{\langle k\rangle} - \gamma\delta\frac{\langle\alpha(k)\varphi_1(k)\rangle}{\langle k\rangle} - \beta(x+\delta+\omega+\mu)\frac{\langle\alpha(k)\varphi_1(k)\rangle}{\langle k\rangle}\} = 0$$

Factors:

$$(x+\tau+\zeta+\mu)(x+\beta+\gamma+\mu)(x+\delta+\omega+\mu) - \gamma(x+\tau+\zeta+\mu)\frac{\langle\alpha(k)\varphi_2(k)\rangle k}{\langle k\rangle} - \gamma\delta\frac{\langle\alpha(k)\varphi_1(k)\rangle}{\langle k\rangle} - \beta(x+\delta+\omega+\mu)\frac{\langle\alpha(k)\varphi_1(k)\rangle}{\langle k\rangle} = 0$$

According to the Routh criterion, all characteristic roots of the system have negative real parts if the following conditions are satisfied:

$$(\tau+\zeta+\mu+\beta+\gamma+\mu+\delta+\omega+\mu)[(\tau+\zeta+\mu)(\beta+\gamma+\mu)+(\beta+\gamma+\mu)(\delta+\omega+\mu)+(\tau+\zeta+\mu)(\delta+\omega+\mu)-\gamma\frac{\langle\alpha(k)\varphi_2(k)\rangle k}{\langle k\rangle}-\beta\frac{\langle\alpha(k)\varphi_1(k)\rangle}{\langle k\rangle}]-\{(\tau+\zeta+\mu)(\beta+\gamma+\mu)(\delta+\omega+\mu)-\gamma(\tau+\zeta+\mu)\frac{\langle\alpha(k)\varphi_2(k)\rangle k}{\langle k\rangle}-\beta(\delta+\omega+\mu)\frac{\langle\alpha(k)\varphi_1(k)\rangle}{\langle k\rangle}-\gamma\delta\frac{\langle\alpha(k)\varphi_1(k)\rangle}{\langle k\rangle}\}>0$$

$$(\tau+\zeta+\mu)(\beta+\gamma+\mu)(\delta+\omega+\mu)-\gamma(\tau+\zeta+\mu)\frac{\langle\alpha(k)\varphi_2(k)\rangle k}{\langle k\rangle}-\beta(\delta+\omega+\mu)\frac{\langle\alpha(k)\varphi_1(k)\rangle}{\langle k\rangle}-\gamma\delta\frac{\langle\alpha(k)\varphi_1(k)\rangle}{\langle k\rangle}>0$$

After simplification, there are:

$$R_1=\frac{\beta}{\Delta 1}\frac{\langle\alpha(k)\varphi_1(k)\rangle}{\langle k\rangle}+\frac{\gamma}{\Delta 2}\frac{\langle\alpha(k)\varphi_2(k)\rangle}{\langle k\rangle}-\frac{\gamma\delta}{(\tau+\zeta+\mu+\beta+\gamma+\mu)\Delta 1}\frac{\langle\alpha(k)\varphi_1(k)\rangle}{\langle k\rangle}<1 \tag{3.3}$$

$$R_0=\frac{\beta}{(\tau+\zeta+\mu)(\beta+\gamma+\mu)}\frac{\langle\alpha(k)\varphi_1(k)\rangle}{\langle k\rangle}+\frac{\gamma}{(\beta+\gamma+\mu)(\delta+\omega+\mu)}\frac{\langle\alpha(k)\varphi_2(k)\rangle}{\langle k\rangle}+\frac{\gamma\delta}{(\tau+\zeta+\mu)(\beta+\gamma+\mu)(\delta+\omega+\mu)}\frac{\langle\alpha(k)\varphi_1(k)\rangle}{\langle k\rangle}<1 \tag{3.4}$$

Here:

$$\Delta 1=(\tau+\zeta+\mu)(\beta+\gamma+\mu)+(\beta+\gamma+\mu)(\delta+\omega+\mu)+(\tau+\zeta+\mu)(\delta+\omega+\mu)+(\delta+\omega+\mu)^2$$
$$\Delta 2=(\tau+\zeta+\mu)(\beta+\gamma+\mu)+(\beta+\gamma+\mu)(\delta+\omega+\mu)+(\tau+\zeta+\mu)(\delta+\omega+\mu)+(\tau+\zeta+\mu)^2$$

there are: $R_1<R_0<1$

So just satisfy the following:

$$R_0=\frac{\beta(\delta+\omega+\mu)+\gamma\delta}{(\tau+\zeta+\mu)(\beta+\gamma+\mu)(\delta+\omega+\mu)}\frac{\langle\alpha(k)\varphi_1(k)\rangle}{\langle k\rangle}+\frac{\gamma}{(\beta+\gamma+\mu)(\delta+\omega+\mu)}\frac{\langle\alpha(k)\varphi_2(k)\rangle}{\langle k\rangle}<1 \tag{3.5}$$

If $R_0 < 1$, all characteristic roots of the system have negative real parts, then the equilibrium state X_0 is asymptotically stable. If $R_0 > 1$, at least one characteristic root of the system has a positive real part, then the equilibrium state X_0 is unstable.

The proof is completed.

Theorem2 If $R_0 < 1$, the COVID-19 free equilibrium $X_0=[\eta_K,0,0,0,0]^T$ of system (2.1) is globally asymptotically stable.

Proof:

We will prove that X_0 is globally asymptotically stable by Lyapunov's second method.

$S_k(t)+E_k(t)+L_k(t)+I_k(t)+R_k(t)=\eta_K$ is a constant, four equations can be taken:

$$\begin{cases}\frac{dE_k(t)}{dt}=\alpha(k)\Phi(t)S_k(t)-\beta E_k(t)-\gamma E_k(t)-\mu E_k(t)\\ \frac{dL_k(t)}{dt}=\gamma E_k(t)-\omega L_k(t)-\delta L_k(t)-\mu L_k(t)\\ \frac{dI_k(t)}{dt}=\beta E_k(t)+\delta L_k(t)-\tau I_k(t)-\zeta I_k(t)-\mu I_k(t)\\ \frac{dR_k(t)}{dt}=\omega L_k(t)+\tau I_k(t)-\mu R_k(t)\end{cases} \tag{3.6}$$

We define the solution vector of the system (3.6) as:

$$X=[E_k(t),\ L_k(t),\ I_k(t),\ R_k(t)]^T \tag{3.7}$$

Let's take the Lyapunov function:

$$V(x)=\frac{1}{\langle k\rangle}\sum\frac{\varphi_1(k)P(k)}{\eta_K}E_k(t)+\frac{1}{\langle k\rangle}\sum\frac{\varphi_1(k)P(k)}{\eta_K}\frac{\delta(\beta+\gamma+\mu)}{\beta(\omega+\delta+\mu)+\gamma\delta}L_k(t)+\frac{1}{\langle k\rangle}\sum\frac{\varphi_1(k)P(k)}{\eta_K}\frac{(\omega+\delta+\mu)(\beta+\gamma+\mu)}{\beta(\omega+\delta+\mu)+\gamma\delta}I_k(t)+\frac{1}{\langle k\rangle}\sum\frac{\varphi_2(k)P(k)}{\eta_K}E_k(t)+\frac{1}{\langle k\rangle}\sum\frac{\varphi_2(k)P(k)}{\eta_K}\frac{(\beta+\gamma+\mu)}{\gamma}L_k(t)$$

Since all parameters are positive, the function V(x) is a positive definite function.

$$\begin{aligned}\frac{dV(x)}{dt}&=\frac{\partial V(x)}{\partial E_k(t)}\frac{dE_k(t)}{dt}+\frac{\partial V(x)}{\partial L_k(t)}\frac{dL_k(t)}{dt}+\frac{\partial V(x)}{\partial I_k(t)}\frac{dI_k(t)}{dt}+\frac{\partial V(x)}{\partial R_k(t)}\frac{dR_k(t)}{dt}\\ &=\frac{1}{\langle k\rangle}\sum\frac{\varphi_1(k)P(k)}{\eta_K}\{\alpha(k)\Phi(t)S_k(t)-(\beta+\gamma+\mu)E_k(t)\}+\frac{1}{\langle k\rangle}\sum\frac{\varphi_1(k)P(k)}{\eta_K}\frac{\delta(\beta+\gamma+\mu)}{\beta(\omega+\delta+\mu)+\gamma\delta}\{\gamma E_k(t)-(\omega+\delta\\ &\quad+\mu)L_k(t)\}+\frac{1}{\langle k\rangle}\sum\frac{\varphi_1(k)P(k)}{\eta_K}\frac{(\omega+\delta+\mu)(\beta+\gamma+\mu)}{\beta(\omega+\delta+\mu)+\gamma\delta}\{\beta E_k(t)+\delta L_k(t)-(\tau+\zeta+\mu)I_k(t)\}+\\ &\quad\frac{1}{\langle k\rangle}\sum\frac{\varphi_2(k)P(k)}{\eta_K}\{\alpha(k)\Phi(t)S_k(t)-(\beta+\gamma+\mu)E_k(t)\}+\frac{1}{\langle k\rangle}\sum\frac{\varphi_2(k)P(k)}{\eta_K}\frac{(\beta+\gamma+\mu)}{\gamma}\{\gamma E_k(t)-(\omega+\delta+\mu)L_k(t)\}\\ &=\frac{1}{\langle k\rangle}\sum\frac{(\varphi_1(k)+\varphi_2(k))P(k)}{\eta_K}\alpha(k)\Phi(t)S_k(t)-\frac{1}{\langle k\rangle}\sum\frac{\varphi_1(k)P(k)}{\eta_K}\frac{(\beta+\gamma+\mu)(\omega+\delta+\mu)}{\gamma}I_k(t)-\end{aligned}$$

$$\frac{1}{\langle k\rangle}\sum\frac{\varphi_2(k)P(k)}{\eta_K}\frac{(\beta+\gamma+\mu)(\omega+\delta+\mu)}{\gamma}L_k(t)-\frac{1}{\langle k\rangle}\sum\frac{\varphi_1(k)P(k)}{\eta_K}(\beta+\gamma+\mu)(\omega+\delta+\mu)\frac{\gamma(\tau+\zeta+\mu)-[\beta(\omega+\delta+\mu)+\gamma\delta]}{\gamma[\beta(\omega+\delta+\mu)+\gamma\delta]}I_k(t)$$
$$=\frac{(\beta+\gamma+\mu)(\omega+\delta+\mu)}{\gamma}\{\frac{1}{\langle k\rangle}\sum\frac{(\varphi_1(k)+\varphi_2(k))P(k)}{\eta_K}\alpha(k)\Phi(t)S_k(t)\frac{\gamma}{(\beta+\gamma+\mu)(\omega+\delta+\mu)}-\frac{1}{\langle k\rangle}\sum\frac{\varphi_1(k)P(k)}{\eta_K}I_k(t)-\frac{1}{\langle k\rangle}\sum\frac{\varphi_2(k)P(k)}{\eta_K}L_k(t)-\frac{1}{\langle k\rangle}\sum\frac{\varphi_1(k)P(k)}{\eta_K}\frac{\gamma(\tau+\zeta+\mu)-[\beta(\omega+\delta+\mu)+\gamma\delta]}{[\beta(\omega+\delta+\mu)+\gamma\delta]}I_k(t)\}$$

At the equilibrium point, the right side of equation(3.6) should be equal to 0.

$$\begin{cases}\alpha(k)\Phi(t)S_k(t)-(\beta+\gamma+\mu)E_k(t)=0\\ \gamma E_k(t)-(\omega+\delta+\mu)L_k(t)=0\\ \beta E_k(t)+\delta L_k(t)-(\tau+\zeta+\mu)I_k(t)=0\\ \omega L_k(t)+\tau I_k(t)-\mu R_k(t)=0\end{cases}$$

We can get the expression: $I_k(t)=\frac{\beta(\omega+\delta+\mu)+\gamma\delta}{(\omega+\delta+\mu)(\beta+\gamma+\mu)(\tau+\zeta+\mu)}\alpha(k)\Phi(t)S_k(t)$

Substituting it into the following:

$$\frac{dV(x)}{dt}=\frac{(\beta+\gamma+\mu)(\omega+\delta+\mu)}{\gamma}\{\frac{1}{\langle k\rangle}\sum\frac{(\varphi_1(k)+\varphi_2(k))P(k)}{\eta_K}\alpha(k)\Phi(t)S_k(t)\frac{\gamma}{(\beta+\gamma+\mu)(\omega+\delta+\mu)}-\frac{1}{\langle k\rangle}\sum\frac{\varphi_1(k)P(k)}{\eta_K}I_k(t)-\frac{1}{\langle k\rangle}\sum\frac{\varphi_2(k)P(k)}{\eta_K}L_k(t)-\frac{1}{\langle k\rangle}\sum\frac{\varphi_1(k)P(k)}{\eta_K}\frac{\gamma(\tau+\zeta+\mu)-[\beta(\omega+\delta+\mu)+\gamma\delta]}{(\omega+\delta+\mu)(\beta+\gamma+\mu)(\tau+\zeta+\mu)}\alpha(k)\Phi(t)S_k(t)\}$$
$$=\frac{(\beta+\gamma+\mu)(\omega+\delta+\mu)}{\gamma}\{\frac{1}{\langle k\rangle}\sum\frac{\varphi_1(k)P(k)}{\eta_K}\alpha(k)\Phi(t)S_k(t)\frac{\beta(\omega+\delta+\mu)+\gamma\delta}{(\omega+\delta+\mu)(\beta+\gamma+\mu)(\tau+\zeta+\mu)}+\frac{1}{\langle k\rangle}\sum\frac{\varphi_2(k)P(k)}{\eta_K}\alpha(k)\Phi(t)S_k(t)\frac{\gamma}{(\beta+\gamma+\mu)(\omega+\delta+\mu)}-\Phi(t)\}$$
$$\le\frac{(\beta+\gamma+\mu)(\omega+\delta+\mu)}{\gamma}\Phi(t)\{\frac{1}{\langle k\rangle}\sum\varphi_1(k)P(k)\alpha(k)\frac{\beta(\omega+\delta+\mu)+\gamma\delta}{(\omega+\delta+\mu)(\beta+\gamma+\mu)(\tau+\zeta+\mu)}+\frac{1}{\langle k\rangle}\sum\varphi_2(k)P(k)\alpha(k)\frac{\gamma}{(\beta+\gamma+\mu)(\omega+\delta+\mu)}-1\}$$
$$=\frac{(\beta+\gamma+\mu)(\omega+\delta+\mu)}{\gamma}\Phi(t)\{\frac{\langle\varphi_1(k)\alpha(k)\rangle}{\langle k\rangle}\frac{\beta(\omega+\delta+\mu)+\gamma\delta}{(\omega+\delta+\mu)(\beta+\gamma+\mu)(\tau+\zeta+\mu)}+\frac{\langle\varphi_2(k)\alpha(k)\rangle}{\langle k\rangle}\frac{\gamma}{(\beta+\gamma+\mu)(\omega+\delta+\mu)}-1\}$$
$$=\frac{(\beta+\gamma+\mu)(\omega+\delta+\mu)}{\gamma}\Phi(t)\{R_0-1\}$$

If $R_0 < 1$, $dV(x)/dt\le 0$, and only if $x = 0$, $\Phi(t)=0$, $dV(x)/dt = 0$.
Therefore, the equilibrium state of origin X = (0,0,0,0) is asymptotically stable.
When $\| x \|\to\infty$, $V(x)\to\infty$, then the system(3.6) is globally asymptotically stable at the origin $X = (0,0,0,0)^T$.
Because $S_k(t)+E_k(t)+L_k(t)+I_k(t)+R_k(t)=\eta_K$ is a constant,
The system(2.1) is globally asymptotically stable at $X_0=[\eta_K,0,0,0,0]^T$
The proof is completed.

IV. EXISTENCE OF COVID-19 PREVAILING EQUILIBRIUM

Theorem3 When $R_0 > 1$, the system (2.1) has a COVID-19 prevailing equilibrium $X_k^* = (S_k^*,E_k^*,L_k^*, I_k^*,R_k^*)^T$.

$$\begin{cases}\frac{dS_k(t)}{dt}=V(k)-\alpha(k)\Phi(t)S_k(t)-\mu S_k(t)\\ \frac{dE_k(t)}{dt}=\alpha(k)\Phi(t)S_k(t)-\beta E_k(t)-\gamma E_k(t)-\mu E_k(t)\\ \frac{dL_k(t)}{dt}=\gamma E_k(t)-\omega L_k(t)-\delta L_k(t)-\mu L_k(t)\\ \frac{dI_k(t)}{dt}=\beta E_k(t)+\delta L_k(t)-\tau I_k(t)-\zeta I_k(t)-\mu I_k(t)\\ \frac{dR_k(t)}{dt}=\omega L_k(t)+\tau I_k(t)-\mu R_k(t)\end{cases}\quad(4.1)$$

Proof:
At the equilibrium point, the right side of equation should be equal to 0.

$$\begin{cases}V(k)-\alpha(k)\Phi^*S_k^*-\mu S_k^*=0\\ \alpha(k)\Phi^*S_k^*-(\beta+\gamma+\mu)E_k^*=0\\ \gamma E_k^*-(\omega+\delta+\mu)L_k^*=0\\ \beta E_k^*+\delta L_k^*-(\tau+\zeta+\mu)I_k^*=0\\ \omega L_k^*+\tau I_k^*-\mu R_k^*=0\\ S_k^*+E_k^*+L_k^*+I_k^*+R_k^*=\eta_K\end{cases}$$

So we get the following：

$$\begin{cases} S_k^* = \frac{(\beta+\gamma+\mu)(\omega+\delta+\mu)}{\gamma\alpha(k)\Phi^*} L_k^* \\ E_k^* = \frac{(\omega+\delta+\mu)}{\gamma} L_k^* \\ I_k^* = \frac{\beta(\omega+\delta+\mu)+\gamma\delta}{\gamma(\tau+\zeta+\mu)} L_k^* \\ R_k^* = \frac{\omega\gamma(\tau+\zeta+\mu)+\tau[\beta(\omega+\delta+\mu)+\gamma\delta]}{\mu\gamma(\tau+\zeta+\mu)} L_k^* \end{cases} \tag{4.2}$$

$$L_k^* = \frac{\eta_K\alpha(k)\Phi^*\mu\gamma(\tau+\zeta+\mu)}{\Delta 3} \tag{4.3}$$

$$I_k^* = \frac{\eta_K\alpha(k)\Phi^*\mu[\beta(\omega+\delta+\mu)+\gamma\delta]}{\Delta 3} \tag{4.4}$$

$$\Delta 3 = \mu(\tau+\zeta+\mu)(\beta+\gamma+\mu)(\omega+\delta+\mu) + \mu(\tau+\zeta+\mu)\alpha(k)\Phi^*(\omega+\delta+\mu) + \mu(\tau+\zeta+\mu)\alpha(k)\Phi^*\gamma + \mu\alpha(k)\Phi^*[\beta(\omega+\delta+\mu)+\gamma\delta] + \alpha(k)\Phi^*\{\omega\gamma(\tau+\zeta+\mu)+\tau[\beta(\omega+\delta+\mu)+\gamma\delta]\}$$

$$\begin{aligned} \Phi^* &= \frac{1}{\langle k\rangle}\sum_{i=1}^{n}\frac{\varphi_1(k)}{\eta_K}P(k)I_k^* + \frac{1}{\langle k\rangle}\sum_{i=1}^{n}\frac{\varphi_2(k)}{\eta_K}P(k)L_k^* \\ &= \frac{1}{\langle k\rangle}\sum_{i=1}^{n}\frac{\varphi_1(k)}{\eta_K}P(k)\frac{\eta_K\alpha(k)\Phi^*\mu[\beta(\omega+\delta+\mu)+\gamma\delta]}{\Delta 3} + \frac{1}{\langle k\rangle}\sum_{i=1}^{n}\frac{\varphi_2(k)}{\eta_K}P(k)\frac{\eta_K\alpha(k)\Phi^*\mu\gamma(\tau+\zeta+\mu)}{\Delta 3} \\ &:= F(\Phi^*) \end{aligned} \tag{4.5}$$

Apparently, $\Phi^*=0$ is a trivial solution of (4.5), i.e., $F(0) = 0$. In order to let (4.5) have a non-trivial solution, i.e., $0<\Phi^*<1$, the right side of (4.5) must satisfy the following conditions:

$$\frac{dF(\Phi^*)}{d\Phi^*}\Big|_{\Phi^*=0} > 1$$

$$\begin{aligned} \frac{dF(\Phi^*)}{d\Phi^*}\Big|_{\Phi^*=0} &= \frac{1}{\langle k\rangle}\sum_{i=1}^{n}\varphi_1(k)\alpha(k)P(k)\frac{[\beta(\omega+\delta+\mu)+\gamma\delta]}{(\tau+\zeta+\mu)(\beta+\gamma+\mu)(\omega+\delta+\mu)} + \frac{1}{\langle k\rangle}\sum_{i=1}^{n}\varphi_2(k)\alpha(k)P(k)\frac{\gamma}{(\beta+\gamma+\mu)(\omega+\delta+\mu)} \\ &= \frac{\langle\alpha(k)\varphi_1(k)\rangle}{\langle k\rangle}\frac{\beta(\omega+\delta+\mu)+\gamma\delta}{(\tau+\zeta+\mu)(\beta+\gamma+\mu)(\omega+\delta+\mu)} + \frac{\langle\alpha(k)\varphi_2(k)\rangle}{\langle k\rangle}\frac{\gamma}{(\beta+\gamma+\mu)(\omega+\delta+\mu)} = R_0 > 1 \end{aligned}$$

So, a nontrivial solution exists if and only if $R_0 > 1$.
Inserting the nontrivial solution into Eq (4.3,4.4), we can obtain I_k^* and L_k^*. By I_k^* and L_k^* we can easily get:
$0 < S_k^* < \eta_k$, $0< E_k^* < \eta_k$, $0< L_k^* < \eta_k$, $0< I_k^* < \eta_k$, $0< R_k^* < \eta_k$
Thus, the COVID-19 prevailing equilibrium $X_k^* = (S_k^*, E_k^*, L_k^*, I_k^*, R_k^*)^T$ is well-defined.
Hence, when $R_0 > 1$, only one positive equilibrium $X_k^* = (S_k^*, E_k^*, L_k^*, I_k^*, R_k^*)^T$ of system (2.1) exists.
The proof is completed.

Remark. The basic reproduction number R_0 depends on some model parameters and the fluctuations of the degree distribution. Next, we discuss the influence of network topology and model parameters on the basic reproduction number R_0.

$$\begin{aligned} R_0 &= \frac{\beta(\delta+\omega+\mu)+\gamma\delta}{(\tau+\zeta+\mu)(\beta+\gamma+\mu)(\delta+\omega+\mu)}\frac{\langle\alpha(k)\varphi_1(k)\rangle}{\langle k\rangle} + \frac{\gamma}{(\beta+\gamma+\mu)(\delta+\omega+\mu)}\frac{\langle\alpha(k)\varphi_2(k)\rangle}{\langle k\rangle} \\ &= \frac{\beta}{(\tau+\zeta+\mu)(\beta+\gamma+\mu)}\frac{\langle\alpha(k)\varphi_1(k)\rangle}{\langle k\rangle} + \frac{\gamma\delta}{(\tau+\zeta+\mu)(\beta+\gamma+\mu)(\delta+\omega+\mu)}\frac{\langle\alpha(k)\varphi_1(k)\rangle}{\langle k\rangle} + \frac{\gamma}{(\beta+\gamma+\mu)(\delta+\omega+\mu)}\frac{\langle\alpha(k)\varphi_2(k)\rangle}{\langle k\rangle} \end{aligned}$$

Obviously, the higher the infection coefficient β and γ, the more serious the virus transmission. $\frac{\langle\alpha(k)\varphi_1(k)\rangle}{\langle k\rangle}$ and $\frac{\langle\alpha(k)\varphi_2(k)\rangle}{\langle k\rangle}$ represent the probability of close contact with virus carriers in susceptible population. The larger $\frac{\langle\alpha(k)\varphi_1(k)\rangle}{\langle k\rangle}$ and $\frac{\langle\alpha(k)\varphi_2(k)\rangle}{\langle k\rangle}$, the larger R_0, which will accelerate the spread of the epidemic. $\frac{\langle\alpha(k)\varphi_1(k)\rangle}{\langle k\rangle}$ and $\frac{\langle\alpha(k)\varphi_2(k)\rangle}{\langle k\rangle}$ represent the connection of social network, we can limit flow of population and isolate close contacts to reduce $\frac{\langle\alpha(k)\varphi_1(k)\rangle}{\langle k\rangle}$ and $\frac{\langle\alpha(k)\varphi_2(k)\rangle}{\langle k\rangle}$, and thus reduce R_0 and the spread of the virus.

The larger τ and ω are, the smaller R_0 is, which will weaken the transmission of the virus. τ and ω represent the cure rate of virus infected people, so it is of great significance to take appropriate treatment to improve the cure rate for epidemic prevention and control.

Next, the global attractivity of the COVID-19 prevailing equilibrium is discussed. The main result is given in the following theorem.

Lemma 1 ([14])
If $a > 0$, $b > 0$, and $dx(t)/dt \le b - ax$, when $t \ge 0$ and $x(0) \ge 0$, we have

$$\lim_{t\to\infty}\sup x(t) \le \frac{b}{a}$$

If $a > 0$, $b > 0$, and $dx(t)/dt \ge b - ax$, when $t \ge 0$ and $x(0) \ge 0$, we have

$$\lim_{t\to\infty}\inf x(t) \ge \frac{b}{a}$$

Theorem 4 $X_k = (S_k, E_k, L_k, I_k, R_k)^T$ is a solution of system (4.1) satisfying initial conditions
$0< E_k < \eta_k$, $0< L_k < \eta_k$, $0< I_k < \eta_k$
If $R_0 > 1$, then
Lim $t\to\infty$ $X_k = (S_k, E_k, L_k, I_k, R_k)^T = X_k^* = (S_k^*, E_k^*, L_k^*, I_k^*, R_k^*)^T$
where $X_k^* = (S_k^*, E_k^*, L_k^*, I_k^*, R_k^*)^T$ is the COVID-19 prevailing equilibrium of (4.1) satisfying (4.2)
for k = 1, 2, . . . , n.

Proof:
In the following, k is fixed to be any integer in (1, 2, . . . , n)
By Theorem 4 $0< E_k < \eta_k$, $0< L_k < \eta_k$, $0< I_k < \eta_k$
there exists a sufficiently small constant $\rho(0 <\rho< 1)$ and a larger enough constant $T > 0$
such that $I_k(t) \geq \rho$ $L_k(t) \geq \rho$ for $t > T$.
Thus for $t > T$

$$\Phi(t) = \frac{1}{\langle k\rangle}\sum_{k=1}^{n}\frac{\varphi_1(k)}{\eta_k}P(k)I_k(t) + \frac{1}{\langle k\rangle}\sum_{k=1}^{n}\frac{\varphi_2(k)}{\eta_k}P(k)L_k(t) \geq \frac{1}{\langle k\rangle}\sum_{k=1}^{n}\frac{\varphi_1(k)+\varphi_2(k)}{\eta_k}P(k)\rho = \Upsilon\rho > 0 \qquad (4.6)$$

$$\Upsilon = \sum_{k=1}^{n}\frac{\varphi_1(k)+\varphi_2(k)}{\eta_k}P(k)$$

Step 1:
Submitting this into the equation of (4.1)

$$\frac{dS_k(t)}{dt} \leq V(k) - \alpha(k)\Upsilon\rho S_k(t) - \mu S_k(t) \qquad t > T$$

By Lemma 1 $\lim_{t\to\infty}\sup S_k(t) \leq \frac{V(k)}{\alpha(k)\Upsilon\rho+\mu}$

For any given constant $0 < \rho_1 \leq \frac{V(k)}{2[\alpha(k)\Upsilon\rho+\mu]}$

There exists $t_1>T$, for $t>t_1$ $S_k(t) \leq X_k^{(1)} - \rho_1$

$$X_k^{(1)} \leq \frac{V(k)}{\alpha(k)\Upsilon\rho+\mu} + 2\rho_1 < \eta_k \qquad (4.7)$$

Step 2:

$$\Phi(t) \leq \frac{1}{\langle k\rangle}\sum(\varphi_1(k) + \varphi_2(k))P(k) := \lambda \qquad (4.8)$$

we obtain from the second equation of system (4.1) that

$$\frac{dE_k(t)}{dt} \leq \alpha(k)\lambda\big(\eta_k - E_k(t)\big) - (\beta + \gamma + \mu)E_k(t) \qquad t > t_1$$

By Lemma 1 $\lim_{t\to\infty}\sup E_k(t) \leq \frac{\alpha(k)\lambda\eta_k}{\alpha(k)\lambda+\beta+\gamma+\mu} < \eta_k$

For any given constant $0 < \rho_2 < min\left\{\frac{1}{2},\ \rho_1,\ \frac{\alpha(k)\lambda\eta_k}{2[\alpha(k)\lambda+\beta+\gamma+\mu]}\right\}$

There exists $t_2> t_1$, for $t>t_2$ $E_k(t) \leq Y_k^{(1)} - \rho_2 < \eta_k$

$$Y_k^{(1)} = \frac{\alpha(k)\lambda\eta_k}{\alpha(k)\lambda+\beta+\gamma+\mu} + 2\rho_2 < \eta_k \qquad (4.9)$$

Step 3:
Then it follows from the third equation of (4.1) that

$$\frac{dL_k(t)}{dt} \leq \gamma\big(\eta_k - L_k(t)\big) - (\omega + \delta + \mu)L_k(t) \qquad t > t_2$$

By Lemma 1 $\lim_{t\to\infty}\sup L_k(t) \leq \frac{\gamma\eta_k}{\gamma+\omega+\delta+\mu} < \eta_k$

For any given constant $0 < \rho_3 < min\left\{\frac{1}{3},\ \rho_2,\ \frac{\gamma\eta_k}{2(\gamma+\omega+\delta+\mu)}\right\}$

There exists $t_3> t_2$, for $t>t_3$ $L_k(t) \leq Z_k^{(1)} - \rho_3 < \eta_k$

$$Z_k^{(1)} = \frac{\gamma\eta_k}{\gamma+\omega+\delta+\mu} + 2\rho_3 < \eta_k \qquad (4.10)$$

Step 4:
Then it follows from the fourth equation of (4.1) that

$$\frac{dI_k(t)}{dt} \leq \beta(\eta_k - I_k(t)) + \delta(\eta_k - I_k(t)) - (\tau + \zeta + \mu)I_k(t) \qquad t > t_3$$

By Lemma 1 $\lim_{t\to\infty}\sup I_k(t) \leq \frac{(\beta+\delta)\eta_k}{\beta+\delta+\tau+\zeta+\mu} < \eta_k$

For any given constant $0 < \rho_4 < min\left\{\frac{1}{4},\ \rho_3,\ \frac{(\beta+\delta)\eta_k}{2[\beta+\delta+\tau+\zeta+\mu]}\right\}$

There exists $t_4> t_3$, for $t>t_4$ $I_k(t) \leq W_k^{(1)} - \rho_4 < \eta_k$

$$W_k^{(1)} = \frac{(\beta+\delta)\eta_k}{\beta+\delta+\tau+\zeta+\mu} + 2\rho_4 < \eta_k \qquad (4.11)$$

Step 5:
On the other hand, we substitute this into the first equation of (4.1)

$\frac{dS_k(t)}{dt} \geq V(k) - \alpha(k)\lambda S_k(t) - \mu S_k(t) \qquad t > t_4$

By Lemma 1 $\lim_{t\to\infty} \inf S_k(t) \geq \frac{V(k)}{\alpha(k)\lambda+\mu}$

For any given constant $0 < \rho_5 < min\left\{\frac{1}{5},\ \ \rho_4,\ \frac{V(k)}{2[\alpha(k)\lambda+\mu]}\right\}$

There exists $t_5 > t_4$, for $t > t_5$ $S_k(t) \geq x_k^{(1)} + \rho_5$

$$x_k^{(1)} = \frac{V(k)}{\alpha(k)\lambda+\mu} - 2\rho_5 \tag{4.12}$$

Step 6:
It follows that

$\frac{dE_k(t)}{dt} \geq \alpha(k)\Upsilon\rho x_k^{(1)} - \beta E_k(t) - \gamma E_k(t) - \mu E_k(t) \qquad t > t_5$

By Lemma 1 $\lim_{t\to\infty} \inf E_k(t) \geq \frac{\alpha(k)\Upsilon\rho x_k^{(1)}}{\beta+\gamma+\mu}$

For any given constant $0 < \rho_6 < min\left\{\frac{1}{6},\ \ \rho_5,\ \frac{\alpha(k)\Upsilon\rho x_k^{(1)}}{2(\beta+\gamma+\mu)}\right\}$

There exists $t_6 > t_5$, for $t > t_6$ $E_k(t) \geq y_k^{(1)} + \rho_6$

$$y_k^{(1)} = \frac{\alpha(k)\Upsilon\rho x_k^{(1)}}{\beta+\gamma+\mu} - 2\rho_6 \tag{4.13}$$

Step 7:
The third equation of (4.1) implies that

$\frac{dL_k(t)}{dt} \geq \gamma y_k^{(1)} - \omega L_k(t) - \delta L_k(t) - \mu L_k(t) \qquad t > t_6$

By Lemma 1 $\lim_{t\to\infty} \inf L_k(t) \geq \frac{\gamma y_k^{(1)}}{\omega+\delta+\mu}$

For any given constant $0 < \rho_7 < min\left\{\frac{1}{7},\ \ \rho_6,\ \frac{\gamma y_k^{(1)}}{2(\omega+\delta+\mu)}\right\}$

There exists $t_7 > t_6$, for $t > t_7$ $L_k(t) \geq z_k^{(1)} + \rho_7$

$$z_k^{(1)} = \frac{\gamma y_k^{(1)}}{\omega+\delta+\mu} - 2\rho_7 \tag{4.14}$$

Step 8:

It follows that $\frac{dI_k(t)}{dt} \geq \beta y_k^{(1)} + \delta z_k^{(1)} - (\tau + \zeta + \mu)I_k(t) \qquad t > t_7$

By Lemma 1 $\lim_{t\to\infty} \inf I_k(t) \geq \frac{\beta y_k^{(1)}+\delta z_k^{(1)}}{\tau+\zeta+\mu}$

For any given constant $0 < \rho_8 < min\left\{\frac{1}{8},\ \ \rho_7,\ \frac{\beta y_k^{(1)}+\delta z_k^{(1)}}{2[\tau+\zeta+\mu]}\right\}$

There exists $t_8 > t_7$, for $t > t_8$ $I_k(t) \geq w_k^{(1)} + \rho_8$

$$w_k^{(1)} = \frac{\beta y_k^{(1)}+\delta z_k^{(1)}}{\tau+\zeta+\mu} - 2\rho_8 \tag{4.15}$$

Step 9:
Due to ρ being a small positive constant, we can derive that

$0 < x_k^{(1)} < X_k^{(1)}, \quad 0 < y_k^{(1)} < Y_k^{(1)}, \quad 0 < z_k^{(1)} < Z_k^{(1)}, \quad 0 < w_k^{(1)} < W_k^{(1)}$

$$h^{(j)} = \frac{1}{\langle k\rangle}\sum_{i=1}^{n}\frac{\varphi_1(i)P(i)}{\eta_i}w_i^{(j)} + \frac{1}{\langle k\rangle}\sum_{i=1}^{n}\frac{\varphi_2(i)P(i)}{\eta_i}z_i^{(j)} \qquad H^{(j)} = \frac{1}{\langle k\rangle}\sum_{i=1}^{n}\frac{\varphi_1(i)P(i)}{\eta_i}W_i^{(j)} + \frac{1}{\langle k\rangle}\sum_{i=1}^{n}\frac{\varphi_2(i)P(i)}{\eta_i}Z_i^{(j)} \tag{4.16}$$

We can easily get $0 < h^{(j)} < \Phi(t) < H^{(j)} < \lambda$ (4.17)

Again, from the first equation of (4.1), we have

$\frac{dS_k(t)}{dt} \leq V(k) - \alpha(k)h^{(1)}S_k(t) - \mu S_k(t) \qquad t > t_8$

By Lemma 1 $\lim_{t\to\infty} \sup S_k(t) \leq \frac{V(k)}{\alpha(k)h^{(1)}+\mu}$

For any given constant $0 < \rho_9 < min\{1/9,\ \ \rho_8\}$

There exists $t_9 > t_8$, for $t > t_9$ $S_k(t) \leq X_k^{(2)}$

$$X_k^{(2)} = \min\left\{X_k^{(1)} - \rho_1\ ,\ \frac{V(k)}{\alpha(k)h^{(1)}+\mu} + \rho_9\right\} \tag{4.18}$$

Step 10:
Then, from the second equation of (4.1), we have

$\frac{dE_k(t)}{dt} \leq \alpha(k)H^{(1)}X_k^{(2)} - \beta E_k(t) - \gamma E_k(t) - \mu E_k(t) \qquad t > t_9$

By Lemma 1 $\lim_{t\to\infty} \sup E_k(t) \leq \frac{\alpha(k)H^{(1)}X_k^{(2)}}{\beta+\gamma+\mu}$

For any given constant $0 < \rho_{10} < min\{1/10, \ \rho_9\}$

There exists $t_{10} > t_9$, for $t > t_{10}$ $\quad E_k(t) \le Y_k^{(2)} = \min\left\{Y_k^{(1)} - \rho_2 \ , \ \frac{\alpha(k)H^{(1)}X_k^{(2)}}{\beta+\gamma+\mu} + \rho_{10}\right\}$ (4.19)

Step 11:

Consequently, from the third equation of (4.1), we have

$\frac{dL_k(t)}{dt} \le \gamma Y_k^{(2)} - (\omega + \delta + \mu)L_k(t) \qquad t > t_{10}$

By Lemma 1 $\quad \lim_{t\to\infty} \sup L_k(t) \le \frac{\gamma Y_k^{(2)}}{\omega+\delta+\mu}$

For any given constant $0 < \rho_{11} < min\{1/11, \ \rho_{10}\}$

There exists $t_{11} > t_{10}$, for $t > t_{11}$ $\quad L_k(t) \le Z_k^{(2)} = \min\left\{Z_k^{(1)} - \rho_3 \ , \ \frac{\gamma Y_k^{(2)}}{\omega+\delta+\mu} + \rho_{11}\right\}$ (4.20)

Step 12:

from the fourth equation of (4.1), we have

$\frac{dI_k(t)}{dt} \le \beta Y_k^{(2)} + \delta Z_k^{(2)} - (\tau + \zeta + \mu)I_k(t) \qquad t > t_{11}$

By Lemma 1 $\quad \lim_{t\to\infty} \sup I_k(t) \le \frac{\beta Y_k^{(2)} + \delta Z_k^{(2)}}{\tau+\zeta+\mu}$

For any given constant $0 < \rho_{12} < min\{1/12, \ \rho_{11}\}$

There exists $t_{12} > t_{11}$, for $t > t_{12}$

$$I_k(t) \le W_k^{(2)} = \min\left\{W_k^{(1)} - \rho_4 \ , \ \frac{\beta Y_k^{(2)} + \delta Z_k^{(2)}}{\tau+\zeta+\mu} + \rho_{12}\right\} \qquad (4.21)$$

Step 13:

Turning back, one has

$\frac{dS_k(t)}{dt} \ge V(k) - \alpha(k)H^{(2)}S_k(t) - \mu S_k(t) \qquad t > t_{12}$

By Lemma 1 $\quad \lim_{t\to\infty} \inf S_k(t) \ge \frac{V(k)}{\alpha(k)H^{(2)}+\mu}$

For any given constant $0 < \rho_{13} < min\left\{\frac{1}{13}, \ \rho_{12} \ , \ \frac{V(k)}{2[\alpha(k)H^{(2)}+\mu]}\right\}$

There exists $t_{13} > t_{12}$, for $t > t_{13}$ $\quad S_k(t) \ge x_k^{(2)} = \max\left\{x_k^{(1)} + \rho_5 \ , \ \frac{V(k)}{\alpha(k)H^{(2)}+\mu} - \rho_{13}\right\}$ (4.22)

Step 14:

It follows that

$\frac{dE_k(t)}{dt} \ge \alpha(k)h^{(1)}x_k^{(2)} - \beta E_k(t) - \gamma E_k(t) - \mu E_k(t) \qquad t > t_{13}$

By Lemma 1 $\quad \lim_{t\to\infty} \inf E_k(t) \ge \frac{\alpha(k)h^{(1)}x_k^{(2)}}{\beta+\gamma+\mu}$

For any given constant $0 < \rho_{14} < min\left\{\frac{1}{14}, \ \rho_{13} \ , \ \frac{\alpha(k)h^{(1)}x_k^{(2)}}{2(\beta+\gamma+\mu)}\right\}$

There exists $t_{14} > t_{13}$, for $t > t_{14}$ $\quad E_k(t) \ge y_k^{(2)} = \max\left\{y_k^{(1)} + \rho_6 \ , \ \frac{\alpha(k)h^{(1)}x_k^{(2)}}{\beta+\gamma+\mu} - \rho_{14}\right\}$ (4.23)

Step 15:

The third equation of (4.1) implies that

$\frac{dL_k(t)}{dt} \ge \gamma y_k^{(2)} - \omega L_k(t) - \delta L_k(t) - \mu L_k(t) \qquad t > t_{14}$

By Lemma 1 $\quad \lim_{t\to\infty} \inf L_k(t) \ge \frac{\gamma y_k^{(2)}}{\omega+\delta+\mu}$

For any given constant $0 < \rho_{15} < min\left\{\frac{1}{15}, \ \rho_{14} \ , \ \frac{\gamma y_k^{(2)}}{2(\omega+\delta+\mu)}\right\}$

There exists $t_{15} > t_{14}$, for $t > t_{15}$ $\quad L_k(t) \ge z_k^{(2)} = \max\left\{z_k^{(1)} + \rho_7 \ , \ \frac{\gamma y_k^{(2)}}{\omega+\delta+\mu} - \rho_{15}\right\}$ (4.24)

Step 16:

It follows that

$\frac{dI_k(t)}{dt} \ge \beta y_k^{(2)} + \delta z_k^{(2)} - (\tau + \zeta + \mu)I_k(t) \qquad t > t_{15}$

By Lemma 1 $\quad \lim_{t\to\infty} \inf I_k(t) \ge \frac{\beta y_k^{(2)} + \delta z_k^{(2)}}{\tau+\zeta+\mu}$

For any given constant $0 < \rho_{16} < min\left\{\frac{1}{16}, \ \rho_{15} \ , \ \frac{\beta y_k^{(2)} + \delta z_k^{(2)}}{2(\tau+\zeta+\mu)}\right\}$

There exists $t_{16} > t_{15}$, for $t > t_{16}$ $\quad I_k(t) \geq w_k^{(2)} = \max\left\{ w_k^{(1)} + \rho_8 \,,\ \frac{\beta y_k^{(2)} + \delta z_k^{(2)}}{\tau+\zeta+\mu} - \rho_{16} \right\}$ (4.25)

Repeating the above analyses and calculations, we get eight sequences:

$$x_k^{(l)} \quad y_k^{(l)} \quad z_k^{(l)} \quad w_k^{(l)} \quad X_k^{(l)} \quad Y_k^{(l)} \quad Z_k^{(l)} \quad W_k^{(l)} \qquad l = 1,2,3 \ldots\ldots \tag{4.26}$$

Due to the first four being monotone increasing sequences and the last four being monotone decreasing ones there exists a sufficiently large positive integer L≥2, such that $l \geq$ L:

$$X_k^{(l)} = \frac{V(k)}{\alpha(k)h^{(l-1)}+\mu} + \rho_{8l-7} \qquad Y_k^{(l)} = \frac{\alpha(k)H^{(l-1)}X_k^{(l)}}{\beta+\gamma+\mu} + \rho_{8l-6}$$
$$Z_k^{(l)} = \frac{\gamma Y_k^{(l)}}{\omega+\delta+\mu} + \rho_{8l-5} \qquad W_k^{(l)} = \frac{\beta Y_k^{(l)} + \delta Z_k^{(l)}}{\tau+\zeta+\mu} + \rho_{8l-4} \tag{4.27}$$
$$x_k^{(l)} = \frac{V(k)}{\alpha(k)H^{(l)}+\mu} - \rho_{8l-3} \qquad y_k^{(l)} = \frac{\alpha(k)h^{(l-1)}x_k^{(l)}}{\beta+\gamma+\mu} - \rho_{8l-2}$$
$$z_k^{(l)} = \frac{\gamma y_k^{(l)}}{\omega+\delta+\mu} - \rho_{8l-1} \qquad w_k^{(l)} = \frac{\beta y_k^{(l)} + \delta z_k^{(l)}}{\tau+\zeta+\mu} - \rho_{8l}$$

We can easily get that

$$x_k^{(l)} \leq S_k(t) \leq X_k^{(l)} \qquad y_k^{(l)} \leq E_k(t) \leq Y_k^{(l)} \qquad z_k^{(l)} \leq L_k(t) \leq Z_k^{(l)} \qquad w_k^{(l)} \leq I_k(t) \leq W_k^{(l)} \tag{4.28}$$

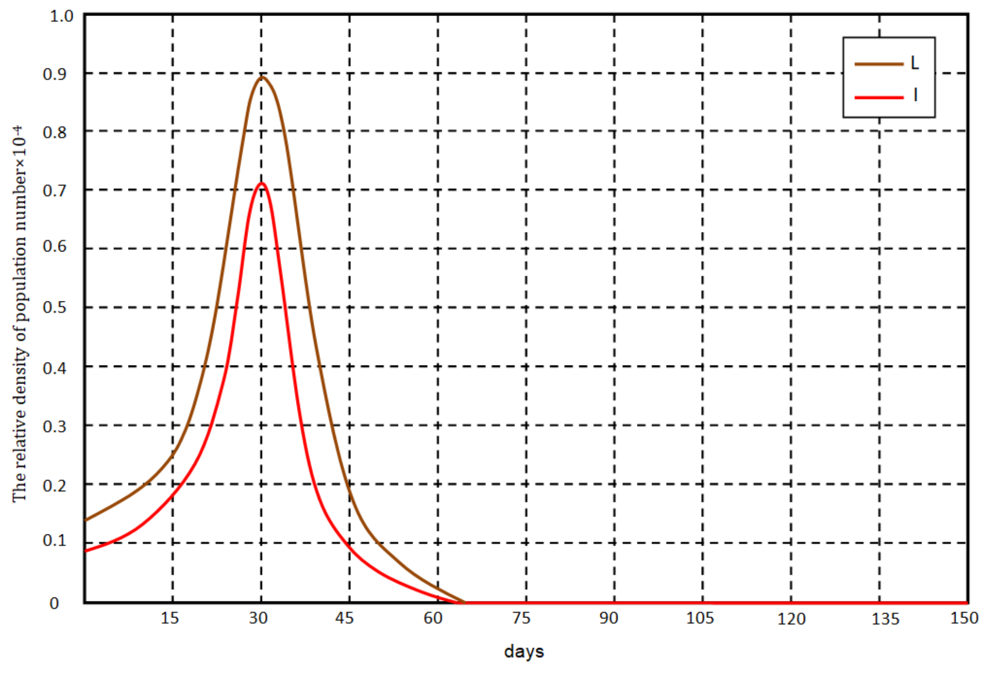

Figure2 Density of L_k and I_k versus time(R_0=0.58 < 1)

Noting that $0 < \rho_l < 1/l$, one has: $l \to \infty$, $\rho \to 0$

taking $l \to \infty$, it follows from (4.27) that

$$X_k = \frac{V(k)}{\alpha(k)h+\mu} \qquad Y_k = \frac{\alpha(k)HX_k}{\beta+\gamma+\mu}$$
$$Z_k = \frac{\gamma Y_k}{\omega+\delta+\mu} \qquad W_k = \frac{\beta Y_k + \delta Z_k}{\tau+\zeta+\mu} \tag{4.29}$$
$$x_k = \frac{V(k)}{\alpha(k)H+\mu} \qquad y_k = \frac{\alpha(k)hx_k}{\beta+\gamma+\mu}$$
$$z_k = \frac{\gamma y_k}{\omega+\delta+\mu} \qquad w_k = \frac{\beta y_k + \delta z_k}{\tau+\zeta+\mu}$$

$$h = \frac{1}{\langle k \rangle}\sum_{i=1}^{n} \frac{\varphi_1(i)P(i)}{\eta_i} w_i + \frac{1}{\langle k \rangle}\sum_{i=1}^{n} \frac{\varphi_2(i)P(i)}{\eta_i} z_i \qquad H = \frac{1}{\langle k \rangle}\sum_{i=1}^{n} \frac{\varphi_1(i)P(i)}{\eta_i} W_i + \frac{1}{\langle k \rangle}\sum_{i=1}^{n} \frac{\varphi_2(i)P(i)}{\eta_i} Z_i \tag{4.30}$$

Further

$$w_k = \frac{\beta}{\tau+\zeta+\mu}\frac{\alpha(k)h}{\beta+\gamma+\mu}\frac{V(k)}{\alpha(k)H+\mu} + \frac{\delta}{\tau+\zeta+\mu}\frac{\gamma}{\omega+\delta+\mu}\frac{\alpha(k)h}{\beta+\gamma+\mu}\frac{V(k)}{\alpha(k)H+\mu} \tag{4.31}$$

$$W_k=\frac{\beta}{\tau+\zeta+\mu}\frac{\alpha(k)H}{\beta+\gamma+\mu}\frac{V(k)}{\alpha(k)h+\mu}+\frac{\delta}{\tau+\zeta+\mu}\frac{\gamma}{\omega+\delta+\mu}\frac{\alpha(k)H}{\beta+\gamma+\mu}\frac{V(k)}{\alpha(k)h+\mu} \quad (4.32)$$

$$z_k=\frac{\gamma}{\omega+\delta+\mu}\frac{\alpha(k)h}{\beta+\gamma+\mu}\frac{V(k)}{\alpha(k)H+\mu} \qquad Z_k=\frac{\gamma}{\omega+\delta+\mu}\frac{\alpha(k)H}{\beta+\gamma+\mu}\frac{V(k)}{\alpha(k)h+\mu} \quad (4.33)$$

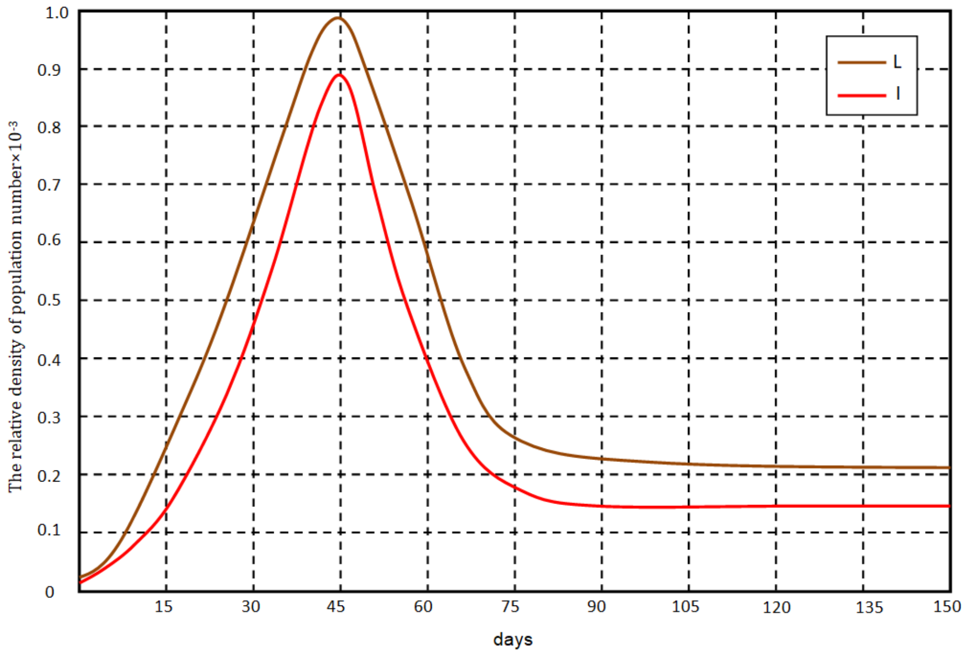

Figure3 Density of L_k and I_k versus time(R_0=1.68 > 1)

Substituting (4.31, 4.32, 4.33) into h and H(4.30), respectively, one has

$$1=\frac{1}{\langle k\rangle}\sum\frac{\varphi_1(k)P(k)}{\eta_k}\frac{\beta}{\tau+\zeta+\mu}\frac{\alpha(k)}{\beta+\gamma+\mu}\frac{V(k)}{\alpha(k)H+\mu}+\frac{1}{\langle k\rangle}\sum\frac{\varphi_1(k)P(k)}{\eta_k}\frac{\delta}{\tau+\zeta+\mu}\frac{\gamma}{\omega+\delta+\mu}\frac{\alpha(k)}{\beta+\gamma+\mu}\frac{V(k)}{\alpha(k)H+\mu}+\frac{1}{\langle k\rangle}\sum\frac{\varphi_2(k)P(k)}{\eta_k}\frac{\gamma}{\omega+\delta+\mu}\frac{\alpha(k)}{\beta+\gamma+\mu}\frac{V(k)}{\alpha(k)H+\mu}$$

$$1=\frac{1}{\langle k\rangle}\sum\frac{\varphi_1(k)P(k)}{\eta_k}\frac{\beta}{\tau+\zeta+\mu}\frac{\alpha(k)}{\beta+\gamma+\mu}\frac{V(k)}{\alpha(k)h+\mu}+\frac{1}{\langle k\rangle}\sum\frac{\varphi_1(k)P(k)}{\eta_k}\frac{\delta}{\tau+\zeta+\mu}\frac{\gamma}{\omega+\delta+\mu}\frac{\alpha(k)}{\beta+\gamma+\mu}\frac{V(k)}{\alpha(k)h+\mu}+\frac{1}{\langle k\rangle}\sum\frac{\varphi_2(k)P(k)}{\eta_k}\frac{\gamma}{\omega+\delta+\mu}\frac{\alpha(k)}{\beta+\gamma+\mu}\frac{V(k)}{\alpha(k)h+\mu}$$

By subtracting the above two equations, we arrive at

$$0=\frac{1}{\langle k\rangle}\sum\frac{\varphi_1(k)P(k)}{\eta_k}\frac{\beta}{\tau+\zeta+\mu}\frac{\alpha(k)V(k)}{\beta+\gamma+\mu}\frac{\alpha(k)(h-H)}{(\alpha(k)H+\mu)(\alpha(k)h+\mu)}+\frac{1}{\langle k\rangle}\sum\frac{\varphi_1(k)P(k)}{\eta_k}\frac{\delta}{\tau+\zeta+\mu}\frac{\gamma}{\omega+\delta+\mu}\frac{\alpha(k)V(k)}{\beta+\gamma+\mu}\frac{\alpha(k)(h-H)}{(\alpha(k)H+\mu)(\alpha(k)h+\mu)}+\frac{1}{\langle k\rangle}\sum\frac{\varphi_2(k)P(k)}{\eta_k}\frac{\gamma}{\omega+\delta+\mu}\frac{\alpha(k)V(k)}{\beta+\gamma+\mu}\frac{\alpha(k)(h-H)}{(\alpha(k)H+\mu)(\alpha(k)h+\mu)}$$

It is obvious that: H=h

$$H-h=\frac{1}{\langle k\rangle}\sum_{i=1}^{n}\frac{\varphi_1(i)P(i)}{\eta_i}(W_i-w_i)+\frac{1}{\langle k\rangle}\sum_{i=1}^{n}\frac{\varphi_2(i)P(i)}{\eta_i}(Z_i-z_i)=0$$

That $W_i=w_i \quad Z_i=z_i \qquad i=1,2,3,\ldots n$

we can arrive at $W_i=w_i \quad X_i=x_i \quad Y_i=y_i \quad Z_i=z_i \qquad i=1,2,3\cdots n$

It follows that:

$$\lim_{t\to\infty}S_k(t)=X_k=x_k \qquad \lim_{t\to\infty}E_k(t)=Y_k=y_k \qquad \lim_{t\to\infty}L_k(t)=Z_k=z_k \qquad \lim_{t\to\infty}I_k(t)=W_k=w_k$$

Finally, substituting h = H into (4.28), in view of (4.2) and (4.29), we obtain:

$S_k=S_k^*$ $E_k=E_k^*$ $L_k=L_k^*$ $I_k=I_k^*$ $R_k=R_k^*$

that is: $X_k=(S_k, E_k, L_k, I_k, R_k)^T=X_k^*=(S_k^*, E_k^*, L_k^*, I_k^*, R_k^*)^T$

The proof is completed

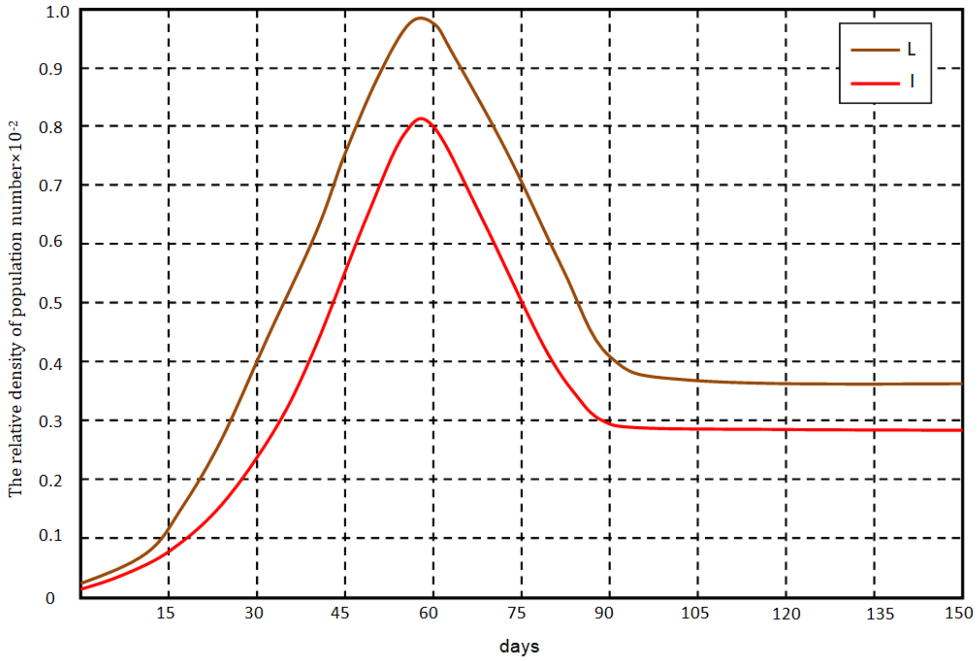

Figure4 Density of L_k and I_k versus time(R_0=3.01 > 1)

V. SIMULATION RESULTS AND ANALYSIS

In this section, we present several numerical simulations to illustrate the analysis results. We take the community degree distribution to be $P(k) = ck^{-l}$ $(2 < l \leq 3)$, in which $l = 3$ and c satisfies $\sum_{k=1}^{n} P(k) = 1$, n = 1000. We choose $\alpha(k) = \alpha k$, $\varphi_1(k) = k$, $\varphi_2(k) = 0.7k$, $V(k) = v/n$.

In Fig. 2, the parameters are chosen as, β= 0.001, γ= 0.0015, μ=0.007, τ=0.9, ζ=0.06, δ= 0.3, ω=0.6, $R_0 = 0.58 < 1$. The L_k peak value of asymptomatic infection reaches 0.89×10^{-4}, and the I_k peak value of symptomatic infection reaches 0.71×10^{-4}. After 65 days, all the cases are cleared and achieve good epidemic prevention effect.

In Fig. 3, the parameters are chosen as, β= 0.006, γ= 0.001, μ=0.007, τ=0.9, ζ=0.06, δ= 0.3, ω=0.5, $R_0 = 1.68 > 1$. the L_k peak value of asymptomatic infection reaches 0.98×10^{-3}, and the I_k peak value of symptomatic infection reaches 0.89×10^{-3}, Infected people always exist, and finally reach equilibrium. The effect of epidemic prevention is not satisfactory. Because the infected people always exist, once the epidemic prevention is relaxed, the epidemic will break out again.

In Fig. 4, the parameters are chosen as, β= 0.1, γ= 0.15, μ=0.007, τ=0.9, ζ=0.06, δ= 0.3, ω=0.5, $R_0 = 3.01 > 1$. the L_k peak value of asymptomatic infection reaches 0.98×10^{-2}, and the I_k peak value of symptomatic infection reaches 0.82×10^{-2}, Infected people always exist. Finally, a balance of high infection rate is achieved.

In Fig. 5, the parameters are chosen as, β= 0.006, γ= 0.001, μ=0.007, τ=0.9, ζ=0.06, δ= 0.3, ω=0.5, $R_0 = 1.68 > 1$. The Fig. 5 describe the time series of the infected individuals I_k with different degree. Obviously, we can see that the positive level will be higher with the increase of degree. The more network connections, the more contacts between people, and the larger R_0, which is conducive to the spread of the COVID-19. We find that the larger degree leads to a larger value of the spreading level. Therefore, reducing the gathering and flow of people is conducive to reducing the spread of the COVID-19.

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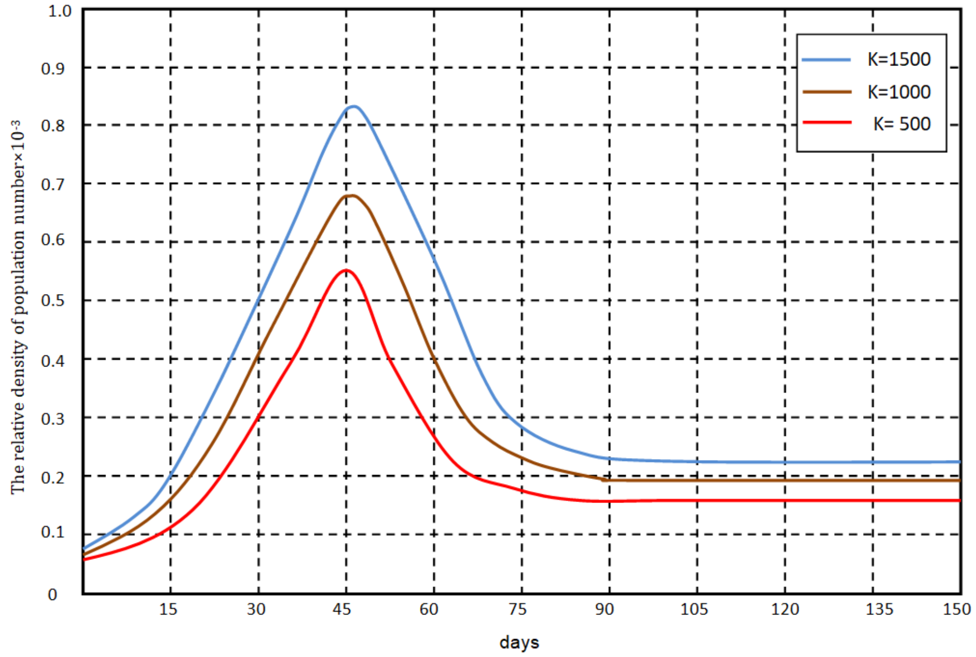

Figure5 Dynamic behavior of I_k with different degree (R_0=1.68 > 1)

VI. CONCLUSION

In this paper, we propose a new SELIR COVID-19 spreading model in complex social networks. With the mean field theory, the spreading dynamics of the model is analyzed in detail. By Lyapunov's first method, it is proved that the COVID-19 free equilibrium X_0 is locally asymptotically stable, and the basic reproduction number R_0 is obtained. The global asymptotic stability of the COVID-19 free equilibrium X_0 is proved with Lyapunov's second method. The basic reproduction number R_0 not only determines the existence of COVID-19 equilibrium, but also determines the global dynamic behavior of the model. If $R_0 < 1$, then X_0 is globally asymptotically stable. No matter what the initial value of the infected individuals, the infected individuals will disappear eventually. If $R_0 > 1$, then the COVID-19 prevailing equilibrium X_k^* is globally asymptotically attractive, it is that the infected individuals will continue to exist and converge to a positive stable level.

We also investigate the influence of some model parameters on COVID-19 spreading. With the increase of degree, the COVID-19 spreading level will be higher. The larger $\frac{\langle\alpha(k)\varphi_1(k)\rangle}{\langle k\rangle}$ and $\frac{\langle\alpha(k)\varphi_2(k)\rangle}{\langle k\rangle}$, the larger R_0, which will accelerate the spread of the epidemic. we can limit population mobility and Gathering activities, isolate close contacts, reduce $\frac{\langle\alpha(k)\varphi_1(k)\rangle}{\langle k\rangle}$ and $\frac{\langle\alpha(k)\varphi_2(k)\rangle}{\langle k\rangle}$, and thus reduce R_0 and the spread of the virus.

The higher the infection coefficient β and γ, the more serious the COVID-19 transmission. The larger τ and ω are, the smaller R_0 is, which will weaken the transmission of the virus. it is of great significance to take appropriate treatment to improve the cure rate for epidemic prevention and control. These results will help to formulate policies to curb COVID-19. The study has important significance for effectively predicting and preventing COVID-19 spreading.

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