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R/S Analysis on Hurst Exponent of Umbilical Artery Blood Flow Time Series

Kaijun Yu[1], Zhongwei Jiang[2]
[1]*(Graduate School of Science Technology for Innovation,Yamaguchi University , Japan)*
[2]*(Micro Mechatronics Laboratory, Yamaguchi University, Japan)*

ABSTRACT: *Aiming at the nonlinear characteristics of umbilical artery blood flow time series featured by both determinacy and randomness, a study was conducted on the precision of R/S analysis for different FGN time series ,while the Hurst exponent of different umbilical artery blood flow time series were researched using R/S theory. The results showed that for the umbilical artery blood flow time series with a length of 4 seconds (4400), the results obtained by Lo analysis were the most accurate. Umbilical artery blood flow time series has the characteristics of "1/f fluctuation" and shows long-term correlation; The time series is highly predictable.*
KEYWORDS -*Hurst exponent, R/S analysis method, Time series, Umbilical cord blood flow*

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I. INTRODUCTION

Umbilical artery blood flow signal is a kind of complicated time series signal with obvious nonlinear characteristics as a medical diagnosis signal.It is an important way to monitor fetal health and prevent maternal and infant diseases by analyzing the signal of umbilical blood flow.

Hurst exponent, as an important indicator of nonlinear characteristic analysis of data signals, is widely used in time series data analysis in the middle of the 20th century.[1]The Hurst exponent can be calculated by R/S method, Whittle method, period graph method and aggregation scale absolute value method. As a non-parametric analysis method, R / S analysis method has been applied to the study of nonlinear systems in many fields. [2]The biggest advantage of this method is that it is not necessary to assume whether the measure of time series is normally distributed or not, and the R/S method can obtain effective and reliable results.R/S analysis method can analyze the fractal characteristics and long-range correlation of time series signals based on Hurst exponent, so as to distinguish the randomness and non-randomness of the time series.[3] Fetal umbilical artery blood flow signals do not have random non-linear time series signals. Therefore, Hurst exponent is used to study them to explore the signal characteristics and predictability of umbilical artery blood flow, which can provide early theoretical reference for using umbilical artery blood flow signal time series to carry out fetal early diagnosis, disease monitoring and early treatment.

II. HURSTEXPONENT

Hurst exponent *(H*) is a statistic between 0 and 1. *H* values in different ranges represent different meanings. It can be used to measure the long-term correlation and self-similarity of time series.[4] Long range correlation reflects the statistical correlation of two time series data within a period of time, while self-similarity reflects the similarity of time series fluctuations.For a time series:

(1) $H > 1$indicates that the time series is an unstable signal;The time series is a random signal similar to Brownian noise when H=1.5.

(2) $H \approx 1$indicates that the process is "1/ f fluctuation". This time series is highly predictable, and data signals in the future can be predicted based on current data signals when $H \approx 1$

(3) $1/2 < H < 1$ indicates that the time series has long-term positive correlation, that is, if the fluctuation characteristic of a time series signal increases in a certain interval, the probability of the next interval also shows an increasing trend, and vice versa;When the H value is between 1/2 and 1, the larger the H value is, the stronger the long range correlation of time series signal is, and vice versa.

(4) $H \approx 1/2$ indicates that the time series signal has no long-range correlation and is a random process similar to "white noise". The time series shows the characteristics of normal distribution, which obeies the standard geometric Brownian motion. The variables are completely random and unrelated. The current data signal has no influence on the future data signal, and the predictability is poor.

(5) $0 < H < 1/2$ indicates that time series has long-term negative correlation, that is, if time series shows a downward trend in a certain interval, it is highly likely to show an increasing trend in the next interval, and vice versa.

III. R/S ANALYSIS METHOD

3.1 CR/S analysis method

H E Hurst proposed CRS analysis method,[5] the CRS analysis principle is as follows:

(1) The length of time series $\{P(t)\}$ is N, which is divided into non-overlapping continuous sub-intervalswith growth degree of $N, I_a(a=1,2,\cdots,A)$, where each element is I_a $p_{k,a}$, $k=1,2,\cdots,n$.

(2) For each sub-interval, calculate its standard deviation S_I, cumulative mean deviation $\{x_{k,a}\}$ and range R_I respectively.

$$S_I = \sqrt{\frac{1}{n}\sum_{k=1}^{n}(P_{k,a}-e_a)^2} \tag{1}$$

$$x_{k,a} = \sum_{i=1}^{k}(P_{i,a}-e_a) \tag{2}$$

$$R_I = \max_{1\le k\le n}(x_{k,a}) - \min_{1\le k\le n}(x_{k,a}) \tag{3}$$

Where, e_a is the mean value of the sequence I_a.

(3) Calculate the rescale range (R_I / S_I) of each sub-interval and the average rescale range $(R/S)_n$ of A interval:

$$(R/S)_n = (1/A)\sum_{a=1}^{A}(R_I / S_I) \tag{4}$$

(4) Change the length *n* in step (1) and repeat Step (1)-(3) to calculate the range $(R/S)_n$ under different sub-interval lengths *n*.There is a linear relationship between $(R/S)_n$ and $\log(R/S)_n$:

$$\lg(R/S)_n = \lg\theta + H\lg n \tag{5}$$

(5) The slope H is obtained by linear fitting the sub-interval length and average range drawn on the log-log-coordinate graph by the least square method,which is the Hurst exponent.

3.2 Method of Lo

In 1991, Loconsidered that the CRS analysis method will be biased if the time series show strong short-term correlation.[6]Therefore, Lo revised CRS analysis by introducing covariance:

$$\frac{R}{S}(n) = \frac{1}{A}\sum_{a=1}^{A}\frac{1}{\hat{S}_{a,q}}([\max_{1\le k\le n}(x_{k,a}) - \min_{1\le k\le n}(x_{k,a})]) = \frac{1}{A}\sum_{a=1}^{A}\frac{R_I}{\hat{S}_{a,q}} \tag{6}$$

$$\begin{aligned}\hat{S}_a^2(q) &= \frac{1}{n}\sum_{k=1}^{n}(m_{k,a}-\bar{X}_a)^2 + \frac{2}{n}\sum_{j=1}^{q}\omega_j(q)[\sum_{k=1}^{n}(m_{k,a}-\bar{X}_a)(m_{k-j,a}-\bar{X}_a)] \\ &= S_a^2 + 2\sum_{j=1}^{q}\omega_j(q)\gamma_j\end{aligned} \tag{7}$$

In the formula, $S_a^2 = \frac{1}{n}\sum_{k=1}^{n}(m_{k,a}-\bar{X}_a)^2$, $\omega_j(q) \equiv 1-\frac{j}{q+1}$ ($q<n$) is called Bartlett weight, σ_a^2 is the variance of the *a*-th a $(a=1,2,3\ldots A)$ sub sample $\{m_{k,a}\}_{k=1}^{n}$,and γ_j is the sample autocovariance of order *j*.

$\{m_{k,a}\}_{k=1}^{n}$. How to select *q* value directly affects the accuracy of R/S analysis results. Therefore, Lo provides the optimal selection formula of *q* :

$$q_{opt} = \text{int}[(\frac{3n}{2})^{1/3}(\frac{2\hat{\rho}}{1\text{-}\hat{\rho}})^{2/3}] \tag{8}$$

$$\frac{R}{S}(n) = \frac{1}{A}\sum_{a=1}^{A}(\frac{1}{A}\frac{Var(S_1^*,\cdots,S_a^*)}{\hat{S}_{a,q}^2}) \tag{9}$$

Where, $\hat{\rho}$ is the estimated value of the first-order auto-correlation function.Lo method is CRS method when $q=0$ .Because the short-term correlation of time series signals is removed, Lo method is better for detecting the long-term correlation of time series.

$$S_k^* = \sum_{j=1}^{a}(m_{k,j} - \bar{X}_j) \tag{10}$$

$$Var(S_1^*,\cdots,S_a^*) = \frac{1}{a}\sum_{n=1}^{N}(S_j^* - \bar{S}_a^*)^2 \tag{11}$$

3.3 V/S method

Some scholars believe that time series with significant long-term correlationcannot be accurately dealed with by Lo analysis method.Therefore, V/S analysis method based on Lo analysis method is proposed:

$$(R/S)_n = \frac{1}{A}\sum_{a=1}^{A}(\frac{1}{A}\frac{Var(S_{k,M_1},\text{L} \ ,S_{k,M_a})}{\hat{S}_{a,q}^2}) \tag{12}$$

IV. FGN AND GOODNESS -OF -FIT TESTS

In order to determine the optimal exponential analysis method of umbilical artery flow Doppler signal, the power spectrum Fast Fourier Transform(FFT) method is used to generate the sequence data of Fractional Gaussian Noise (FGN) with known exponent. Three R/S analysis methods were compared to determine the best method to calculate the Hurst exponent of umbilical arterytime series in our research.

4.1 FGN

J.w. Vanness and Mandelbrot put forward the complete long memory model in 1968.[7]Thesequence of fractal Gaussian noise (FGN) is defined as the first-order difference process of fractal Brownian motion with Hurst exponent $B_H(t)$, and its self-covariance function of order k is: $\{y_t \,|\, y_t = B_H(t+1) - B_H(t)\}_{t=0}^{\infty}$ k

$$\gamma_y(k) = \frac{\sigma^2}{2}(|k+1|^{2H} - 2|k|^{2H} + |k-1|^{2H}) \tag{13}$$

Where, $k = \ldots$,-1, 0, 1, $\ldots, \sigma^2 = Var(y_t)$ is any positive number, and *H* is the Hurst exponent between 0 and 1.

FGN sequence can be generated by power spectrum FFT: firstly, the density function of Fractional Brownian Motion (FBM) is constructed, and the corresponding FBM sequence is obtained by inverse transformation of the density function.Then the first order difference is adopted for FBM sequence and the Hurstexponent (*H*) is selected reasonably to realize the simulation of FGN sequence.

4.2 Evaluation results of R/S analysis method based on FGN sequence

In our research, FGN time series with known Hurstexponent is reverse-generated to simulate real umbilical artery blood flow signals, and the three methods are verified to obtain the best method for analyzing Hurstexponent of umbilical artery blood flow signals.Five kinds of FGN time series with *H* values between 0.5 and 0.9 are generated by power spectrum FFT method, and the accuracy of Hurst exponent calculation of three kinds of analytical methods is compared and studied.In our research, the time series length of each umbilical artery flow Doppler signal is 44,000, so FFT simulation is used to generate FGN sequences of the same length. Since FGN sequences are randomly generated, 100 FGN sequences are generated under each specific H value to evaluate the three R/S analysis methods respectively. Then calculate the mean value of Hurstexponent of these 100 FGN sequences, and the calculation formula is as follows:

$$\bar{H} = \frac{1}{100}\sum_{i=1}^{100}\hat{H}_i \tag{14}$$

Where, $\hat{H}_i$ is the Hurst estimation value of 100 randomly generated FGN sequences.The results are as shown in Figure 1.

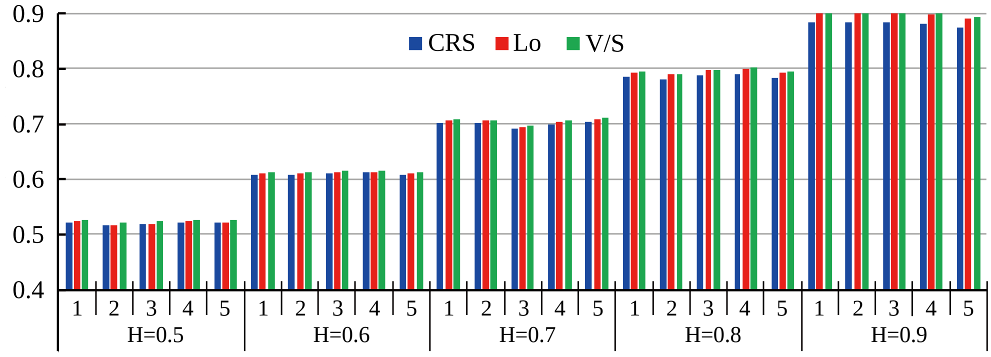

Fig. 1 Mean value of Hurst exponent for FGN sequence

In the above figure, the *X* coordinate is the number of runs and the ideal Hurst exponent, and the *Y* coordinate is the average Hurst exponent. According to the Figure 1 , when the length of FGN time series is 44,000, the Hurst exponent obtained by three R/S analysis methods is as follows: when *H*=0.5 and *H*=0.6, the average Hurst exponent obtained by the three methods is slightly larger than the true value, but CRS method is the closest to the true value. Lo method and V/S method both have large errors, among which V/S method has the largest error.When H=0.7, the Hurst exponent of the three methods are both lower than the true value and greater than the true value, but the calculation results of CRS method are still better and closer to the true value,when H=0.8 and H=0.9, the Hurst exponent of CRS method are both smaller than the true value and have the largest error, while the calculation results of Lo method and V/S method are close to the true value and there is little difference between the two methods, that is to say: If the Hurst exponent of the time series of umbilical artery flow Doppler signal is between 0.5 and 0.7, the CRS method is used to analyze the Hurst exponent.However, if the Hurst exponent of the time series of umbilical artery flow Doppler signal is around 0.8~0.9, the Lo method is more accurate.When calculating Hurst exponent of time series of umbilical artery blood flow Doppler signal with length of 44,000, it is found that its value is around 1. Therefore, Lo method is selected in this research.

V. HURST EXPONENT ANALYSIS OF DOPPLER SIGNALS OF UMBILICAL ARTERY FLOW

Based on the above FGN time series and the verification results of three R/S analysis methods, it can be seen that Lo method is more accurate for Hurst index analysis of 44000-length umbilical artery signal time series.Therefore, Lo method was adopted in this section to study umbilical artery blood flow signals and goodness -of -fit tests of the calculation process to ensure the accuracy of the calculation results.

According to the Hurst exponent analysis and goodness -of -fit tests mentioned above, Hurst exponent analysis was performed on time series of 12 normal umbilical artery blood flow, and the results are shown in Figure 2.

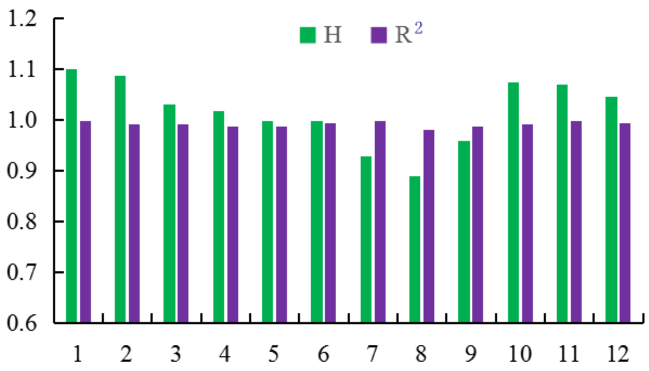

Fig. 2 R/S analysis and goodness of fit test results

In Figure 2, the X coordinate is the blood flow time series of 12 umbilical arteries, and the Y coordinate is the Hurst exponent. It can be seen from Figure 2 that the Hurst exponent of time series of umbilical artery blood flow signal based on Lo method is different from each other, the maximum is 1.0898, the minimum is 0.87919. However, the Hurst index of all the time series of umbilical artery flow signals is close to 1.The results showed that the time series of signal of umbilical artery blood flow showed a long range positive correlation and was consistent with the characteristics of "1/*f* fluctuation".In addition, goodness of fit test was carried out for 12 umbilical artery blood flow samples. If the fitting results of Hurst exponent calculation process were better, the value of R^2 is between 0.990 and 0.999.It indicates that the time series of umbilical artery signal has obvious self-similarity.

VI. CONCLUSION

In this research, Hurst exponent analysis is performed on the time series of 12 umbilical artery flow signals based on Lo method.The results show that the time series of signal of umbilical artery flow is non-stationary and shows obvious "1/ *f* fluctuation" characteristic, which is highly predictable.

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