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Geometrical Dynamics of Volume Implies Schrödinger Equation

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ABSTRACT: *Today, quantum physics [1,2] and general relativity, GR, [3,4] are the two essential fundamental theories in physics [5–7]. However, as we live in one world, these two theories should be unified [8]. In this paper, we show that the volume is an essential dynamical quantity, the dynamic volume, DV. And we derive the volume dynamics, VD. For it, we use the dynamics of GR. From the VD, we derive a cornerstone of the unification, the Schrödinger equation [9, Eq. 1.1]. Hereby, the DV is a part of reality [5], as its amount and its energy density u_{vol} (dark energy) [10–12] can be observed. Therefrom, we derive the deterministic dynamical equation of quanta - the Schrödinger equation [13–16] - and we derive the stochastic dynamics of quanta [17–19], as well as the Hilbert space structure of quanta [9,20]. On that basis, the postulates of quanta [21–23] and the energy density of dark energy [21,24,25] have been derived. All results are in precise accordance with observation, whereby neither a hypothesis is introduced, nor an ununified fit parameter is proposed, nor a fit is executed, nor a universal constant of nature is modified. Thus, a high unifying power is achieved [8].*

KEYWORDS – *general relativity, gravity, quantum physics, unification*

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1. INTRODUCTION

In 1905, Einstein [26] introduced special relativity, SR, describing a system or frame moving at a velocity v relative to a rest frame, whereby no gravity or acceleration are present. As results, time dilation, length contraction, the Lorentz transformation and the Minkowski metric have been derived, for instance. In 1915, Einstein [3] introduced general relativity, GR, describing a system with gravity or acceleration. As a result, a curvature of spacetime is obtained. It can be described by the Einstein field equation [3,4,27–30]. In particular, in the vicinity of a mass M, the Schwarzschild metric [31] is derived. Based on GR, Einstein [32] showed in 1917 that the universe could expand in the course of time. Indeed, Hubble observed galaxies and thereby, he confirmed such an expansion of space [33]. Moreover, such an expansion of space has been confirmed by many quite different observations [34–47]. Furthermore, SR and GR have been confirmed by many observations [48]. In 1900, based on observation, Planck [2] discovered the quantization and the Planck constant h of quantization. In 1926, Schrödinger [13] proposed a differential equation for quanta, the Schrödinger equation, SEQ. The SEQ turned out to describe the basic dynamics of quanta. Moreover, the SEQ became the basis for many quantum systems [49,50]. Accordingly, the SEQ is inherent to the postulates of quantum physics [20–23,51,52]. In 1935, Einstein [5] proposed that quantum theory would be incompatible with relativity. However, as we live in one world only, quantum physics and relativity should be unified [8], whereby no ad hoc hypothesis or unified fit parameter should be used, if possible. How can this be achieved? Perlmutter [10], Riess [11] and Smoot [12] discovered the accelerated expansion of the universe. According to Einstein [32], such an accelerated expansion can be explained by GR with help of an additional constant, which he named cosmological constant Λ. That constant corresponds to an energy density u_{vol} of the corresponding density $\rho_{\text{vol}} = \frac{u_{\text{vol}}}{c^2}$ of volume [29, Eq. 15.4, p. 389]. The energy $\delta E_{\text{vol}} = u_{\text{vol}} \cdot \delta V$ of the energy density u_{vol} has been called dark energy [53, abstract]. In the present paper, the term volume describes that volume with the energy density u_{vol}. According to Einstein, Rosen and Podolski [5], a physical quantity that can be observed or confirmed by observation should be regarded as an element of physical reality. In this sense, volume is an element of physical reality, as volume has been observed in the form of its amount δV, of its energy density u_{vol} and of its energy $\delta E_{\text{vol}} = u_{\text{vol}} \cdot \delta V$. According to observation and derivation, the energy density u_{vol} amounts to approximately 67 % of all energy or mass of the universe [21,24,46]. In this paper, we apply volume in order to derive an essential dynamical basis of quantum physics, the Schrödinger equation, SEQ. Note that by using that derivation of the SEQ, all postulates of quantum physics have been derived, and a unification of quantum physics, relativity and gravity has been elaborated [21–23,54]. Thereby, no ad hoc hypothesis has been proposed, no ununified fit parameter has been introduced, no fit has been executed, no universal constant of nature has been modified, and precise accordance with observation has been achieved. The paper is organized as follows. We present possible measures of observable distances and their application to the Schwarzschild metric in section two. We derive additional volume as well as relative additional volume, which explains curvature, gravitational fields and gravitational potentials, see sections 3.1 and 3.2. In section 3.3, we derive the propagation, including the formation, of relative additional volume. We describe the dynamics of relative additional volume with a Lorentz scalar, and we discover the general nature of that scalar in part 3.4. In sections 3.5 and 3.6, we derive waves of relative additional volume, and we derive the SEQ. We conclude our findings in part four. We provide a glossary for some technical terms in section five.

In this text, if we use units, then we use SI units only, see e. g. [55]. In this paper, we use a spaceship as a probe mass that is in the vicinity of a mass M. Thereby, the influences of the probe mass upon the curvature of spacetime, upon the gravitational field and upon the gravitational potential are negligible, as usual, see e. g. [3,27,29,30,56].

In order to provide a transparent and exact basis for our investigation, we clarify the concept of volume in more detail in the following sections 1.1 towards 1.3.

1.1 Definition of the dynamical density of volume in nature

In this section, we define the energy density u_{vol} of the volume, and we distinguish it from energy densities of vacuum. By definition, the energy density of a portion of volume δV is the ratio of the energy inherent to the volume δE_{vol} and δV:

$$u_{vol} = \frac{\delta E_{vol}}{\delta V}$$

In cosmology, an energy density u is usually described by a corresponding dynamical density or density ρ. Thereby, the dynamical density ρ is equal to energy density u divided by the square of the velocity of light c^2, see e. g. sections 15.1 and 15.2 in [29]:

$$\rho = \frac{u}{c^2}$$

Accordingly, we derive the dynamical density of volume in nature:

$$\rho_{vol} = \frac{u_{vol}}{c^2} = \frac{\delta E_{vol}/c^2}{\delta V}$$

As the volume in nature is a geometrical quantity, and as the volume in nature should describe the extension of space at all times in a coherent manner, we define that the dynamical density of volume ρ_{vol} does not change as a function of time. Accordingly, and altogether, we define that the energy density u_{vol} of a portion of volume δV is the energy δE_{vol} that is inherent to that portion δV and that does not change as a function of time.

In contrast, several different energy densities of vacuum have been proposed in physics. For instance, an energy density vacuum caused by the electromagnetic field has been proposed, see e. g. section 19.3 in [51]. For example, the vacuum expectation value, vev, has been proposed in the theory of the weak interaction, see e. g. sections 10 and 11 in [55]. Accordingly, we make a difference between the dynamical density of the volume in nature and the various proposed densities of the vacuum. According to this distinction, we derive the geometrical dynamics of volume, as announced in the title of the paper.

1.2 Identification of the dynamical density of volume

In this section, we analyze the expansion of space since the Big Bang, in order to identify the energy density u_{vol} of the volume. In a very good and popular approximation, the expansion of space since the Big Bang can be described in an isotropic manner, see e. g. [29]. Accordingly, the time evolution of that expansion can be described by the time evolution of a radius $R(t)$ of a prototypical ball of the universe, see e. g. [29] or [24]. Usually, the time derivative $\dot{R}$ per radius R is analyzed, that ratio is called Hubble parameter H:

$$H = \frac{\dot{R}}{R}$$

In a very good and popular approximation, the expansion can be modeled by a homogeneous universe, see e. g. [24] or [29]. Note that the consequences of heterogeneity have been analyzed and are in precise accordance with observation, see e. g. [24]. Moreover, in another very good and popular approximation, the global curvature of space is zero, see e. g. [24,29,46]. A space with zero curvature is called flat. Altogether, in an isotropic and homogeneous universe with zero global curvature, the Hubble parameter H is the following function of the density ρ, whereby the gravitational constant is marked by G [24,29]:

$$H^2 = \frac{8\pi G}{3} \cdot \rho$$

For the purpose of an analysis of the time evolution of the expansion of the universe, the density is classified into three components: The density of matter ρ_m is proportional to R^{-3}, the density of radiation ρ_r is proportional to R^{-4}, and the density of volume ρ_{vol} is constant (proportional to R^{-0}), see e. g. [29]:

$H^2 = \frac{8\pi G}{3} \cdot (\rho_r + \rho_m + \rho_{vol})$, with $\rho_m \propto R^{-3}$ and $\rho_r \propto R^{-4}$ and ρ_{vol} is constant

Note that the constant density ρ_{vol} is denoted by ρ_Λ in [29], and it is named density of vacuum, see p. 389 in [29]. Application of the time derivative to the above equation provides the following result:

$$\frac{R \cdot \ddot{R}}{\dot{R}^2} = \frac{4\pi G}{3H^2} \cdot (-2\rho_r - \rho_m + 2\rho_{vol})$$

In 1998, observations showed that the universe expands in an accelerated manner, so that the above fraction $\frac{R \cdot \ddot{R}}{\dot{R}^2}$ is positive [10-12,46]. The above equation shows that the density of volume must be positive, as a consequence. More precisely, the observation shows that the density of volume ρ_{vol} amounts to ca. 67 % of the whole density of the universe. Additionally, this result has been confirmed theoretically, see e. g. [24].

Altogether, as the density of volume is constant, it can cause an accelerated expansion of the universe (see above Eq.). Observations show that the constant density amounts to ca. 67 % of the whole density of the universe. Analysis shows that the constant density amounts to 66,7 % of the density of the universe [24]. The observed energy density $u_{vol} = \rho_{vol} \cdot c^2$ has been named dark energy [53]. However, the volume and vacuum are not distinguished in [53]. Thus, we identify the density of volume with the density that causes an accelerated expansion according to the above equation. Moreover, we realize that many people call the energy density causing the accelerated expansion dark energy, whereby they do not distinguish between volume and vacuum. That distinction is essential, as the densities of proposed forms of vacuum [51,55] differ by dozens of orders of magnitude from the observed constant density ρ_{vol}.

1.3 Analysis of the global dynamics of volume

In the case of the isotropic and homogeneous universe with zero global curvature, the rate $\dot{V}$ of formation of volume is derived from the above equation $H^2 = \frac{8\pi G}{3} \cdot \rho$. In that case, the volume of the prototypical ball is described by the usual term of Euclidean geometry:

$$V = \frac{4\pi}{3} \cdot R^3$$

As a consequence, volume is formed in an isotropic manner. Thereby, the rate of formation of relative additional volume $\frac{\dot{V}}{V}$ can be derived by analyzing the derivative. Hereby, the following rate is derived [24]:

$$\frac{\dot{V}}{V} = \frac{3\dot{R}}{R} = 3H$$

However, in the vicinity of a mass M, the space is neither homogeneous nor flat. As a consequence, the additionally formed volume and its rate are quite different from the above global formation of volume. The local formation of volume is analyzed in the following. For it, the formation of volume in the vicinity of a mass M is analyzed with help of the Schwarzschild metric, see sections II and III. Later, the obtained results are generalized, and the volume dynamics, VD, are derived, see sections 3.3 and 3.4. Finally, we show that the VD implies the Schrödinger equation, the corner stone of quantum physics, see section 3.5. More generally, the VD is an essential basis for the unification of spacetime, gravity and quanta [21-23].

II. SCHWARZSCHILD METRIC

In this section, we summarize the Schwarzschild metric. In general, a metric can be described with help of a line element ds in four dimensional spacetime [3,4,26,27,29,56–58]. For instance, flat spacetime can be described in terms of spherical polar coordinates $(ct, R, \vartheta, \varphi)$ as follows [26,27,29,56]:

$ds^2 = -c^2dt^2 + dR^2 + R^2 \cdot d^2\vartheta + R^2 \cdot \sin^2\vartheta \cdot d^2\varphi$

note: the opposite sign convention is always possible

$-ds^2 = +c^2dt^2 - dR^2 - R^2 \cdot d^2\vartheta - R^2 \cdot \sin^2\vartheta \cdot d^2\varphi$ (2.1)

In particular, if a mass M is at the origin of a frame of four dimensional spacetime, and if the frame is described with the above spherical polar coordinates, then the metric is described by the Schwarzschild metric [27,29,31,56] as follows:

$ds^2 = -c^2dt^2 \cdot \varepsilon_E^2 + \frac{dR^2}{\varepsilon_E^2} + R^2 \cdot d^2\vartheta + R^2 \cdot \sin^2\vartheta \cdot d^2\varphi$ with $\varepsilon_E = \sqrt{1 - \frac{R_S}{R}}$

and with the Schwarzschild radius $R_S = \frac{2GM}{c^2}$ and with the gravitational constant G (2.2)

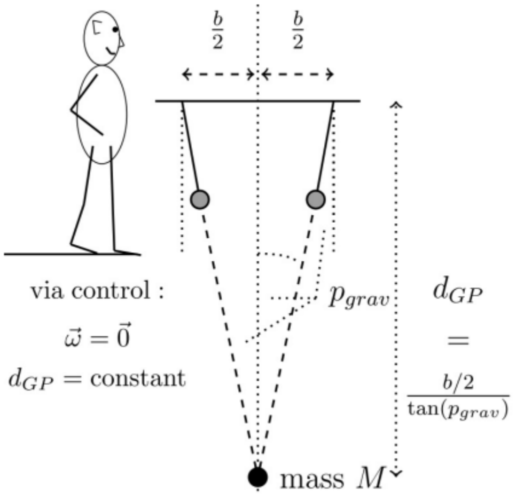

Figure 1. Measurement of a well-defined gravitational parallax distance d_{GP} between an observer in a spaceship and a mass M.

2.1 Two measures of observable distance

In general, there are various methods for the measurement of a physical quantity. For instance, if you want to measure a mass, then you can use a beam balance or a spring balance. If you measure one kilogram of bread, then both balances will provide the same result at Earth. However, they will provide significantly different results at the Moon. Thereby, the beam balance provides the correct value of the mass at Earth and at the Moon. In contrast, the spring balance provides the correct mass at Earth and a wrong value for the mass at the Moon. Of course, the spring balance basically measures a force. And the spring balance provides the correct values of the force at Earth and at the Moon, if it is calibrated for the measurement of forces. Moreover, the difference of the values provided by the two balances provides essential information about the local properties of gravity at Earth and at the Moon. Similarly, we use two distance measures in order to obtain essential information about volume, see e. g. [21-23,54].

2.2 Light-travel distance

In this section, we summarize the light-travel distance d_{LT} [59]. If an object emits a light signal at a time t_{em}, and if that light signal propagates in volume, and if that light signal is observed at a second object at a time t_{obs}, then the light signal propagated the light-travel distance $d_{LT} = c \cdot (t_{obs} - t_{em})$.

An alternative procedure of measurement is as follows: If an object emits a light signal at a time t_{em}, and if that light signal propagates in volume, and if that light signal is reflected at a second object, and if the reflected signal is observed at the first object at a time t_{obs}, then the light-travel distance between the objects is as follows $d_{LT} = c \cdot \frac{t_{obs}-t_{em}}{2}$.

The measured distances can be represented in a map of curved spacetime, as the light-travel distance is used in GR, and as GR provides a curved spacetime. For an illustration see the upper map in Fig. (2).

2.3 Gravitational parallax distance

In this section, we propose a second distance measure, the gravitational parallax distance d_{GP}, see Fig. (1), and we show that this distance is well defined.

Theorem 2.1. If a mass M is in an empty environment, and if M is neither accelerated, nor rotating nor charged. And if a spacecraft is in the vicinity, and if that spacecraft navigates with a gyro-sensor so that it does not rotate, $\vec{\omega} = \vec{0}$, and if the spacecraft has two hand leads at the ends of a rod of length b, and if the spacecraft navigates via feedback control so that each hand lead encloses the same angle α with the rod, and so that the spacecraft keeps the angle α fixed, then the angle $90^o - \alpha = p_{grav}$ is called gravitational parallax angle, then following holds:

(1) A well-defined gravitational parallax distance d_{GP}, also called R, between the spacecraft and the mass M can be measured and evaluated as follows:

$$R = d_{GP} = \frac{b/2}{\tan p_{grav}}$$

(2) The measured gravitational parallax distance d_{GP} does not depend on the value of M.

(3) In the zero-mass limit, the light-travel distance d_{LT} is equal to the measured gravitational parallax distance and to the light-travel distance $d_{LT,flat}$ in flat space:

$$d_{LT,flat} = \lim_{M\to\infty} d_{LT}(M) = d_{GP}$$

(4) At a location with $R > R_S$, the radius R can be measured by using the light-travel distance d_{LT} only. For it, the circumference $C = 2\pi R$ of a circle with M at the center can be measured. With it, the *circumferential radial coordinate* $R = \frac{C}{2\pi}$ is determined [30, p. 107].

In order to prove part (1) of the above theorem, we have to show that the measurement of d_{GP} is well defined. For it, we show that the measurement provides the same value, whenever the procedure of measurement is applied at the same geometric situation. For it, we realize that there is no inertial force at the spaceship:
Firstly, there acts no centripetal force at the spaceship, as the spaceship does not rotate.
Secondly, there acts no linear inertial force at the spaceship, as the distance to M is constant, and as M does not experience any inertial force, since M is not accelerated or M is in a state of free fall.
Thirdly, there acts no force at the spaceship that is based on a drag, because M does not rotate [60].
Furthermore, except the gravitational interaction, there acts no long-range force upon the hand leads or the spaceship. This is a consequence of the following facts:

Firstly, M is not charged.

Secondly, the spaceship is in a vicinity of M consisting of volume without any other content.

Thirdly, long range forces, except the gravitational interaction, cancel according to shielding or to a zero average, as a result of the relative homogeneity of the universe [24,61].

Altogether, the gravitational parallax distance d_{GP} is well defined.
In order to prove part (2) of the above theorem, we realize that the gravitational parallax angle p_{grav} does not depend on the particular value of the mass M. Thus, the obtained distances $d_{GP} = \frac{b/2}{\tan p_{grav}}$ do not depend on the particular nonzero value of the mass M.
In order to prove part (3) of the above theorem, we analyze the limit of the nonzero mass M to zero. In that limit, the Schwarzschild metric (Eq. 2.2) is the same as the metric (Eq. 2.1) of flat space. Thus, the light-travel distance becomes equal to the light-travel distance at $M = 0$ of flat space, $d_{LT,flat} = \lim_{M\to\infty} d_{LT}(M)$. Moreover, in that limit, the gravitational parallax distance is the same as in flat space, $d_{LT,flat} = \lim_{M\to\infty} d_{LT}(M) = d_{LT}(M)$, as the gravitational parallax distance does not depend on M, see part (2). As the gravitational parallax distance is based on a triangle with angle sum 180° (see Fig. 2 and the angle-sum theorem [62, theorem 1.2]), $d_{LT,flat} = d_{GP,flat} = d_{GP}$.
In order to prove part (4) of the above theorem, we provide a procedure of measurement. Using the light-travel distance, a set of spaceships can navigate to positions at equal light-travel distance to M and at the same plane with M in that plane. Thus, the spaceships are at a circle around M at a common radius R. According to the Schwarzschild metric (Eq. 2.2), the circumference of the circle has the length measured by light-travel distances of $C = 2\pi R$. Hence, the radius is measured on the basis of the light-travel distance only, $R = \frac{C}{2\pi}$ [30, Eq. 9.5].
Altogether, we proved all parts of the theorem.

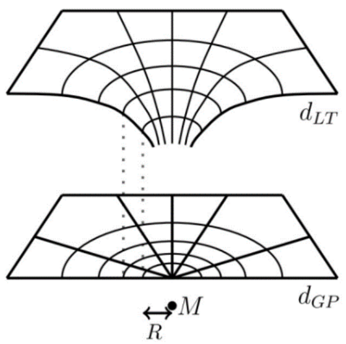

Figure 2. In the vicinity of a mass M, the measured light-travel distances can be represented in a map of curved spacetime, according to the Schwarzschild metric. In the vicinity of the same mass M, the measured gravitational parallax distances can be represented in a map of flat spacetime, according to theorem (2.1). Thereby, each event can be represented in both maps, see dotted lines.

Corollary 2.1. As the gravitational parallax distance provides distances in flat space, these distances can be marked in an illustrative map or d_{GP}-map of flat space (Fig. 2). As the light-travel distance provides distances in curved spacetime according to the Schwarzschild metric, these distances can be marked in an illustrative map or d_{LT}-map of curved spacetime (Fig. 2). For similar illustrative maps see for instance [30, Fig. 11.3], [29, 13.1], [56, Fig. 4.1].

2.4 A mass causes a change of the metric

As described in section (2.1), the comparison of two measurement procedures can provide interesting information about physics. In this sense, we analyze the Schwarzschild metric in the vicinity of a mass M. Thereby, we compare the two distance measures described, proposed or derived in section (2) and illustrated in Fig. (2). For it, we compare the shell with a radius R and with an incremental thickness dR in the d_{GP} - map on one hand with the shell with the same radius R and the corresponding thickness dL in the d_{LT} - map on the other hand (for an illustration see Fig 3).

Theorem 2.2. If a mass M fulfills the conditions of theorem (2.1), then the following holds:

(1) The mass M causes a change of the metric from the flat space metric (Eq. 2.1) to the Schwarzschild metric (Eq. 2.2).

(2) In particular, the mass M causes a change of the radial increment dR. Thereby, dR ranges from an event E_1 at R to an event E_2 at $R + dR$ at zero dt, $d\vartheta$ and $d\varphi$ in the d_{GP} - map. The increment dR changes to the corresponding radial increment dL ranging from the event E_1 to the event E_2 in the d_{LT} - map (for an illustration see Fig 3). Thereby, the increments dR and dL are related as follows:

$$dL = \frac{dR}{\varepsilon_E} \text{ at } dt = 0, d\vartheta = 0 \text{ and } d\varphi = 0 \quad (2.3)$$

In order to prove part (1) of the above theorem, we apply part (3) in theorem (2.1). Thus, in the limit M to zero, the metric is the flat space metric, while at an arbitrary mass M, the metric is the Schwarzschild metric. In this sense, the mass M causes the change from the flat space metric to the Schwarzschild metric.

In order to prove part (2) of the above theorem, we apply part (1) in this theorem and the Schwarzschild metric (Eq. 2.2). Thus, at constant time and polar angles, the mass M causes a change from dR to $\frac{dR}{\varepsilon_E}$ in Eq. (2.2). Altogether, we proved both parts of the theorem.

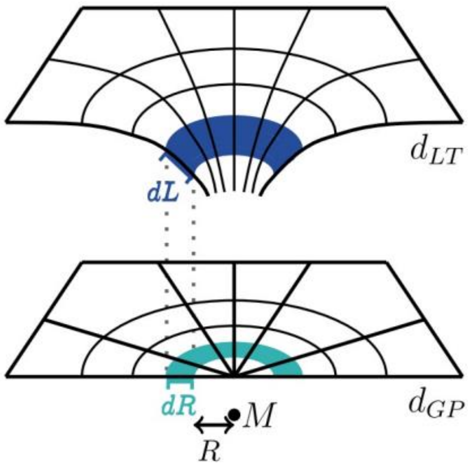

Figure 3. Corresponding shells in the two maps (see Fig. 2) of the vicinity of M. The thicknesses dL and dR are related according to Eq. (2.3).

III. ADDITIONAL VOLUME

In this section, we analyze the volumes of the shells in theorem (2.2), see Fig. (3). Hereby, we apply various well established mathematical and physical concepts.

3.1 Mass causes additional volume

In this section, we analyze the difference of the volumes of the shells in theorem (2.2), see Fig. (3).

Theorem 3.1. If a mass M fulfills the conditions of theorem (2.2), then the following holds (for an illustration see Fig. 3):

(1) At a radius R, the mass M causes the following additional radial increment:

$$\delta R := dL - dR, \text{ thus, } \delta R = dR \cdot \left(\frac{1}{\varepsilon_E} - 1\right) \quad (3.1)$$

(2) As a consequence, the mass M causes an increase of the volume dV_R of a shell with the center M, a radius R and a thickness dR in the d_{GP} - map. If the mass changes from the limit M to zero to an arbitrary value M, then the thickness of the above shell increases from dR to $dL = \frac{dR}{\varepsilon_E}$. Thus, the volume of the above shell increases from dV_R to a value dV_L. Thence, the volumes are as follows: $dV_R = 4\pi R^2 dR$, and $dV_L = 4\pi R^2 dL$, with the additional volume $\delta V: dV_L - dV_R$ and with the *relative additional volume* $\varepsilon_L := \frac{\delta V}{dV_L}$, thus, $\varepsilon_L = 1 - \varepsilon_E$ (3.2)

In order to prove part (1) of the above theorem, we apply part (2) in theorem (2.2), and we factorize dR. We prove part (2) of the above theorem as follows: The incremental volume dV_R of the shell is equal to its surface $4\pi R^2$ multiplied by the thickness dR. The incremental volume dV_L of the shell is equal to its surface $4\pi R^2$ multiplied by the thickness dL. By definition, the difference of these volumes is $\delta V = dV_L - dV_R$. By definition, the difference of these volumes δV relative to the complete volume dV_L is $\varepsilon_L = \frac{\delta V}{dV_L}$. In the definition $\varepsilon_L = \frac{\delta V}{dV_L}$, we insert the terms for δV, dV_R and dV_L. Then we cancel dV_L and $4\pi R^2$ in order to derive $\varepsilon_L = 1 - \frac{dR}{dL}$. Next, we apply part (2) in theorem (2.2). Thus, we derive part (2) in the above theorem. Altogether, we proved all parts of the theorem.

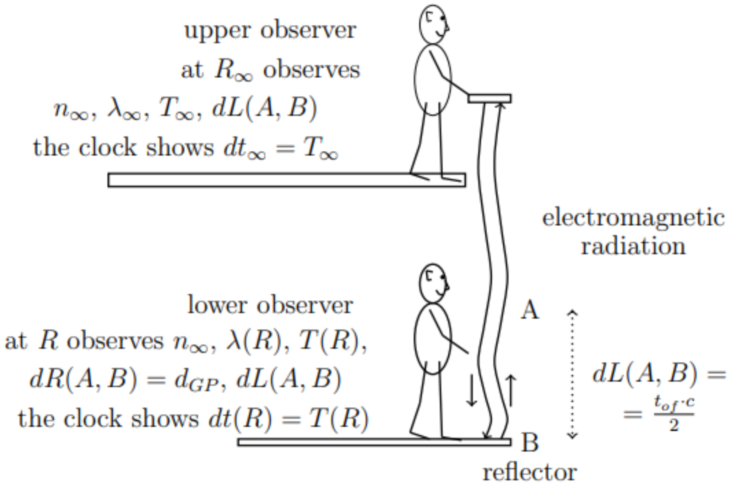

Figure 4. Two observers above a mass M measure the gravitational redshift and time dilation: An upper observer at R_∞ measures the light travel distance $dL(A, B)$ between two locations A and B with help of the time of flight t_{of}. A lower observer additionally measures the gravitational parallax distance $dR(A, B)$ between the same locations.

3.2 Relative additional volume provides curvature, potential and field

In this section, we show that the relative additional volume derived in theorem (3.1) provides the observed curvature of spacetime of the Schwarzschild metric as well as the gravitational potential and the gravitational field as a function of the gravitational parallax distance $d_{GP} = R$.

Theorem 3.2. If a mass M fulfills the conditions of theorems (2.1, 2.2, 3.1), then the following holds:

(1) At each location with $R \geq R_S$, and in the d_{GP} - map, the gravitational field $\vec{G}^*$ is antiparallel to the radial unit vector $\vec{e}_L$ and the field has the following absolute value, see e. g. [30, Eq. 9.9]:

$$|\vec{G}^*| = \frac{G \cdot M}{R^2}, \qquad \text{thus,} \qquad \vec{G}^* = \frac{G \cdot M}{R^2} \cdot \vec{e}_L \tag{3.3}$$

(2) At each location with $R \geq R_S$, the gravitational potential $\Phi_L(\vec{R})$ is equal to the relative additional volume multiplied by $-c^2$, and the field is the gradient of the potential:

$$\Phi_L(\vec{R}) = -c^2 \cdot \varepsilon_L(\vec{R}), \text{ thus, } \vec{G}^* = -\frac{\partial}{\partial L}\Phi_L \cdot \vec{e}_L, \text{ thence, } \vec{G}^* = -grad_L \Phi_L \cdot \vec{e}_L$$

or with $\vec{\partial}_L := grad_L$ follows

$$\vec{G}^* = -\vec{\partial}_L \Phi_L \tag{3.4}$$

(3) The mass M causes a gravitational redshift as follows. For it, we analyze a wave emitted at R_∞ (that is the limit $R \to \infty$) with a wavelength λ_∞ and a periodic time T_∞. We derive the corresponding wavelength $\lambda(R)$ and periodic time $T(R)$ at a radius R:

$$\lambda(R) = \lambda_\infty \cdot \varepsilon_E(R) \quad \text{with} \quad \lambda_\infty = \lambda(R_\infty),$$
$$T(R) = T_\infty \cdot \varepsilon_E(R) \quad \text{with} \quad T_\infty = T(R_\infty)$$

The time corresponding time increments are conventionally obtained from atomic clocks, which are based on periodic times of light:

$$dt_\infty = T_\infty \quad \text{and} \quad dt(R) = T(R)$$

If both observers use the periodic time T_{H_α} of the H_α – line as the unit of time, for instance, then they observe the same light travel distance.

In order to prove part (1), we use the well-known fact that the gravitational field in the d_{GP} - map is equal to its Newtonian value [30, Eq. 9.9]. That value can be used in the d_{LT} - map as well, according to the exact correspondence of events, see Fig. (3).

In order to prove part (2), we use $\varepsilon_L = 1 - \varepsilon_E$ in part (2) in theorem (3.1), and we apply the derivative $\frac{\partial}{\partial L}$ to ε_L. For it, we substitute $\frac{\partial}{\partial L} = \frac{\partial R}{\partial L} \cdot \frac{\partial}{\partial R}$. By evaluating the expressions and using $R_S = \frac{2GM}{c^2}$, we derive $\vec{G}^* = \frac{G \cdot M}{R^2} \cdot \vec{e}_L$. Furthermore, the derivative $\frac{\partial}{\partial L}\Phi_L$ multiplied by the radial unit vector $\vec{e}_L$ is equal to the gradient of that potential, as each derivative of the potential Φ_L with respect to a direction orthogonal to the radial direction is zero. Altogether, we showed that the relative additional volume provides the gravitational potential and field, according to part (2).

In order to prove part (3), we analyze the observations made by the upper and lower observer in Fig. (4): The local observer at the radius R in Fig. (4) measures the local wavelength $\lambda(R)$ and the increment $dR(A, B) = dR$ with help of the gravitational parallax distance, see Theorem (2.1). When the electromagnetic radiation propagates from A to B, then the radiation exhibits a number n_∞ of wavelengths. This is the case for both observers. Thus, the following holds:

$$n_\infty = \frac{dL}{\lambda_\infty} \text{ and } n_\infty = \frac{dR}{\lambda(R)}, \qquad \text{thus,} \qquad \lambda(R) = \lambda_\infty \cdot \frac{dR}{dL} \tag{3.5}$$

With it, theorem (2.1) implies the following relation describing a gravitational blue shift or an (inverse) gravitational redshift, more generally:

$$\lambda(R) = \lambda_\infty \cdot \varepsilon_E \tag{3.6}$$

As a consequence of the invariance of the velocity of light, the following holds:

$$\frac{\lambda_\infty}{T_\infty} = c = \frac{\lambda(R)}{T(R)}$$

Solving for $T(R)$ and using Eq. (3.6) implies the following relation of the periodic times:

$$T(R) = T_\infty \cdot \varepsilon_E \tag{3.7}$$

As the periodic time of light represents a typical reference interval dt of time, e. g. at a clock, see [64], the relative additional volume ε_L causes a decrease of the reference interval of time dt_∞ by the factor ε_E, resulting in the gravitational time dilation:

$$dt(R) = dt_\infty \cdot \varepsilon_E \tag{3.8}$$

If an observer uses the periodic time T_{H_α} of the H_α – line (line of the Balmer series with the lowest energy) as the unit of time, then the time of flight is as follows:

$$t_{of} = 2n_\infty \cdot T_{H_\alpha}$$

Thus, the light travel distance is as follows:

$$d_{LT} = dL(A, B) = c \cdot \frac{t_{of}}{2} = n_\infty \cdot c \cdot T_{H_\alpha} = n_\infty \cdot \lambda_{H_\alpha}$$

As a consequence, if both observers use the periodic time T_{H_α} of the H_α – line as the unit of time, then they observe the same light travel distance, see Fig. (4). Altogether, we proved all parts of the theorem.

3.3 Propagation of relative additional volume

In this section, we analyze the propagation of relative additional volume.

Theorem 3.3. If a mass M fulfills the conditions of theorems (2.1, 2.2, 3.1, 3.2), and if faster-than-c signals are excluded (causality is not provided automatically in the case of faster-than-c signals [65, § 3], [66, section 3.1], however, causality should be provided, at least in natural volume), then the following holds:

(1) A portion of relative additional volume propagates at a velocity $v \geq c$. As faster-than-c signals are excluded, see above, relative additional volume propagates at c. Of course, faster-than-c correlations are possible.

(2) A portion of relative additional volume propagates parallel or antiparallel to the radial direction. In the following, we model relative additional volume propagating parallel to the radial direction, that is the outward propagation:

$L(\tau_0 - \tau) = L(\tau_0) + c \cdot \tau$ with $d\tau = dt \cdot \varepsilon_E$, thus,

$$\frac{\partial L}{\partial \tau} = c, \text{ with direction vector } \vec{e}_L \text{ of propagation} \tag{3.9}$$

(3) The propagation of relative additional volume is driven by the potential. Thus, unidirectional propagation is as follows:

$\frac{\partial}{\partial L}\Phi_L = -c^2 \frac{\partial \tau}{\partial L} \cdot \frac{\partial}{\partial \tau}\varepsilon_L$, thus, we derive the DEQ:

$$\frac{\partial}{\partial L}\Phi_L = -c \cdot \frac{\partial}{\partial \tau}\varepsilon_L, \text{ with direction vector } \vec{e}_L \text{ of propagation} \tag{3.10}$$

In order to prove part (1), we realize that each portion of volume has zero rest mass, $m_0 = 0$. Hence, the energy momentum relation $E^2 = m_0^2 c^4 + p^2 c^2$ takes the form $E^2 = p^2 c^2$, corresponding to $v = c$ [27, § 9]. Thus, volume does not propagate at a velocity smaller than the velocity of light c.

In order to prove part (2), we realize that relative additional volume in the vicinity of a mass M represents a steady state of propagating volume. According to symmetry, the net propagation of the relative additional volume must

be parallel or antiparallel to the radial direction. If the radial position of a portion of relative additional volume is $L(\tau_0)$ at a time τ_0 according to the d_{LT} - map, then its time evolution is given by Eq. (3.9).
In order to prove part (3), we use the true equation $\frac{\partial}{\partial L}\Phi_L = \frac{\partial}{\partial L}\Phi_L$, we insert the potential in part (2) in theorem (3.2), and we substitute via $\frac{\partial}{\partial L} = \frac{\partial \tau}{\partial L}\cdot\frac{\partial}{\partial \tau}$. Next, we apply Eq. (3.9), in order to derive the differential equation, DEQ. Altogether, we proved all parts of the theorem.

3.4 A four-scalar of relative additional volume

In this section, we derive four-scalars and four-vectors [27, § 6] describing the relative additional volume. Each four-scalar is invariant under any rotation of the four-dimensional coordinate system, in particular under Lorentz transformations [27, § 6]. A four-scalar is called a Lorentz invariant or a Lorentz scalar. With the four-scalar, we provide a generalization.

Theorem 3.4. If a mass M fulfills the conditions of theorems (2.1, 2.2, 3.1, 3.2, 3.3), then the following holds:

(1) Relative additional volume is described by the following equation:

$$0 = -c^2\cdot\left(\frac{\partial}{\partial\tau}\varepsilon_L\right)^2 + \left(\vec{G}^*\right)^2 \quad \text{or}$$

$$0 = -c^2\cdot\left(\frac{\partial}{\partial\tau}\varepsilon_L\right)^2 + (G_x{}^*)^2 + \left(G_y{}^*\right)^2 + (G_z{}^*)^2 = RGS_L \tag{3.11}$$

Hereby, the term denoted by RGS_L is a Lorentz scalar. It is formed by the rate of change of relative additional volume $\frac{\partial}{\partial\tau}\varepsilon_L$ and by the gravitational field $\vec{G}^*$. Accordingly, it is named rate gravity scalar, RGS_L. It is the four-vector scalar product, also called four-scalar, [27, paragraph 6] of the following rate gravity four-vector, whereby the sign convention [27, paragraph 2] with a negative sign at the zeroth component is used and represented by the tensor (η_{ik}) in Eq. (3.13):

$$RGV_L = \begin{pmatrix} c\frac{\partial}{\partial\tau}\varepsilon_L \\ G_x{}^* \\ G_y{}^* \\ G_z{}^* \end{pmatrix} \tag{3.12}$$

The rate gravity scalar is obtained with the sign convention (η_{ik}) as follows:

$$(\eta_{ik}) = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad \text{and} \quad RGS_L = \sum_{i=0}^{3}\sum_{k=o}^{3} RGV_{L,k}\cdot\eta_{ik}\cdot RGV_{L,i} \tag{3.13}$$

Hereby, (η_{ik}) represents the matrix notation.

(2) The rate gravity scalar RGS_L is a function of the gravitational field only. Thus, the volume dynamics, VD, does not only hold in the vicinity of a mass M. Instead, the VD holds generally, thus the VD is not restricted to the vicinity of a mass M or to a radius or distance R.

(3) In local orthonormal coordinates including L, the corresponding gradient $\vec{\partial}_L$ can be applied to the potential Φ_L, see part (2) in theorem (3.2). The spatial coordinates can be named L_1, L_2 and L_3. The RGS_L can be expressed in terms of the derivatives of the potential Φ_L, and the resulting Lorentz invariant scalar can be named slope four-scalar, SFS_L [24]:

$$0 = -c^2\cdot\left(\frac{\partial}{\partial\tau}\varepsilon_L\right)^2 + \left(\frac{\partial}{\partial L_1}\Phi_L\right)^2 + \left(\frac{\partial}{\partial L_2}\Phi_L\right) + \left(\frac{\partial}{\partial L_3}\Phi_L\right) = SFS_L \tag{3.14}$$

In order to prove part (1), we apply part (2) in theorem (3.2) to the DEQ in theorem (3.3). Then we apply the square, and we simplify so that zero is at the left-hand side. This shows Eq. (3.14). Next, we derive the invariance of Eq. (3.14) under Lorentz transformations:
Firstly, Eq. (3.14) holds for each $R \geq R_S$, as we derived the equation independently of the particular value of $R \geq R_S$. Similarly, Eq. (3.14) holds for each time τ and for each angular coordinate ϑ and φ.
Secondly, we show that Eq. (3.14) holds for each motion at a constant velocity: The field $\vec{G}^*$ is measured via the acceleration in the gravitational field, see section (2). That acceleration does not depend on a constant velocity, according to the equivalence principle. A motion at a constant velocity does not change the own time $d\tau$. A motion at a constant tangential velocity does neither change the field nor the relative additional volume. Thus, for such a motion, Eq. (3.14) is invariant. A motion at a constant radial velocity changes the field and the relative additional volume $\varepsilon_L = 1 - \varepsilon_E = 1 - \frac{dR}{dL}$ (remind that dR and dL can be measured in a local manner, see section (2)), so that Eq. (3.14) is invariant. Altogether, a motion at a constant velocity does not change Eq. (3.14). Thus, the RGS_L is invariant under a boost. Hence, the RGS_L is invariant under a Lorentz transformation. Hence, the RGS_L is a Lorentz invariant and a Lorentz scalar. In order to prove part (2), we remind that the Lorentz scalar RGS_L is a Lorentz invariant. Thus, frames can be transformed accordingly. Moreover, the RGS_L does not depend on M, so

the RGS_L holds generally, not restricted to the vicinity of a mass M. Correspondingly, the RGS_L does not depend on R, so the RGS_L holds generally, not restricted to a distance or radial coordinate or distance R.
In order to prove part (3), we apply the procedure described in part (3). As that procedure does not change the effect of a Lorentz transformation, the SFS_L is a Lorentz scalar and a Lorentz invariant. Altogether, we proved all parts of the theorem.
Corollary 3.1. The RGS_L and the SFS_L are invariant under Lorentz transformations. Similarly, local Lorentz invariance is analyzed in several systems of GR [67]. Moreover, the RGS_L and the SFS_L are valid for all M and R. Furthermore, the RGS_L and the SFS_L hold generally, not restricted to the vicinity of a mass M or distance R. Additionally, we emphasize that the RGS_L and the SFS_L exhibit a very clear and unique structure, which is a well-known advantage [1, p. 237], [2, p. 237].
3.5 Waves of relative additional volume
In this section, we derive plane waves of relative additional volume. Moreover, we derive a corresponding differential equation.
Theorem 3.5. If a mass M fulfills the conditions of theorems (2.1, 2.2, 3.1, 3.2, 3.3, 3.4), then the following holds:

(1) The following plane waves are solutions of the DEQ $\frac{\partial}{\partial L}\Phi_L = -c \cdot \frac{\partial}{\partial \tau}\varepsilon_L$ of relative additional volume in Eq. (3.10), whereby the waves propagate towards positive values of L (it is marked by the subscript $L,+$ in Eq. (3.17):
$\varepsilon_{L,\omega} = \hat{\varepsilon}_{L,\omega} \cdot \exp(i \cdot \omega \cdot \tau - i \cdot k \cdot L)$ and
$\Phi_{L,\omega} = \widehat{\Phi}_{L,\omega} \cdot \exp(i \cdot \omega \cdot \tau - i \cdot k \cdot L)$, with direction vector $\vec{e}_L$ of propagation (3.15)

(2) Inserting of the waves into the DEQ yields the following relations:
$\widehat{\Phi}_{L,\omega} = \hat{\varepsilon}_{L,\omega} \cdot c^2$ and $\Phi_{L,\omega} = \varepsilon_{L,\omega} \cdot c^2$ and the DEQ for relative additional volume
$-c\frac{\partial}{\partial L}\varepsilon_{L,\omega} = \frac{\partial}{\partial \tau}\varepsilon_{L,\omega}$ (3.16)

(3) All finite, discrete infinite or continuously infinite sufficiently converging linear combinations are also solutions of the DEQ in part (2). Thus, even non-periodic solutions are included. Accordingly, the only particular property of these solutions is the propagation in the positive direction of the unit vector $\vec{e}_L$. Correspondingly, we mark these solutions by a plus sign:
$-c\frac{\partial}{\partial L}\varepsilon_{L,+} = \frac{\partial}{\partial \tau}\varepsilon_{L,+}$, with direction vector $\vec{e}_L$of propagation (3.17)

In order to prove parts (1) and (2), we insert the proposed solutions in Eq. (3.15) into the DEQ in Eq. (3.10). We simplify the resulting equation, in order to derive the relation of amplitudes $\widehat{\Phi}_{L,\omega} = \hat{\varepsilon}_{L,\omega} \cdot c^2$. We insert that relation into the proposed solutions, in order to derive the relation of solutions $\Phi_{L,\omega} = \varepsilon_{L,\omega} \cdot c^2$. We insert that relation into the original DEQ in Eq. (3.10), in order to derive the DEQ for these solutions of waves $-c\frac{\partial}{\partial L}\varepsilon_{L,\omega} = \frac{\partial}{\partial \tau}\varepsilon_{L,\omega}$.
In order to prove part (3), we realize that the DEQ in part (2) is a linear DEQ. Altogether, we proved all parts of the theorem.
3.6 Relative additional volume implies the Schrödinger equation
In this section, we use the DEQ of relative additional volume in part (3) in theorem (3.5), in order to derive the SEQ.
Theorem 3.6. If a mass M fulfills the conditions of theorems (2.1, 2.2, 3.1, 3.2, 3.3, 3.4, 3.5), then the following holds:

(1) The time derivative is applied to the DEQ of relative additional volume in Eq. (3.17), and the DEQ is multiplied by $i\hbar$. Thus, the following DEQ describes relative additional volume:
$i\hbar\frac{\partial}{\partial \tau}\dot{\varepsilon}_{L,+} = -i\hbar c\frac{\partial}{\partial L}\dot{\varepsilon}_{L,+}$ with $\dot{\varepsilon}_{L,+} := \frac{\partial}{\partial \tau}\varepsilon_{L,+}$ (3.18)

(2) We introduce the product of the rate of relative additional volume $\dot{\varepsilon}_{L,+}$ and of a normalization factor t_n as the wave function $\Psi(t,\vec{R})$. As a consequence, the absolute square $|\Psi(t,\vec{R})|^2$ of the wave function is proportional to the probability density $f_R(t,\vec{R})$ that a portion of relative additional volume can be observed at $(t,\vec{R})$. Thus, the wave function fulfills Born's statistical interpretation of quantum physics [17–19]. Accordingly, the wave function fulfills the postulates about the statistical properties of quantum physics, including entanglement [20–23,51,52,68,69]:
$f_R(t,\vec{R})$: = probability density,
t_n: = normalization factor and
$\Psi(t,\vec{R})$: = $\dot{\varepsilon}_{L,+}(t,\vec{R}) \cdot t_n$ = wave function, consequently,
$|\Psi(t,\vec{R})|^2 \propto f_R(t,\vec{R})$ (3.19)

(3) The DEQ (3.18) for the relative additional volume implies the DEQ for wave functions:

$$i\hbar\frac{\partial}{\partial\tau}\Psi = -i\hbar c\frac{\partial}{\partial L}\Psi \qquad (3.20)$$

(4) The solutions of the DEQ for wave functions form a linear vector space. Hereby, a scalar product can be introduced so that the solutions form a Hilbert space H [20–23,51,52]. Accordingly, operators in H correspond to observables in reality [21–23,51,52]. So, the operator of momentum p_L, the operator of energy $E = p_L \cdot c$ and the Schrödinger equation, SEQ, are as follows [13–16,21,22,51,52]:

$\hat{p}_L := -i\hbar\frac{\partial}{\partial L}$ = momentum operator, $\hat{H} = c \cdot \hat{p}_L$ = Hamilton operator and

$$i\hbar\frac{\partial}{\partial\tau}\Psi = \hat{H}\Psi \text{ is the SEQ} \qquad (3.21)$$

In order to prove part (1), we apply the operations described in part (1).

In order to prove part (2), we realize that the square of the rate of relative additional volume $\dot{\varepsilon}_{L,+}(t,\vec{R})$ is proportional to the absolute square of the gravitational field, according to the RGS_L in theorem (3.4):

$$|\dot{\varepsilon}_{L,+}(t,\vec{R})|^2 \propto \left|\vec{G}^*(t,\vec{R})\right|^2$$

The absolute square of the gravitational field $\left|\vec{G}^*(t,\vec{R})\right|^2$ is proportional to the positive part of the energy density of the considered relative additional volume, see e. g. [21–24]. Note that the positive part of the energy density of the gravitational field can be interpreted as a kinetic energy, see e. g. [24]. Note that the energy density of the gravitational field has a similar mathematical structure as the energy density of the electric field, whereby the signs are different [21,24,27,70,71,80,81]. Our algebraic analysis is based on the Schwarzschild metric only. As the Schwarzschild metric fulfills the law of energy conservation, see e. g. [27, Eq. 88.9], and as our analysis fulfills exactly that conservation of energy, for details, see e. g. [21–24], we can apply the law of energy conservation. Thus, we can analyze a longer accumulation of observations taking place at a constant rate [72]. Thereby, the amount of relative additional volume observed at (t, $\vec{R}$) is proportional to the positive part of the energy density, which is proportional to $\left|\vec{G}^*(t,\vec{R})\right|^2$. Hence, the probability density of an observation of relative additional volume at (t, $\vec{R}$) is proportional to $\left|\vec{G}^*(t,\vec{R})\right|^2$ and to $|\dot{\varepsilon}_{L,+}(t,\vec{R})|^2$ and to $|\Psi(t,\vec{R})|^2$. In order to prove part (3), we realize that the DEQ in part (2) is a linear DEQ. In order to prove part (4), we insert the wave function in Eq. (3.19) in the DEQ for the rate $\dot{\varepsilon}_{L,+}(t,\vec{R})$ in Eq. (3.18). Altogether, we proved all parts of the theorem.

Corollary 3.2. The SEQ derived in theorem (3.6) is interpreted as follows:

(1) The SEQ in theorem (3.6) is derived for the case of volume.

(2) Matter forms from volume via a phase transition, see e. g. [54,73,74].

(3) If objects undergo a phase transition, then the fundamental dynamics of the objects does not change, see e. g. [75, chapter 1], as the phase transition only represents a breaking of symmetry [76,77].

(4) Accordingly, when an object with a rest mass m_0 forms from volume via a phase transition, then the SEQ derived for volume in theorem (3.6) holds for the rest mass m_0 as well.

(5) If the momentum of the rest mass m_0 is relatively small compared to $m_0 \cdot c$, then the DEQ (3.21) takes the following particular form, according to the relativistic momentum energy relation $E^2 = m_0^2c^4 + p^2c^2$, and in linear order in $\frac{p}{m_0 \cdot c}$:

$$i\hbar\frac{\partial}{\partial\tau}\Psi = \sqrt{m_0^2c^4 + \hat{p}^2c^2}\,\Psi \doteq m_0c^2\Psi + \frac{\hat{p}^2}{2m_0}\Psi \qquad (3.22)$$

In non-relativistic physics, the term m_0c^2 of the rest energy is subtracted from the Hamiltonian [78, § 17]. Thus, the non-relativistic SEQ is derived:

$$\hat{H}_{non-relativistic} \coloneqq \hat{H} - m_0c^2, \text{ thus, } i\hbar\frac{\partial}{\partial\tau}\Psi = \frac{\hat{p}^2}{2m_0}\Psi \qquad (3.23)$$

(6) The relativistic SEQ in theorem (3.6) is a generalization of the non-relativistic SEQ in Eq. (3.23) or [9, Eq. 1.1].

(7) The DEQs of additional relative volume in theorem (3.4) are Lorentz invariant generalizations of the relativistic SEQ in in theorem (3.6). For Lorentz invariants, see e. g. [29, p. 137], [27, § 6]. Moreover, the DEQs of relative additional volume hold in general, these DEQs are not restricted to the vicinity of a mass or dynamical mass M or to a radial coordinate or distance R.

(8) Furthermore, there are even more general DEQs for relative additional volume with a general tensor structure [22,24,74,79,80,81]. These can provide physics of cosmology, fundamental interactions and of matter.

(9) The DEQ (3.23) can be supplemented by a term describing an electromagnetic, electroweak or chromodynamic interaction or by using any less fundamental interaction [55,74,79,80,81].

IV. CONCLUSION

In the present paper, we realize that the volume and the various forms of vacuum proposed in physics should be distinguished. Moreover, we realize that the concept of volume in nature should provide a description of the space of the universe at each time, and thus the definition of volume is the same for each time, while the amount and tensor structure (isotropic or unidirectional or anisotropic) of the volume is consequence of the dynamics. Thus, we arrive at the concept of a dynamical volume, DV. On that basis, we analyze the volume dynamics, VD. As a first example, we analyze the isotropic formation of volume in an isotropic, homogeneous and globally flat universe. As a second example, we analyze the formation of volume in the vicinity of a mass or dynamical mass M. For it, we apply the Schwarzschild metric. As a result, we arrive at the local formation of anisotropic unidirectional volume forming in the radial direction only. Note that this local and unidirectional formation of volume adds up to the global and isotropic formation of volume, see e. g. [80].

Moreover, we show that the volume dynamics, VD, exactly implies the curvature of spacetime as well as the gravitational field and the gravitational potential. Thus, the VD exactly explains the curvature of spacetime at a semiclassical level of description. In this manner, the VD exactly explains the gravitational field and the gravitational potential at the level of a semiclassical description. Hereby, the VD explains the propagation of the gravitational interaction. Note that a propagation of gravity had been proposed in terms of a hypothetical graviton [82]. Furthermore, we derive the Schrödinger equation from the VD.

The VD derived here are the basis for many further derived results: For instance, on the basis of the VD, the postulates of quantum physics have been derived [21–23,80]. Furthermore, the VD have been used in order to derive the theoretical value of the energy density of volume, $u_{\text{vol,theo}}$, whereby no hypothesis has been introduced, no fit parameter has been proposed, and no fit has been executed, see e. g. [21,24,80,83]. Additionally, the observed H_0 tension [37] has been solved by using the VD [21,24,25,80], whereby neither a hypothesis has been proposed nor a fit has been executed. Also, the transition from volume to matter, as proposed by the Higgs mechanism [73] and as confirmed by observation [55,84,85], has been derived on the basis of the VD [54,74,79,80,81], whereby the observed mass of the Higgs boson has been derived and neither a hypothesis has been proposed nor a fit has been executed. On the basis of that transition, the quantum postulates derived from the VD here are transferred to elementary particles and to matter in general. Likewise, the six parameters of the standard model of cosmology have been derived from the VD [74], whereby neither a hypothesis has been proposed nor a fit has been executed. Similarly, the fundamental electromagnetic, weak and electroweak interactions have been derived from the VD, including the respective charges, coupling constants and the weak angle, whereby neither a hypothesis has been proposed nor a fit has been executed [79,80,81]. Altogether, the VD provide many deep results in fundamental physics. Thereby, a precise accordance with observation is achieved.

Thus, the VD provide a high evidentiality [8]. Moreover, in the above listed derivations from the VD, neither a hypothesis, nor a fit, nor an ununified parameter, nor a change of fundamental or universal constants of nature G, c and h [86, p. 561] have been proposed. Thence, these derivations from the VD exhibit a low metaphysical weight [8]. Hence, the above listed derivations from the VD exhibit a large unifying power, which is the ratio of the evidentiality divided by the metaphysical weight [8]. In contrast, the standard model of elementary particles uses approximately 26 ununified parameters [55,87], the standard model of cosmology exhibits six ununified parameters [46,55], and other proposed theories of dark energy use ad hoc hypotheses and proposed fit parameters [88].

In summary, the present derivation of the Schrödinger equation from the VD is a good example for unification in modern physics [8]. The present derivations are exact, as no approximation has been used. This fact represents an additional advantage of the present paper, see e. g. [1, p. 245] or [2].

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REFERENCES

[1]. Max Planck, Zur Theorie des Gesetzes der Energieverteilung im Normalspectrum, Verhandl. Dtsch. Phys. Ges. at December 14th 2, 1900, 237–245.

[2]. Max Planck, On the Theory of the Energy Distribution Law of the Normal Spectrum, Verhandl. Dtsch. Phys. Ges. at Dezember 14th 2, 1900, 237–245.

[3]. Albert Einstein, Die Feldgleichungen der Gravitation. Sitzungsberichte der Königlich Preußischen Akademie der Wissenschaften, 1915, 844–847.

[4]. David Hilbert, Die Grundlagen der Physik, Mathematische Annalen, 1928, 1–30.

[5]. Albert Einstein, B. Podolski, N. Rosen, Can the quantum-mechanical description of physical reality be considered complete? Phys. Rev. 47, 1935, 777–780.

[6]. Richard P. Feynman, The Character of Physical Law (Boston: MIT Press, 1965).

[7]. Steven Weinberg, The Trouble with Quantum Mechanics. The New York Review of Books, 2017.

[8]. Avril Styrman, Economical Unification in Philosophy of Science Before and After Ernst Mach, in Stadler, F. (Ed.), Ernst Mach − Life, Work, Influence, (Switzerland: Springer Nature, 2019).
[9]. David J. Griffiths, Introduction to Quantum Mechanics (Upper Saddle River: Prentice Hall, 1994).
[10]. Samuel Perlmutter et al., Discovery of a Supernova Explosion at Half the Age of the Universe, Nature 391, 1998, 51–54.
[11]. Adam G. Riess et al., Tests of the Accelerating Universe with Near-Infrared Observations of a High-Redshift Type Ia Supernova, The Astrophysical Journal 536, 2000, 62–67.
[12]. George F. Smoot, Nobel Lecture: Cosmic microwave background radiation anisotropies: Their discovery and utilization, Review of Modern Physics 79, 2007, 1347–1379.
[13]. Erwin Schrödinger, Quantisierung als Eigenwertproblem I. Annalen der Physik 79, 1926, 361–376.
[14]. Erwin Schrödinger, Quantisierung als Eigenwertproblem II. Annalen der Physik 79, 1926, 489–527.
[15]. Erwin Schrödinger, Quantisierung als Eigenwertproblem III. Annalen der Physik 80, 1926, 437.
[16]. Erwin Schrödinger, Quantisierung als Eigenwertproblem IV. Annalen der Physik 81, 1926, 109–139.
[17]. Max Born, Zur Quantenmechanik der Stoßvorgänge, Zeitschrift für Physik 37, 1926 , 863–867.
[18]. Max Born, Heisenberg Werner, Jordan P., Zur Quantenmechanik. II, Zeitschrift für Physik 35, 1926, 557–615.
[19]. Max Born, Statistical Interpretation of Quantum Mechanics, Science 122, 1955, 675–679.
[20]. David Hilbert, L. Nordheim, John von Neumann, Über die Grundlagen der Quantenmechanik, Nachrichten von der Königlichen Gesellschaft der Wissenschaften zu Göttingen, Math-Physik. Klasse, November 1915, 395–407.
[21]. Hans-Otto Carmesin, Quantum Physics Explained by Gravity and Relativity (In H.-O. Carmesin, Ed., Universe: Unified from Microcosm to Macrocosm - Volume 7, 1–148. Berlin: Verlag Dr. Köster, 2022).
[22]. Hans-Otto Carmesin, Unification of Spacetime, Gravity and Quanta, (In H.-O. Carmesin, Ed., Universe: Unified from Microcosm to Macrocosm - Volume 9, 1–258. Berlin: Verlag Dr. Köster, 2022).
[23]. Hans-Otto Carmesin, Explanation of Quantum Physics by Gravity and Relativity, PhyDid B, 425-438, 2022.
[24]. Hans-Otto Carmesin, Quanta of Spacetime Explain Observations, Dark Energy, Graviton and Nonlocality. (In H.-O. Carmesin, Ed., Universe: Unified from Microcosm to Macrocosm - Volume 4, pp. 1–273. Berlin: Verlag Dr. Köster, 2021).
[25]. Hans-Otto Carmesin, Physical Explanation of the H0-Tension, International Journal of Engineering and Science Invention 10 (8) II, 2021, 34–38.
[26]. Albert Einstein, Zur Elektrodynamik bewegter Körper, Annalen der Physik 17, 1905, 891–921.
[27]. Lev Landau, Evgenii Lifschitz, Course of Theoretical Physics II - The Classical Theory of Fields (Oxford: Pergamon Press 3. Edition, 1971).
[28]. Hans-Otto Carmesin, Grundideen der Relativitätstheorie (Berlin: Verlag Dr. Köster, 1996).
[29]. Michael Paul Hobson, George P. Efstathiou, A. N. Lasenby, General Relativity (Cambridge: Cambridge University Press, 2006).
[30]. Thomas A. Moore, A General Relativity Workbook (Mill Valley, CA: University Science Books, 2013).
[31]. Karl Schwarzschild, Über das Gravitationsfeld eines Massenpunktes nach der Einstein'schen Theorie, Sitzungsberichte der Deutschen Akad. d. Wiss., 1916, 186–196.
[32]. Albert Einstein, Kosmologische Betrachtungen zur allgemeinen Relativitätstheorie, Sitzungsberichte der Königlich Preußischen Akademie der Wissenschaften, 1917, 142–152.
[33]. Edwin Hubble, A relation between distance and radial velocity among extra-galactic nebulae, Proc. of National Acad. of Sciences 15, 1929, 168–173.
[34]. A. Penzias, R. W. Wilson, A measurement of excess antenna temperature at 4080 Mc/s, Astrophysical Journal Letters 142, 1965, 419–421.
[35]. D. W. Pesce et al., The Megamaser Cosmology Project: XIII. Combined Hubble Constant Constraints. Astrophysical Journal Letters 891, 2020, L1.
[36]. Adam G. Riess et al., Cosmic Distances Calibrated at 1 % Precision with Gaia EDR3 Parallaxes and Hubble Space Telescope Photometry of 75 Milky Way Cepheids Confirm Tension with ΛCDM. The Astrophys. J. Lett. 908: L6, 2021, 1 – 11.
[37]. Adam G. Riess et al., A Comprehensive Measurement of the Local Value of the Hubble Constant with 1 km s−1 Mpc−1 Uncertainty from the Hubble Space Telescope and the SHOES Team, The Astrophys. J. Lett. 934: L7, 2022, 1 – 52.
[38]. C. Vogl, Cosmological distances of type II supernovae from radiative transfer modeling. Doctoral diss., Technical University Munich, Munich, 2020.
[39]. T. de Jaeger et al., A 5% measurement of the Hubble constant from Type II supernovae. MNRAS 000, 2022, 1–9.
[40]. S. Cao, J. Ryan, B. Ratra, Cosmological constraints from H II starburst galaxy, quasar angular size, and other measurements. MNRAS 000, 2021, 1–13.
[41]. O. H. E. Philcox, Mikhail Ivanov M. Simonovic, M. Zaldarriaga, Combining Full-Shape and BAO Analyses of Galaxy Power Spectra: A 1.6% CMB-Independent Constraint on H0. Journal of Cosmology and Astroparticle Physics, 2020, 1–42.
[42]. G. E. Addison D. J. Watts, C. L. Bennett, M. Halperin, G. Hinshaw, J. L. Weiland. Elucidating ΛCDM: Impact of Baryon Acoustic Oscillation Measurements on the Hubble Constant Discrepancy. ApJ 853, 2018, 1–12.
[43]. T. M. C. Abbott et al., Dark Energy Survey Year 1 Results: Cosmological Constraints from Galaxy Clustering and Weak Lensing. Phys. Rev. D 102, 2020, 1–34.
[44]. John P. Blakeslee et al., The Hubble Constant from Infrared Surface Brightness Fluctuation Distances. The Astrophysical Journal 911(65), 2021, 1–12.
[45]. Joseph B. Jensen et al., Infrared Surface Brightness Fluctuation Distances for MASSIVE and Type Ia Supernova Host Galaxies. Astrophysical Journal Supplement Series 255/21, 2021, 1–17.
[46]. Planck-Collaboration, Planck 2018 results. VI. Cosmological parameters. Astronomy and Astrophysics, 641(A6), 2020, 1–73.
[47]. Ben J. Helmcke, Hans-Otto Carmesin, Lennert Sprenger, Paul Brüning. Three methods for the observation of the Big Bang with our school telescope. PhyDid B, 2018, 55–60.
[48]. Clifford M. Will, The Confrontation between General Relativity and Experiment. Living Revies in Relativity 17/4, 2014, 1–117.
[49]. Jun John Sakurai, Jim Napolitano, Modern Quantum Mechanics. (New York - London - Delhi: Addison-Wesley, 1994).
[50]. Paul A. M. Dirac, The Quantum Theory of the Electron. Proc. Roy. Soc. 117, 1928, 610–624.
[51]. Leslie E. Ballentine, Quantum Mechanics (London and Singapore: World Scientific Publishing, 1998).
[52]. Ajit Kumar, Fundamentals of Quantum Mechanics (Cambridge: Cambridge University Press, 2018).
[53]. Dragan Huterer, Michael S. Turner, Prospects for probing the dark energy via supernova distance measurements. Phys. Rev. D 60, 1999, 081301–081306.
[54]. Hans-Otto Carmesin, How Excitations of Vacuum Form Matter, PhyDid B, 2022, 447-458.
[55]. R. L. Workman et al., Review of Particle Physics (by Particle Data Group), Progr. Theor. Exp. Phys., 2022, 083C01, 1–2270.
[56]. Norbert Straumann, General Relativity (Dordrecht - Heidelberg - New York - London: Springer 2. Edition, 2013).

[57]. Bernhard Riemann, Über die Hypothesen, welche der Geometrie zu Grunde liegen. Abhandlungen der Königlichen Gesellschaft der Wissenschaften zu Göttingen 13, 1868, 133–150.
[58]. Hans-Otto Carmesin, Neuronal Adaptation Theory (Frankfurt: Peter Lang Verlag, 1996).
[59]. J. J. Condon, A. M. Mathews, ΛCDM Cosmology for Astronomers. Publications of the Astronomical Soc. of the Pacific 130, 2018, 1–17.
[60]. R. Kerr, Gravitational field of a spinning mass as an example of algebraically special metrics. Physical Review Letters 11, 1963, 237–238.
[61]. Isaac Newton, Mathematical Principles of Natural Philosophy (first published 1686, translated into English by Andrew Motte. New York: Daniel Adee, 1846).
[62]. John M. Lee, Riemannian Manifolds: An Introduction to Curvature (New York: Springer Verlag, 1997).
[63]. F. L. Pedrotti, L. S. Pedrotti, L. M. Pedrotti, Introduction to Optics (Cambridge: Cambridge University Press, 2017)
[64]. Physikalisch Technische Bundesanstalt, Optische Atomuhren. PTB, 2007, 1–27.
[65]. Albert Einstein, Über die vom Relativitätsprinzip geforderte Trägheit der Energie, Annalen der Physik 328, 1907, 371–384.
[66]. S. Liberati, S. Sonego, M. Visser, Faster-than-c Signals, Special Relativity and Causality. Annals of Physics 298, 2002, 167–185.
[67]. Claus Lämmerzahl, Special Relativity and Lorentz Invariance. Annals of Physics (Leipzig) 14, 2005, 71–102.
[68]. John S. Bell, On the Einstein-Podolski-Rosen paradox. Physics 1(3), 1964, 195–200.
[69]. Alain Aspect, Philippe Grangier, Gerard Roger, Experimental Realization of the Einstein-Podolski-Rosen Bohm Gedankenexperiment: A new Violation of Bell's Inequalities, Phys. Rev. Lett. 49, 1982, 91–94.
[70]. James C. Maxwell, A dynamical theory of the electromagnetic field. Philosophical Transactions of the Royal Society of London 155, 1865, 459–512.
[71]. John David Jackson, Classical Electrodynamics (New York: John Wiley, 1975).
[72]. Emmy Noether, Invariante Variationsprobleme, Nachrichten der Königlichen Gesellschaft der Wissenschaften zu Göttingen, Math-phys. Klasse, 1918, 235–257.
[73]. P. W. Higgs, Broken Symmetries, Massless Particles and Gauge Fields, Phys. Lett., 1964, 132– 133.
[74]. Hans-Otto Carmesin, Cosmological and Elementary Particles Explained by Quantum Gravity (In H.-O. Carmesin, Ed., Universe: Unified from Microcosm to Macrocosm - Volume 5, pp. 1–198. Berlin: Verlag Dr. Köster, 2021).
[75]. J. D. van der Waals, Over de Continuiteit van den gas- en vloeistoftoestand. (Leiden: Sijthoff, 1873).
[76]. Lev Landau, On the Theory of Phase Transitions, Zh. Eksp. Toer. Fiz. 7, 1937, 19–32.
[77]. Lev Landau, Evgenii Lifschitz, Course of Theoretical Physics V - Statistical Physics (Oxford: Pergamon Press 3. Edition, 1980).
[78]. Lev Landau, Evgenii Lifschitz, Course of Theoretical Physics III - Quantum Mechanics (Oxford: Pergamon Press 2. revised edition, 1965).
[79]. Hans-Otto Carmesin, The Elementary Charge Explained by Quantum Gravity (In H.-O. Carmesin, Ed., Universe: Unified from Microcosm to Macrocosm - Volume 6, pp. 1–149. Berlin: Verlag Dr. Köster, 2021).
[80]. Hans-Otto Carmesin, The Geometrical and Exact Unification of Spacetime, Gravity and Quanta – The Theory of the Dynamics of Volume is Derived from Physical Principles (In H.-O. Carmesin, Ed., Universe: Unified from Microcosm to Macrocosm - Volume 10, pp. 1–345. Berlin: Verlag Dr. Köster, 2023).
[81]. Hans-Otto Carmesin, The Electroweak Interaction Explained by and Derived from Gravity and Relativity (In H.-O. Carmesin, Ed., Universe: Unified from Microcosm to Macrocosm - Volume 8, pp. 1–168. Berlin: Verlag Dr. Köster, 2022).
[82]. D. I. Blokhintsev, F. M. Galperin, Neutrino hypothesis and conservation of energy. Pod Znamenem Marxisma 6, 1934, 147–157.
[83]. Hans-Otto Carmesin, A Unifying Derivation of Dark Energy, SciPost, 2023, Preprint.
[84]. G. Aad et al., Observation of a new particle in the search for the Standard Model Higgs boson with the ATLAS detector at the LHC, Phys. Lett. B 716, 2012, 1-37.
[85]. S. Chatrchyan et al., Observation of a new boson at a mass of 125 GeV with the CMS experiment at the LHC, Phys. Lett. B 716, 2012, 30-61.
[86]. Max Planck, Über das Gesetz der Energieverteilung im Normalspectrum, Annalen der Physik 309, 1901, 553–663.
[87]. J. Barranco, Some Standard model problems and possible solutions, Journal of Physics: Conference Series 761, 2016, 012007-012011.
[88]. Marc Kamionkowski, Adam G. Riess, The Hubble Tension and Early Dark Energy, Ann. Rev. Nucl. Part. Sci. 2023, 1–34.