

On The Fundamental Flaws of Qubit Concept for General-Purpose Quantum Computing

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ABSTRACT: *For general-purpose quantum computing, the addition rules must be imposed and implemented first. An examination of this condition shows that the use of qubits is fundamentally flawed. The reasons are elaborated here. When two qubits are used for an addition operation of two bits, it is not the four states in superposition that are relevant for the addition. Rather it is the four symbolic substitution rules that are derived after the two qubits are collapsed that are to be used as a processor. This fundamental quantum processor possesses the capabilities of executing four instructions and a storage of two data. Thus general-purpose quantum computing is shown to be rule-based, rather than logic-gate (or truth-table) based and with implemented spatial relations. The consequence of replacing the 4 states in superposition with four symbolic substitution rules brings the addition operation into the architecture of one-dimensional cellular automaton with a dual-bit in each cell. They are equivalently transformed into a 16 specific right-nearest neighbor interaction rules for each cell. When the quantum processors are not connected correctly in cellular automata with relaxed long range spatial relations, a new kind of science appears and explained.*

KEYWORDS :

*Cellular Automata
Quantum Computing
Aharonov-Bohm Effect
Symbolic Substitution
Turing Machine*

I. INTRODUCTION

Employing qubit concept for quantum computing has become the main-stream method by researchers for decades now [1-14] and qubit itself indeed appears to be a necessary ingredient for a new parallel computing scheme. In essence, a large wave packet containing many weakly entangled '0's and '1's in superposition could possibly be manipulated to perform complicated space and time-saving computations. So many researchers have thought of this possibility very generally. Yet for a general-purpose quantum computing, the suitability of such qubit concept has been challenged [15-18]. A general-purpose computer must be able to compute anything that is computable, from the computing nature that is 100 percent sequential to that of 100 percent parallel. We had addressed this question in Reference [17]. Here we would like to elaborate more with our arguments at a very elementary level and to explain why even a two-qubit system cannot be used for an addition operation of two bits and why the reversibility of computing is not as stringent as a sequence of unitary operations on some superposition of states implies. After challenging the qubit concept, we would like to convince the readers the sound foundation of our new approach [17] for a general-purpose quantum computing. That is: we would like to show what are the proper quantum processor and its associated computing architecture. Finally, we note the qubit approach could still remain valid for some special-purpose computing, very much like Fourier optics for classical computing, if a large set of qubits could be assembled and manipulated non-interactively.

II. THE FLAWS OF QUBITS FOR GENERAL-PURPOSE QUANTUM COMPUTING.

The existence of quantum superposition of states, say, for a two-qubit system to perform a computing, originates from the nature of coupled harmonic oscillators of the electron waves. The superposition of states in a larger structure can be maintained only at a very delicate entanglement arrangement. Only then, a quantum computing can be executed. Take a flux qubit, for example, the angular momentum vector of a nano-size metal ring or smaller possessed with spin-less coherent electron waves can point in the direction of up or down by the value of the applied magnetic flux inside the ring and it can be determined through the directions of its persistent currents. This is the case of an isolated Aharonov-Bohm (AB) ring [19-20], which is an internally-closed system. The fact that an AB ring forms a ring of N coupled harmonic oscillators, or an artificial atom, implies there is an associated flux periodicity.

Note that here, N, the number of oscillators, is always a finite number and when N is an even or an odd number will have different consequences with respect to the external scattering probes attached and hence the transmission characteristics. At a certain flux value, which appears periodically, the angular momentum vector can be simultaneously up and down, due to the oscillating behavior of a ring of harmonic oscillators and hence the existence of a superposition. Similarly, in spintronics, the spin direction can be simultaneously up and down from the same argument. For quantum computing purpose, a single AB ring is already capable of performing truth/false logic operation, the lowest level of a computing that includes the “INVERT” and “IF-THEN” logics [19]. However, any measurement requires some external couplings, or some scattering events, to the ring. First, because an AB ring is a ring of coupled harmonic oscillators, any external coupling must be in the form of a chain of harmonic oscillators of the same strength only. Stronger or weaker coupling will not work better [15-16]. This is the same argument of a classical inductor-capacitor (LC) circuit where external probing excitation must be tuned at the LC natural oscillating frequency. In other words, any scattering probes to measure the ring must be made of harmonic oscillators of the same type where the coupling strength is the strongest. It is important to note that for an isolated AB ring, there are 4 different types of rings, depending on the number of harmonic oscillators, N, existed on the ring. Therefore even when only two probing terminals are attached to perform the AB effect, there exist three fundamental modes, or three classes of AB rings, such that even and odd numbers of the harmonic oscillators will have different transmission characteristics [20]. This is very similar to the fundamental microwave modes exist in a waveguide. The difference is that in AB rings, each mode is related to a specific class of scalable ring geometries, while each mode in microwave waveguide is a sub-division on the length scale for a given geometry [15-16, 19-20].

Secondly, where to place those scattering probes, or the locations of the external terminals, are themselves part of the computation scheme and cannot be considered as a mere measurement. In other words, internal system and external perturbation must be integrated as one complete system for the computing and there can be no separation. That is also to say that one cannot have an isolated LC circuit and then attempting to probe the circuit with an unmatched perturbation. So here we would like to point out even at single flux-qubit level, a truth/false measurement cannot be made at the flux value where the superposition of the angular momentum vectors is occurring. This is because the locations of the external-coupling terminals and the suitable flux value combined are the integral part of the truth/false measurement and the superposition of the two states needs to be destroyed first. Thus a single flux qubit is not suitable for the logic operation of truth/false [15-16]. Now let us just consider how a basic addition operation of two bits can be made before any general-purpose quantum computing can be further considered. This is clearly the first road block that no general-purpose quantum computer can avoid to by-pass it and this step needs to be solved from the very beginning. For a main-stream method [5-14, 21], one would have to employ two entangled qubits made of two coupled AB rings that possess four superposition of states. But in order to have the superposition of four states, the two flux qubits must be maintained at a point-contact entanglement. This is shown in Fig. 1b, where the superposition of 00,01,10,11 states, corresponding four possible angular momentum pairs of the configuration, are available. Here the 00 pair can then be used when operand A=0 and B=0 and so on. Thus all four possible combinations of operand pairings are available for computing at the two-qubit level. For convenience, we denote the corresponding four operand pairs from those states as being S_1, S_2, S_3 and S_4 . That means: S_1 is when operands A=0 and B=0; S_2 for A=0 and B=1; S_3 for A=1 and B=0 and S_4 for A=1 and B=1. Thus, for example, the addition operation of 5+3, with A=101 and B=011, has an initial configuration $S_3 S_2 S_4$ from the least significant bit pair to the most significant bit pair. This configuration is to be transformed by a proper sequence of unitary operators into the final configuration $S_2 S_1 S_1 S_1$, which contains the correct result of 1000, or a decimal value of 8, at the bottom row, where the operand B is being replaced with. This certainly needs not to be a probabilistic computing, but superposition of 4 states appears to be space and possibly time saving if a proper algorithm can be made.

It is important to note that addition operation of any two bit strings is a pure 100 per cent sequential in the nature of computing because of the “ripple carry” requirement, while Fourier transform is a pure 100 per cent parallel operation in a massive quantum Turing machine. Thus any parallel quantum computer must show first how a computing of a pure sequential nature can be executed there with the same number of computing steps as in a sequential computer and with no advantage gained by using a Turing machine [2-3]. It is much easier to show how powerful a Fourier transform type of computation can be executed in a quantum parallel machine. But if the sequential nature of the addition operation cannot be performed there, then such a parallel quantum computing machine remains a special-purpose one, not a general-purpose one, which must be able to

compute anything that is computable. To answer this question, we need to examine how addition of two bits could possibly be performed first from two qubits and if that were possible at all. Because the four states from the two qubits can be in superposition only at the point-contact condition, even if someone could possibly find the proper sequence of unitary operations on the initial quantum configuration state to obtain the proper final state, it is a very fragile closed system. This forces all the scattering probes to be very weak, a condition not suitable for any readout measurement. This is similar to the situation of perturbing a LC circuit with a frequency far away from the nature frequency and trying to measure a possible response. Clearly we have a dilemma of whether to increase the strength of the external perturbation at the expense of destroying the superposition of four states or not. We would like to point out that, first of all, superposition of those four states are totally irrelevant for the addition operation of quantum computing to begin with. Rather, it is the associated four symbolic substitution rules that need to be implemented (Fig.2).

Secondly, internal coupling, or the entanglement strength, must cooperate with the external coupling, the readout measurement, not the other way around as many researchers had attempted unfruitfully. In other words, if an internal quantum computing system is so constructed such that it cannot respond readably to an external probing, it is not to reduce the perturbation strength of the readout probing. Rather it is to discard the entire internal setup of the two-qubit system. All we need to find out is the alternatives: if the two-qubit system were able to perform the addition operation and be measurable successfully, what one could expect from those reversible results and then we go and find those results directly before the measurement. This is what we will elaborate here. Since any scattering probes must use the harmonic oscillators of the same strength as the electron waves in the AB ring, internal coupling strength must be changed and increased in cooperation with the external coupling. Secondly, one must look at the external couplings are part of the computing. In other words, where the readout probes are to be located are closely related to the internal system and is thus part of the computing scheme. We have shown earlier [17-18] that after increasing the entanglement strength to destroy the superposition of four states and with measurement probes set up properly at three correct locations, the four states are transformed into four symbolic substitution rules. The change of the internal coupling and the needed three strong external probes are shown in Fig.1c. Since a flux-qubit or an AB ring is an artificial atom, to remain in a superposition condition, the two qubits are to remain as two atoms. Here we showed in Fig. 1c that the two artificial atoms must form a diatomic molecule with a strong entanglement in response to the three properly located external probes. A test pulse of electron wave packet originated from S-terminal will be elastically scattered into C, D or S terminals according to the four flux combinations of (Φ_1, Φ_2) , the inputs of the two operands. The scattering characteristics have been investigated [18] using the quantum network theory developed [20] and are shown in Fig 3. This half-adder processor can perform four symbolic substitution rules is shown in Fig.2. The integration of internal and external couplings is such that when the (Φ_1, Φ_2) flux pair corresponding to the operand pair of (1,1) is applied, the majority of the test-pulse wave is scattered into C-terminal. Thus we have C=1 (carry=1) and S=0 (sum=0). This is rule 1. Similarly the flux pair of the operand pair of (0,1) or (1,0) is such that the test wave will be scattered(or reflected back) into the S-terminal and thus S=1 (sum=10 and C=0 (carry =0)). This is rule2 and rule 3. Finally, the flux pair for the operand pair (0,0) is such that the test-pulse will be scattered into the D-terminal (dump terminal) so that C=0 and S=0 (carry=sum=0). This is rule 4. (See Fig.3 and Table A)

The fundamental basis for such four symbolic substitution rules to exist in the strongly-coupled double AB rings comes strictly from the extension of the transmission behavior of a quantum circulator [19]. This is a class of single AB rings with three terminals where a test pulse from any one terminal will be transmitted totally only to one of the two other terminals circularly. In this situation, there is only one input flux value to modulate correctly the phase of electron wave function to produce such an output characteristic. Now if we generally attach a second AB ring to it with a center common path, we will have two input fluxes (for the two operands) to make a similar transmission behavior to happen. This is because now we have two fluxes to jointly modulate the phase of the electron wave function on the center common path for a similar result as from a quantum circulator. Now we see even if two qubits could be initialized and somehow manipulated through a series of unitary operations for the addition operation of two bits and the subsequent readout processes were possible at all, the results could not be better than what are described above from Fig. 3. Thus it is not the superposition of four states that is relevant for the addition operation of two bits. It is the four symbolic substitution rules generated after superposition of the four states are collapsed first that are needed. When the internal system does not fit for a readout measurement, it is to change the qubit concept and replace with the concept of symbolic substitutions, which are pure quantum mechanics based. Since AB effect derived its results strictly from the quantum gauge invariance, the computing mechanism described here is based purely on quantum mechanics and it relies on the phase of the electron wave function to provide computational result.

Therefore qubit approach cannot perform algebraic operation properly because the condition of entanglement (internal coupling) is in conflict with the readout process (the external coupling). The four symbolic substitutions (Fig. 2) are rule-based as opposed to truth-table based, where all logic gates are based on. It is a search-and-replace process and those rules impose the spatial relations. This is a great departure of our computing concept and it is the starting point for a general-purpose quantum computing that we would like to emphasize here. Symbolic substitution rules distinguish left and right or up and down, the spatial relations. We note if one of the operands is saved after the elastic scattering event, then the entire process is reversible and a CNOT gate is preserved. Qubit-based quantum computing does not impose spatial relation. When two qubits are entangled, it is just one enlarged complex wave packet as if left and right have no implication in the computing even though the space occupied by the wave packet is now enlarged twice as much. But for the addition operation of two long bit strings, it absolutely needs to distinguish left and right because “ripple carry” has to move from the least significant bit pair to the most significant bit pair. This is another flaw of the current qubit computing method. Once we show how two bits can be added by destroying two qubits first, a suitable quantum computing architecture remains to be described.

III. THE NATURAL CONSEQUENCE OF A PROPER QUANTUM COMPUTING ARCHITECTURE BY ITS PROCESSORS.

For this purpose, the magnetic AB effect described earlier will now be replaced by the counterpart of an electric AB effect for conveniently implementing a quantum computing architecture based on cellular automata (CA). The half-adder processor based on an electric AB effect is shown in Fig. 1d. The two square rings are drawn with two charging cylinders, V_1 and V_2 . In the figure, a quantum circulator, QC, is used to separate the incoming test-pulse charge with the reflected wave out of the S-terminal. A discharging reset sequence is to remove the previous charge on the two charging cylinders before the arrival of the wave packet of the next charging sequence. In Table A, we show the connections of the bit pair with the flux pair or with the voltage polarities of the electric AB effect. The outputs at C, S and D terminals are determined by the charge transport through such a quantum networks as shown in Fig.3 [18]. This is valid with a particular class of the networks that has been investigated earlier [15-20]. In other words, the problems of electron transport through quantum networks are divided into several classes, similar to the existence of allowed propagation modes in microwave wave-guides. The major difference is that each class of quantum networks belongs to a particular geometry and the scaled up versions that are magnified by any odd number of times so as to maintain the same transmission behavior. While in microwave waveguides, each propagation mode is a sub-division of the geometrical lengths to fit more half-integer wavelength, in a rectangular waveguides for example. We will now point out that the quantum processor (in Fig.1d) possess the capabilities of performing four instructions and a storage of two data at the two charging cylinders. Thus any addition operation of two bit strings can be performed in a one-dimensional cellular automaton (CA) that is constructed with each cell inserted with such a quantum processor. The electron coherence needs only to be maintained only within the cell. This is shown in Figure 4-I (a) with an array of interconnected processors of Fig.1d inserted in each cell. Note that C-terminal must be connected to the left cell and S-terminal connected to the cell itself and D-terminal is not connected to any cell. This is the canonical connections and any addition operation can be performed through such a CA. CA has been investigated extensively and many applications, including the parallel computing possibility, have been proposed [23-30]. Note that the strategy of parallel computing is to trade space with time or vs versa. Therefore there is no need to use a full adder in a parallel machine. Whenever a full adder is needed, one simply uses a half adder two times, instead of using two half adders in space as were employed in Reference [21, 26].

Qubit-based quantum computing can automatically guarantee the reversibility through its unitary operation [21]. But we would like to point out that the reversibility in quantum computing is not as strict as unitary transformation implies. In the quantum processor shown in Fig. 1c or Fig. 1d, the reversibility on quantum computing requires only the elastic scattering process and the saving of one of the inputs. Those two requirements are less stringent than that of applying a sequence of unitary operations on the superposition states that are imposed by the qubit approach. It is clear that qubit-based quantum computing is stopped at two-qubit level for a general-purpose computing and any higher order qubit schemes are not needed because of the CA architecture used. That means in the place of 2^N superposition of states, we have the same number of the exponential growth on the symbolic substitution rules in a CA. However, if many qubits were to be assembled successfully, those huge bundles of ‘0’s and ‘1’s in superposition could be initialized very much like a bundle of photons passing through a lens to performing a Fourier transform in a classical 100 percent parallel computing. But it will not be able to compute anything that is computable, including the computing of a pure sequential nature. Therefore as long as those entangled qubits can be transformed and are not to depend on each

other, a special purpose of Fourier-type parallel computing is possible. Shor's algorithm [4] is one of such examples.

IV. CONSEQUENCES OF EMPLOYING ADDITION-RULE V.BASED CELLULAR AUTOMATA

Here we further show that once a half-adder processor is imbedded in each cell for any addition operation, the CA has only right nearest neighbor interaction rule and the right cell is actually wired into each cell as shown in Fig. 4-I (a) [31]. The two-bit per cell CA can be illustrated through the use of a vertical dual-rail system. The four computing states with operands A and B are $S_1 = (B,A)=(0,0)$, $S_2 = (0,1)$, $S_3 = (1,0)$ and $S_4 = (1,1)$ with the notations shown in Fig. 2, where black (or red) color designates bit '0' and white (or clear) color for bit '1'. The four symbolic substitution rules are then expanded in the framework of the CA into 16 transition rules. This is illustrated in the set of 16 rules in the upper half of Fig.5, corresponds to the connections scheme of Fig.4-I (a). The lower set of 16 rules in Fig.5 corresponds to the connections scheme of Fig.4-I (b), the time reversal version.

The symbols of the upper row on the upper half of Fig 5 are the configurations for each parent cell and its right neighbor and the symbols for the lower row are for the child cell itself after the iteration. Thus we note this particular set of the transition rules in the upper half of Fig.5 (for the Fig. 4-I(a) interconnections) is only one set out of a total of $4^{16} = 4,294,967,296$ sets available that can perform addition operation. To find such a set directly from searching all the available sets in the CA would be like finding a needle in a big haystack. Let us illustrate this addition-rule-based CA with an example. Let a 6-digit operand A=101011 and operand B=010101. The 7-digit result of this addition operation is 1000000. That is the decimal addition of $43 + 21 = 64$ operation. This is illustrated in our CA example here. According to the 16 CA transition rules we have in upper half Fig.5, the two operands have the states in the configuration of $S_2S_3S_2S_3S_2S_4$ and are located at the cell locations, labeled as 6,5,4,3,2 and 1 on the top horizontal axis as shown in Table B and is designated as the original parent configuration (labeled "0" on the left vertical axis). The rest of the 1-D space are then filled with S_1 's from cell 0, -1, -2 -3 and so on as well as from cell 7,8, 9 and so on. After the first iteration, cell 1 state is automatically changed into S_1 because the original S_4 parent state has a right neighbor of S_1 state and according to the rule #16 in the upper Fig. 5, the child state is S_1 . Similarly, at cell 2 site, the S_2 parent state automatically has the child state of S_4 after the pulse because S_2 is next to S_4 on the right and from the rule #8 in the upper Fig. 5, the child state becomes S_4 . So after the first iteration, the new configuration becomes $S_2S_2S_2S_2S_4S_1$ at the locations from cell 6 down to cell 1 (labeled as iteration "1" on the vertical axis). Repeatedly using the rules in the upper Fig.5 for the iteration scheme, after the 7th iteration we have the result located from cell 7 down to cell 1 as $S_2S_1S_1S_1S_1S_1S_1$ and it remains the same configuration after further iteration (Table B). The lower row is thus the result, which is the sequence of 1000000, which are located at V_1 's from cell 7 to cell 1 (the result of all "sum" locations while the rest of space are all 0's).

Note that each cell contains four computing instructions as well as the storage of data at V_1 and V_2 , and the inter-cell and intra-cell interconnections for parallel computing have to be the canonical connections, as we have shown in Fig.4-I (a) in order to obtain the correct result of an addition operation.

If the 16 transition rules of the lower half of Fig.5 are used on the same initial configuration, corresponding to the interconnection scheme of Fig.4-I (b), then a stable final configuration is also obtained. This result corresponds to a situation where the least significant bit-pair and the most significant bit-pair are interchanged, a time reversal situation. Since the quantum processor has three terminals and only two terminals are connected, there are several different connections available. Some interconnection examples are shown here in Fig.4- II through Fig.4-V. Here we would like to illustrate some of our contrasting interpretations of the results with Wolfram's [22-24]. For example, when Fig. 4 -II (a) or (b) are connected, the same initial configuration will result in two oscillating X and Y configurations with $X=S_4S_1$ for (a) and $S_1S_4S_2S_2S_2S_1S_4$ for (b) while $Y=S_3S_1$ for (a) and $S_1S_3S_2S_2S_2S_1S_3$ for (b). When Fig.4- IV (a)

or (b) is connected, the same initial configuration will turn into a moving configuration. When Fig.4-V (a) or (b) is connected, the initial configuration will turn into two oscillating and moving configurations [31]. Wolfram [23-24] has asserted that CA with glider-like structures, which have been classified as class 4 CA, is characterized by the capability for universal computation. This is not true in view of our 1-D CA results, which are class 2 CA. The glider-like structures of class 4 CA are actually not addition-rule compatible. Other classes, such as those exhibit chaotic configurations, are not found here and are attributed to the fact that the bit or bits possessed inside each cell do not have the half-adder processor capabilities before any cell-to-cell interconnection scheme is imposed. The use of addition-rule compatibility to classify CA at the fundamental level can thus provide better understanding and better physical insights.

V. CONCLUSIONS.

We show the fundamental flaws of qubit approach for a general-purpose quantum computing from the examples of employing flux qubits. This is because superposition of two flux qubits can exist only in a point-contact condition. That means two harmonic oscillator rings are to be very weakly coupled. But the readout processes require that the internal system be coupled strongly to an external harmonic perturbation. Therefore we have two conflicting requirements. This is the first fundamental flaw of using qubits for general-purpose quantum computing. To resolve this problem, it has been shown that internal coupling (the entanglement) must cooperate with external coupling (the readout process), instead of the other way around. Furthermore, the locations of the readout probes are part of the computing because the number of coupled harmonic oscillators in the AB ring dictates the probing locations for robust readouts. That is the second flaw of the qubit concept. We then show that it is not the superposition of the four states in the two flux qubits that are relevant for the addition of two bits, rather it is the four symbolic superposition rules derived from the two collapsed qubits in the form of a diatomic-molecule, which is a strongly coupled double-rings, that becomes the fundamental processor for a general-purpose quantum computing. Whatever the manipulations of the two qubits that one hopes to achieve for the addition of two bits, it is already realized from the four symbolic substitution rules derived. Thus quantum computing is not to be logic-gate-based, or truth-table-based, as many researchers have pursued. Rather, the quantum computing is shown to be ruled-based that is capable of imposing the spatial relations. This is a necessary requirement for computing the most elementary operation of addition of two long bit strings and this is a pure sequential operation that has to be executed by a massive parallel quantum Turing machine. The fundamental quantum processor in each cell of CA can execute four instructions and with the capability of storing two data. The reversibility in the quantum computing here is not as strict as some unitary operators on the initialized state of the qubits implies. The elastic scatterings through the three probe terminals are reversible by the Buttiker symmetry rule and as long as one of the operand is saved, the computation is reversible even though there is a dissipation of energy at each charging-discharging cycle and is thermodynamically irreversible.

CA are usually investigated [23-25] based on one single bit per cell. There are two basic flaws of such approach to be pointed out also as a consequence of the employing the fundamental quantum processor. First, with a one-bit processor in each cell, the local computing power inside the cell is too small because each cell can only compute truth/false and there is no implementation of the needed spatial relations inside the cell. Secondly, it is not possible to comb through billion numbers of the inter-cell interaction rules to find out a suitable one for the parallel quantum computing. Addition-rule compatible CA exists as the only one set out of roughly 4.3 billion sets (or 4^{16}) to be found. It is shown that each cell must be composed of two bits possessed with the capabilities of executing four symbolic substitution rules and the storage of two data. This increases the computation power within the cell two times as compared to the one-bit-per-cell based CA. A quantum parallel computing further requires that all cells must be wired together according to the same addition-rule based connections in order to provide the proper spatial relations. We show this correct inter-cell and intra-cell connections, the canonical connections, to form the one-dimensional CA as shown in Fig.4-I (a) for a stable output configuration. Repeated usage of those one-dimensional CA chains will then lead to all algebraic operations in a two dimensional CA [17]. When the canonical interconnections are altered (such as those interconnections shown in Figs.4-II, 4-III, 4-IV and 4-V), the results will lead to either the oscillations between two configurations or in the form of some moving signals [31]. Those results are thus not addition-rule compatible and may be classified as *a new kind of science* [23, 26-30]. So when the spatial relations in a CA are not compatible with the addition rules, different or strange results will come out and they are labelled as a new kind of science. We must emphasize that all physical phenomena are clearly associated with addition-rule compatible CA's because physical phenomena are interpreted through the computations that are imposed by the use of addition rules.

Therefore those results derived from the altered interconnections (Fig. 4-II, 4-III, 4-IV and 4-V) will look odd or look new as measured by the rules that are addition-rule based. Those addition rule non-compatible CA's have the long range spatial relations relaxed while maintain the local half-adder capability within the cell. Therefore the addition-rule-based CA's are valid in Euclidean space, while addition-rule non-compatible CA's are for Non-Euclidean space and those results thus appear to be very odd or new only through the view that is Euclidean based. Reversible quantum computing implies reversible CA and it is a weaker requirement than that of unitary operations on the initial states before the readouts.

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FIGURE LEGENDS

FIG. 1: (a) A chain of N point-contacted AB rings with 2^N superposition states. (b) Every pair of them is grouped together into one cell with 4 superposition states to perform addition operations and with an attempted scattering probe set-up for the readouts in each cell. (c) Two AB rings in each cell are collapsed to form an artificial diatomic molecule with three external scattering probes attached. Transformation of (b) to (c) is made so that the 4 superposition states existed in (b) is converted into 4 symbolic substitution rules existed in (c) at the readout. This is illustrated in magnetic AB effect with two fluxes, Φ_1 and Φ_2 , as the bit-inputs with a test pulse from the S terminal. The scattered electron waves exit in three possible terminals, C, S and D [18]. (d) An enlarged version of (c) in the corresponding electric AB effect. Here the electric charge on V_1 and V_2 metal cylinders are the bit-inputs [17]. QC is a quantum circulator [19]. The alternating charging and discharging reset pulse sequences are shown [17]. One-dimensional CA with a processor of Figure 1(d) in each cell thus transforms 2^N superposition states of the N qubits in (a) into an equal number of symbolic substitution rules for quantum parallel computing of addition operations (FIG. 4-I (a)).

FIG.2: The four “search-and-replace” symbolic substitution rules for a binary addition of two bits are shown on the left half. The corresponding computing states and the designated dual-rail notations are shown on the right half. The ‘sum’, S, replaces the second addend at the current bit position, while the ‘carry’, C, replaces the first addend at the next most significant bit position.

FIG.3: Transmission characteristics for a test charging signal pulse from S terminal when the two fluxes, (Φ_1, Φ_2) , from Fig.1c are anti-symmetric (a) and symmetric (b). The four rules corresponding to the four flux-pairing configurations are labelled on the figure, with S denoted as the sum and C as the carry. See also Table A for the bit-pair mappings.

FIG. 4: A 1-D CA is shown in three consecutive cells (each cell is derived from Figure 1(d)) for various interconnection schemes: Five different cases are investigated. Case I (a) is for the correct addition-rule based connections, the canonical connections, when C terminal is connected to left cell at V_2 location and S terminal is connected to itself at V_1 location of Fig. 1(d). In Case I (b), C terminal is connected to the right cell, the situation of a backward addition. In Case II (a), C and D terminal are interchanged from Case I (a) and Case II (b) is the corresponding interchange from Case I (b). In Case III (a), S and D terminals are interchanged from Case I (a) and the corresponding interchanged of Case I (b) is in Case III (b). In Case IV (a), C and S terminals are interchanged from Case I (a). Similarly Case IV (b) for Case I (b). In Case V (a), D terminal is connected to the left cell and S terminal is connected to the right, resulting in a three cell rules that is summarized in the text. In Case V (b), D terminal is connected to the right and S terminal is connected to the left. The CA’s successive iteration results for Fig.4-I(a) are shown in Table B.

FIG. 5: The 16 CA transition rules: The 16 parent-child cell transition rules for the cell-to-cell interconnecting scheme of FIG. 4-I (a) are shown on the upper half. The child states are lined up with the parent states on the left side because of FIG.4-I (a) connections. Similarly the lower half is from FIG.4-I (b) interconnecting scheme. The child states are now lined up to the parent states on the right side. This is the time-reversal version.

Table A: The addition rules for a two-bit addition. Four possible bit-pairings are mapped into four combinations of the two fluxes in a magnetic AB effect or of the two electric charge polarity states on the metal cylinders V_1 and V_2 in an electric AB effect. The capabilities of the half-adder quantum processor of FIG. 1(d) are characterized by the transmission probabilities to C, S and D terminals when a test pulse of electron wave is originated from the S terminal. The high-low ratio is about 9 in the example shown in Reference [18].

Table B: The CA configurations after successive iterations. The original starting configuration $S_2S_3S_2S_3S_2S_4$ located at cell locations from #6 through #1. The rest of the cells are filled with S_1 states. Iteration sequences are labeled on the left columns and the cell locations are marked at the top rows. Table B shows the successive configurations for the CA using FIG. 4-I (a) connections. This is the case for the addition operation of operand A=101011 and operand B=010101. The result of $43+21=64$ is read from the binary bits at the 7th iteration configuration, $S_2S_1S_1S_1S_1S_1S_1$, (underlined) through the polarities of V_1 ’s located from cell 7 to cell 1. The charge polarity sequence at the V_1 ’s is then - ++++++ or the result of 1000000

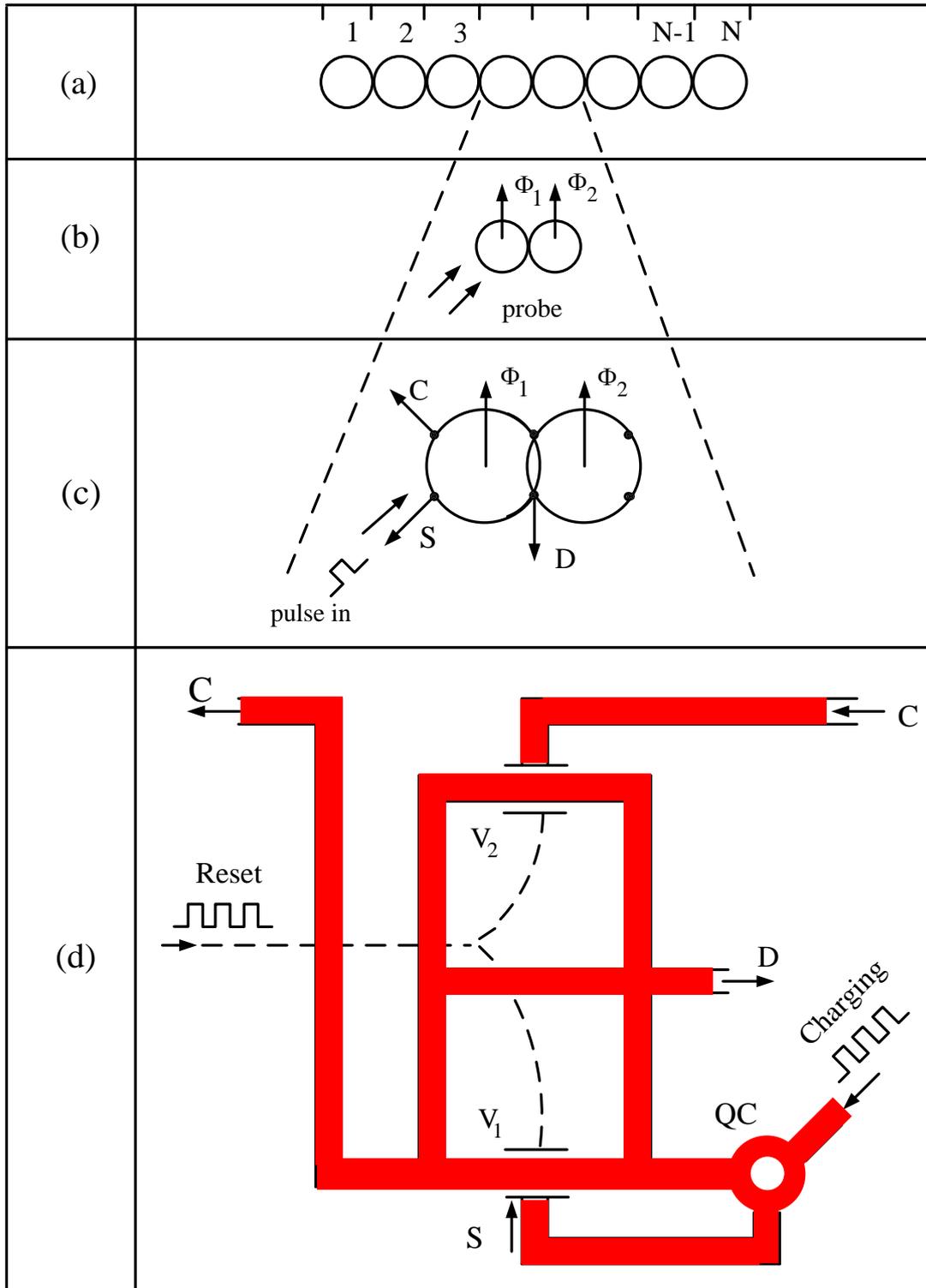


Figure 1

Rule	Symbolic substitution	State	Dual-rail notation
#1	$\begin{matrix} B & \boxed{0} \\ A & \boxed{0} \end{matrix} \rightarrow \begin{matrix} \boxed{0} & C \\ & \boxed{0} \end{matrix} S$	S_1	
#2	$\begin{matrix} B & \boxed{0} \\ A & \boxed{1} \end{matrix} \rightarrow \begin{matrix} \boxed{0} & C \\ & \boxed{1} \end{matrix} S$	S_2	
#3	$\begin{matrix} B & \boxed{1} \\ A & \boxed{0} \end{matrix} \rightarrow \begin{matrix} \boxed{0} & C \\ & \boxed{1} \end{matrix} S$	S_3	
#4	$\begin{matrix} B & \boxed{1} \\ A & \boxed{1} \end{matrix} \rightarrow \begin{matrix} \boxed{1} & C \\ & \boxed{0} \end{matrix} S$	S_4	

Figure 2

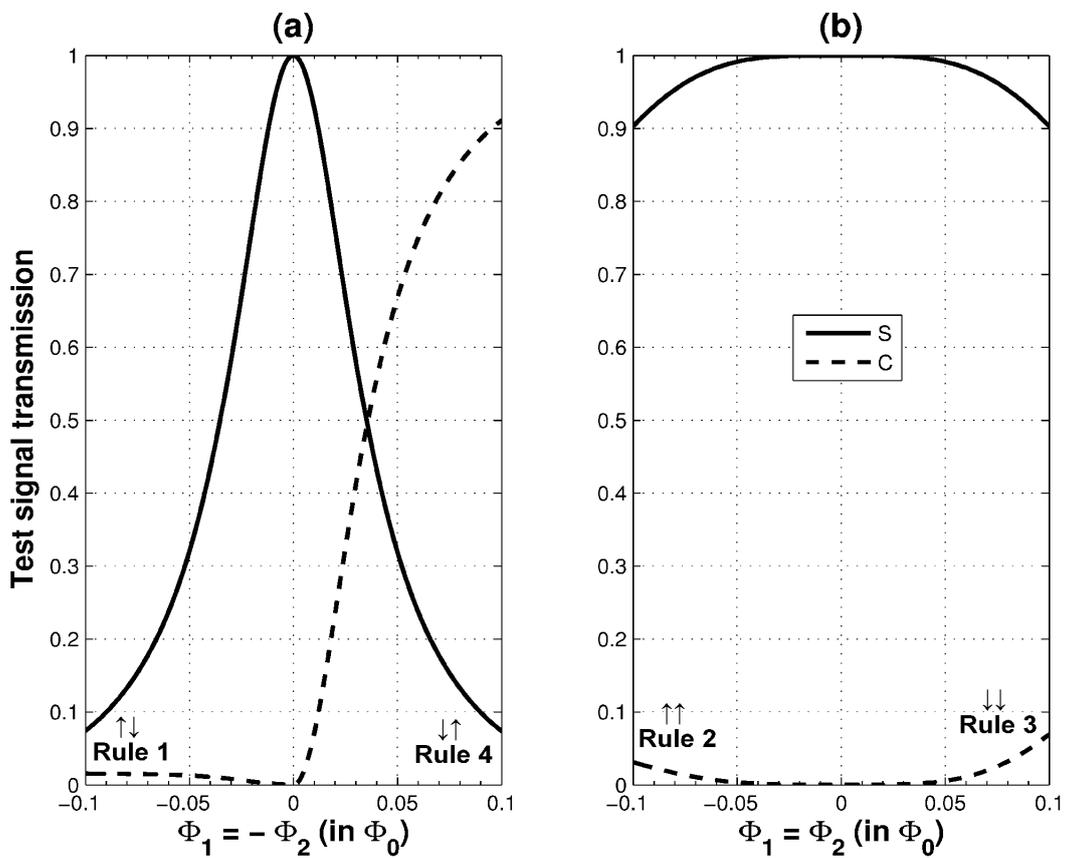


Figure 3

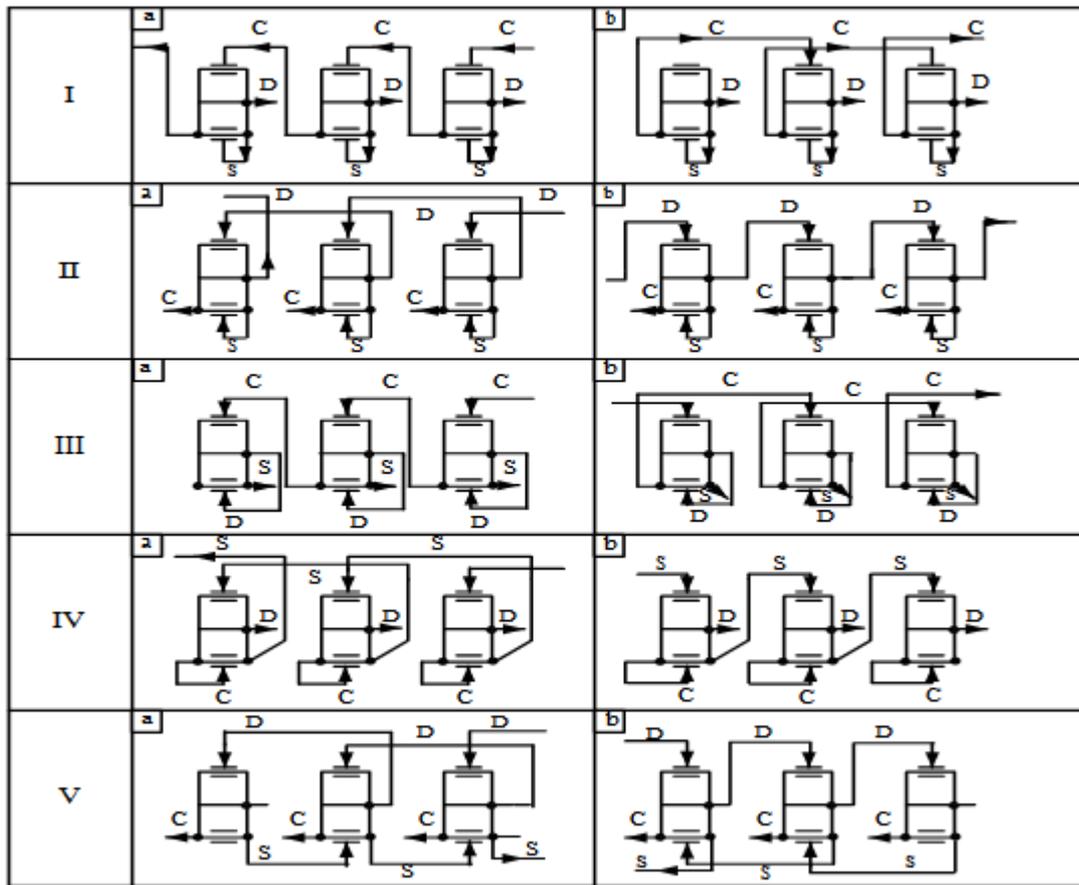


Figure 4



Figure 5

State	Bit Pair (B, A)	Flux directions		Voltage polarities		Output at Terminals		
		Φ_1	Φ_2	V_1	V_2	C	S	D
S_1	(0,0)	↑	↓	+	-	low	low	high
S_2	(0,1)	↑	↑	+	+	low	high	low
S_3	(1,0)	↓	↓	-	-	low	high	low
S_4	(1,1)	↓	↑	-	+	high	low	low

Table A

	8	7	6	5	4	3	2	1	0	-1	-2
0	S_1	S_1	S_2	S_3	S_2	S_3	S_2	S_4	S_1	S_1	.
1	S_1	S_1	S_2	S_2	S_2	S_2	S_4	S_1	S_1	.	.
2	S_1	S_1	S_2	S_2	S_2	S_4	S_1	S_1	.	.	.
3	S_1	S_1	S_2	S_2	S_4	S_1	S_1	S_1	.	.	.
4	S_1	S_1	S_2	S_4	S_1	S_1	S_1	S_1	.	.	.
5	S_1	S_1	S_4	S_1	S_1	S_1	S_1	S_1	.	.	.
6	S_1	S_3	S_1	S_1	S_1	S_1	S_1	S_1	.	.	.
7	S_1	S_2	S_1	S_1	S_1	S_1	S_1	S_1	.	.	.
8	S_1	S_2	S_1	S_1	S_1	S_1	S_1	S_1	.	.	.

Table B