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# Fluctuating Flow of Vescoelastic Fluids between Two Coaxial Cylinders

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**ABSTRACT:** *In this paper the flow of a viscoelastic fluid where the motion is due to the fluctuations of the outer cylinder has been studied. The governing equations are expressed in cylindrical polar coordinates. The influence of elastic parameter on velocity distribution and skin friction has been studied and the velocity profiles are shown in graphs.*

**KEYWORDS:** *Fluctuating flow, Viscoelastic fluid, cylindrical polar coordinates, skin friction, stress tensor.*

## I. INTRODUCTION

Flow of both Newtonian and non-Newtonian fluids through pipes and concentric cylinders has wide technological importance and has attracted the attention of many researchers. In everyday life we encounter such type of problems. Such types of problems are of great importance both in engineering and bio-physical fields. Pulsating flow due to pressure gradient in the direction of flow has been considered by Uchida (1956). The motion of the Newtonian fluids between two circular cylinders where the flow is due to the oscillation of the inner cylinder has been studied by Khamrui (1957). Chandrasekhar (1960) has studied the hydrodynamic stability of viscid flow between coaxial cylinders and got interesting results. Rath and Jena (1979) have investigated the flow of a viscous fluid generated in response to fluctuation in the axial velocity of the outer cylinder. They obtained a closed form solution and analysis has also been extended to low and high frequency approximations. Rajgopal et al. (1985) studied the flow of viscoelastic fluid between two coaxial cylinders. Biswal (1985) considered the above fluid which has fluctuating flow between two coaxial cylinders. Gupta et al. (1996) investigated the steady flow of an elastico-viscous fluid in porous coaxial circular cylinders. Irene Dris et al. (1998) studied the flow of a viscoelastic fluid between eccentric cylinders both theoretically and experimentally. Fetecau (2004) has obtained an analytical solution for non-Newtonian fluid flows in pipe like domain. Unsteady rotating flows of a viscoelastic fluid with the fractional Maxwell model between coaxial cylinders has been investigated by Qi and Jin (2006) and got interesting results. Erdogan et al. (2007) have discussed on the steady flow of a second grade fluid between two coaxial porous cylinders. Similar types of flow have been investigated by Kamra et al. (2010), Nazar et al. (2010) and Khandelwal et al. (2014) and got interesting results.

Samal et al. (2015) studied the fluctuating flow of a second order fluid between two coaxial circular pipes and got the analytical solution using Fourier transforms. In the present paper we have extended the above problem using viscoelastic fluids where the motion is due to the fluctuations of the outer cylinder. The velocity distributions for different values of elastic parameter have been studied.

## II. BASIC EQUATIONS

The governing equation for the flow of viscoelastic fluid model considered here is

$$p'^{ik} = 2\eta_0 e^{ik} - 2k_0 \tilde{e}^{ik} \tag{1}$$

where $p'^{ik}$ is the stress tensor, $\eta_0$ is the coefficient of viscosity, $k_0$ is the elastic parameter of the fluid and $e^{ik}$ is the rate of strain tensor. The term $\tilde{e}^{ik}$ appearing in equation (1) is given by

$$\tilde{e}^{ik} = \frac{\partial \tilde{e}^{ik}}{\partial t} + e^{ik}{}_{,\alpha} v^{\alpha} - e^{\alpha k} v^{k}{}_{,\alpha} - e^{\alpha k} v^{i}{}_{,\alpha} + e^{ik} v^{\alpha}{}_{,\alpha} \tag{2}$$

where $v^{\alpha}$ is the velocity vector.

The equation of motion and continuity are given by

$$\rho\left(\frac{\partial v^{i}}{\partial t}+v^{j}v^{i}{}_{,j}\right)=-p_{,j}+p^{ij}_{,j} \tag{3}$$

and $v^{\alpha}{}_{,\alpha}=0$ (4)

where $\rho$ is the density of the medium and $p$ is an arbitrary isotropic pressure.

## III. FORMULATION OF THE PROBLEM

We work through the cylindrical polar coordinates $(r,\theta,z)$ and z-axis coinciding with the common axis of the cylinders. Let $u,v,w$ be the velocity components of the fluid in the radial, azimuthal and axial directions respectively. The space between two cylinders is filled with viscoelastic fluid and the outer cylinder moves in axial direction along with a velocity $w=w_0\left(1+\varepsilon e^{iwt}\right)$, where $w_0,\varepsilon$ and $w$ are constants. As the cylinders are infinitely long and have axial symmetry, the velocity functions are considered to be independent of $\theta$ and $z$, hence functions of $t$ and $r$ respectively. Since there is no force to generate acceleration in the radial and azimuthal directions, we assume that $u=v=0$ in the entire flow region.
The momentum equation (3) reduces to

$$\frac{\partial w}{\partial t}=-\frac{1}{\rho}\frac{\partial p}{\partial z}+v_0\left(\frac{\partial^2 w}{\partial r^2}+\frac{1}{r}\frac{\partial w}{\partial r}\right)+v_1\frac{\partial}{\partial t}\left(\frac{\partial^2 w}{\partial r^2}+\frac{1}{r}\frac{\partial w}{\partial r}\right) \tag{5}$$

Where $v_0=-\frac{\eta_0}{\rho}$ the coefficient of kinematic viscosity is, $v_1=-\frac{k_0}{\rho}$ represents the coefficient of viscoelasticity and $\rho$ is the density of the fluid. The above equation is to be solved under the boundary conditions

$$\left.\begin{aligned} r&=b;\, w=0\\ r&=a;\, w=w_0\left(1+\varepsilon e^{iwt}\right)\end{aligned}\right\} \tag{6}$$

Here 'a' and 'b' are the outer and inner cylinder respectively.

## IV. SOLUTION OF THE PROBLEM

To solve the momentum equation (5), we assume

$$w(r,t)=G_1(r)+\varepsilon G_2(r)e^{iwt} \tag{7}$$

as the solution where the second term is the fluctuating part associated with an arbitrary parameter $\varepsilon$ of small magnitude.

Now from momentum equation in r-direction, we have

$$-\frac{1}{\rho}\frac{\partial p}{\partial r}=0 \tag{8}$$

Hence $p$ and $\frac{\partial p}{\partial r}$ can be taken as functions of 'z' and't'. From equation (5), we conclude that $\frac{\partial p}{\partial z}$ will be a function of't' alone as other terms do not depend on 'z'. Hence we suggest that

$$-\frac{1}{\rho}\frac{\partial p}{\partial r}=i\varepsilon we^{iwt} \tag{9}$$

Now using equations (7) and (9) in equation (5) and equating the terms independent of $\varepsilon$ and harmonic terms separately to zero, we get

$$f_1'' + \frac{1}{r} f_1' = 0 \tag{10}$$

$$v_0\left( f_1'' + \frac{1}{r} f_1' \right) + v_1 iw \left( f_1'' + \frac{1}{r} f_1' \right) + \left( w_0 - f_2 \right) iw = 0 \tag{11}$$

Where the primes denotes the differentiation with respect to ‘r’.

The boundary conditions to be satisfied are

$$\left. \begin{array}{l} r = b;\ f_1 - f_2 = 0 \\ r = a;\ f_1 = f_2 = w_0 \end{array} \right\} \tag{12}$$

We introduce the following dimensionless quantities

$$\left. \begin{array}{l} \eta = \dfrac{r}{a},\ w' = \dfrac{w_a}{w_0},\ t' = \dfrac{rw_0^2}{\nu} \\ F_1(\eta) = \dfrac{f_1(r)}{w_0};\ F_2(\eta) = \dfrac{f_2(r)}{w_0} \end{array} \right\} \tag{13}$$

Thus equations (10) and (11) reduce to

$$F_1'' + \frac{1}{\eta} F_1' = 0 \tag{14}$$

$$\left( F_2'' + \frac{1}{\eta} F_2' \right) + iK \left( F_2'' + \frac{1}{\eta} F_2' \right) + i(1 - F_2) = 0 \tag{15}$$

Where $L = \dfrac{v_0}{w_0 a w'},\ K = \dfrac{v_1}{a^2}$

The corresponding boundary conditions are

$$\left. \begin{array}{l} \eta = h;\ F_1 - F_2 = 0 \\ r = 1;\ F_1 = F_2 = 1 \end{array} \right\} \tag{16}$$

Where $h = \dfrac{b}{a}$

From equation (14), we have

$$F_1 = \frac{\log\left( h / \eta \right)}{\log\ h} \tag{17}$$

To solve equation (15), we take

$$F_2 = F_2 + iF_{2i} \tag{18}$$

And equation (15) yields

$$L\left( F_{22}'' + \frac{1}{\eta} F_{2r}' \right) - K\left( F_{2i}'' + \frac{1}{\eta} F_{2i}' \right) + F_{2i} = 0$$
(19)

$$L\left( F_{22}'' + \frac{1}{\eta} F_{2i}' \right) + K\left( F_{2r}'' + \frac{1}{\eta} F_{2i}' \right) + (1 - F_{2r}) = 0 \tag{20}$$

The corresponding boundary conditions become

$$\left. \begin{array}{l} \eta = h; F_{2r} = F_{2i} = 0 \\ r = 1; F_{2r} = 1, F_{2i} = 0 \end{array} \right\} \tag{21}$$

The solutions of equation (19) and (20) are

$$F_{2r} = 1 + A_1 Ber\ \lambda\eta + A_2 Bei\ \lambda\eta + A_3 Ker\ \lambda\eta + A_4 Kei\ \lambda\eta + \ldots \tag{22}$$

$$F_{2i} = -A_2 Ber\ \lambda\eta + A_1 Bei\ \lambda\eta + A_3 Kei\ \lambda\eta - A_4 Ker\ \lambda\eta + \ldots \tag{23}$$

Where $\lambda = \dfrac{L}{L^2 + K^2}$,

$$A_1 = -\frac{PKer\ \lambda + QKei\ \lambda}{P^2 + Q^2},\ A_2 = -\frac{PKei\ \lambda - QKer\ \lambda}{P^2 + Q^2},$$

$$A_3 = -\frac{PBer\ \lambda + QBei\ \lambda}{P^2 + Q^2},\ A_4 = -\frac{PBei\ \lambda - QBer\ \lambda}{P^2 + Q^2},$$

$$P = \left( Ker\ \lambda Ber\ \lambda h - Kei\ \lambda Bei\ \lambda h \right) - \left( Ker\ \lambda h Ber\ \lambda - Kei\ \lambda h Bei\ \lambda \right)$$

$$Q = \left( Kei\ \lambda Ber\ \lambda h + Ker\ \lambda Bei\ \lambda h \right) - \left( Kei\ \lambda h Ber\ \lambda + Ker\ \lambda h Bei\ \lambda \right)$$

## V. SKIN FRICTION

The response of skin-friction at the wall to the fluctuation in velocity is of immense practical importance. The skin-friction in the case of outer cylinder is given by

Skin friction (outer cylinder) $= \eta_0 w_0 \left[ -\dfrac{1}{\log\ h} + \lambda\varepsilon\alpha_1 \cos(wt' + \beta_1) \right]$

Where $\alpha_1 = \left[ f_{2r}'^2(1) + f_{2i}'^2(1) \right]^{1/2}$ and $\beta_1 = \tan^{-1}\left( \dfrac{f_{2i}'^2(1)}{f_{2r}'^2(1)} \right)$

The skin-friction in the inner cylinder is

Skin friction (inner cylinder) $= \eta_0 w_0 \left[ -\dfrac{1}{h\log\ h} + \lambda\varepsilon\alpha_2 \cos(wt' + \beta_2) \right]$

Where $\alpha_2 = \left[ f'^{2}_{2r}(h) + f'^{2}_{2i}(h) \right]^{1/2}$ and $\beta_2 = \tan^{-1}\left( \dfrac{f'^{2}_{2i}(h)}{f'^{2}_{2r}(h)} \right)$

## VI. CONCLUSION

In this note, we have studied the flow of viscoelastic fluids between two coaxial cylinders. The flow is due to the fluctuation of the outer cylinder which moves with the velocity $w = w_0\left(1 + \varepsilon e^{iwt}\right)$. The present study gives the following conclusions.

Figure-1 depicts the velocity distribution for $\varepsilon = 0.5, \lambda = 5$ and for different values of the elastic parameter and $wt$. Here we observe that with the increase of the elastic parameter, the velocity of the fluid increases. Also as $wt$ increases with passage of time, there is a slowdown of the flow. The skin friction is also influenced considerably by the elasticity of the fluid. It is observed that by increasing the velocity fluctuations of the outer cylinder, the rate of flow of the fluid through the annulus of the pipes could be increased. This result could be profitably employed in polymer industries where the pumping in or pumping out of the fluid is involved.

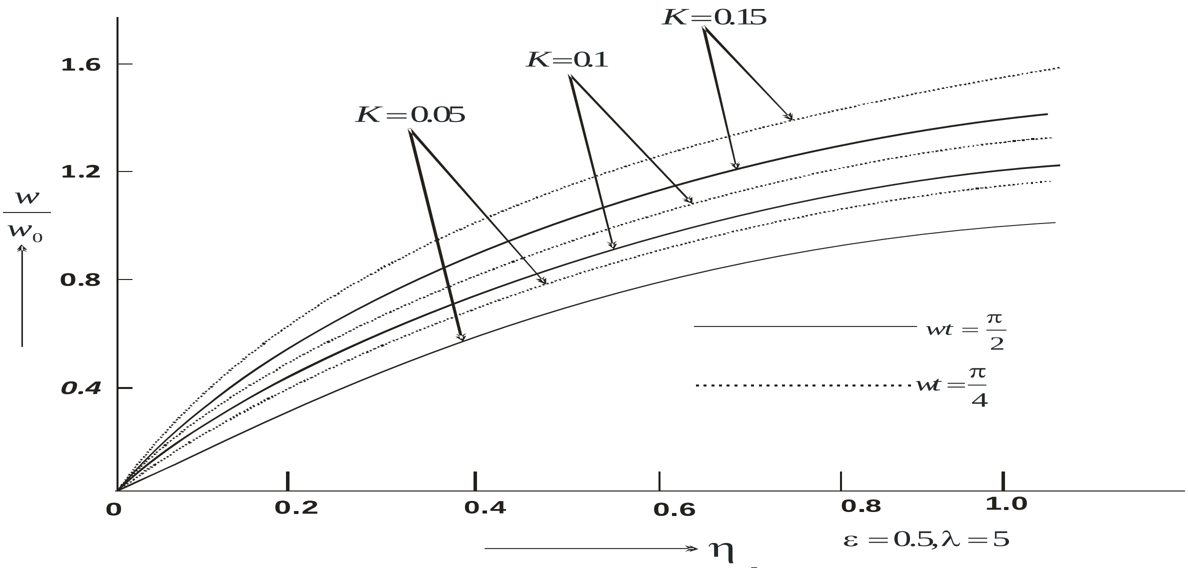


Figure-1: Velocity distribution for different values of $k$

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