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Snap-through Buckling Analysis of FGM Shallow Spherical Shells due to Temperature Variations

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ABSTRACT: *The snap-through buckling of shallow spherical shells made of functionally graded material (FGM) induced by temperature variations is studied in this paper. The volume fractions of the material constituents are assumed to be in the form of power exponential function of the coordinate along the thickness direction, and the physical property of FGM is obtained by Voight mixing rate model. The governing equations for nonlinear deformation of FGM shallow spherical shells are presented for the case of axisymmetrical deformation based on the thin shell theory. The analytical solution for the nonlinear characteristic relation between the temperature variation and central deflection is derived by using the asymptotic iteration method. Numerical examples are given and comparison of the present results with finite element simulation shows accuracy and validity of the theoretical model. The resulting solution can be used readily to do parametric analysis of FGM shell structures.*

KEYWORDS - *functional graded materials (FGM); shallow spherical shell; temperature variation; snap-through buckling, asymptotic iteration method*

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I. INTRODUCTION

Nowadays it becomes more imperative that some requirements of heat resistance, corrosion resistance and designability should meet for engineering materials and their structures under various special environmental conditions for the purpose of safe use. As a type of multi-component materials, functionally graded materials (FGM) can be chosen to accord with these demands, and have been used widely as structural elements in mechanical, aeronautical and aerospace engineering, etc.

Buckling analysis of FGM structures under temperature condition is of great importance to evaluation of safety. Shahsiah and Eslami gave an analysis of thermal buckling of simply-supported FGM cylindrical shell based on the first-order shell theory and Sanders' equations [1]. Using Donnell's theory, Wu, Jiang and Liu studied thermal buckling problem of the FGM cylindrical shell and critical buckling loading was obtained by applying the method of critical equilibrium together with the prescribed mode of buckling [2]. Due to the Donnell–Mushtari–Vlasov assumption and Sanders' equations, Shahsiah, Eslami and Naj [3] solved the thermal stability problem of FGM shallow spherical shell. Prakash, Sundararajan and Ganapathi [4] adopted Mindlin's theory to analyze the nonlinear axisymmetrical dynamic buckling of clamped FGM spherical shells. Hafezalkotob and Eslami [5] presented the critical buckling temperature by a corresponding solution for eigen-value problem of simply supported shallow FGM spherical shells in which component materials are dependent of temperature. Boroujerdy and Eslami derived an analytical solution for thermal buckling of piezo-FGM shallow spherical shells under conditions of three types of thermal loadings for the constant driving voltages [6]. Introducing first-order shear deformation theory and geometrical nonlinear theory, Tung [7] deduced the relationship of critical loading with geometrical and material parameters of FGM shallow spherical shells resting on elastic foundations with temperature-dependent material. Anh, Bich and Duc [8] tackled nonlinear stability problem of thin FGM annular spherical shells on elastic foundations under a combined action of external pressure and thermal loads by Galerkin's method. Further, Anh and Duc [9] solved the nonlinear stability of a sigmoid functionally graded material (S-FGM) shallow spherical shell incorporating effect of transverse shear deformation. Moosaie and Panahi-Kalus investigated the thermal stability of an incompressible FGM spherical shell [10].

Recently most existing studies on the thermal buckling and stability of FGM shallow shell structures have focused on the case of combined action of mechanical loading and temperature variation. For the case of only temperature variation, related work is concentrated on treatment of bifurcation buckling mainly induced by in-plane compressive loading. In addition, some specific forms of modes of buckling, e.g., trigonometric function, etc. have been prescribed in the process of solution, which makes problems easy to solve, especially for nonlinear analysis. So far there are relatively few studies on thermal snap-through buckling (limited point buckling) of shallow shell structures [11]. This work aims to present an analytical solution for snap-through buckling of FGM shallow spherical shells under temperature loading by an asymptotic iteration method. The method has been used widely to solve nonlinear problems of plates and shells and proved to have good convergence in computation [12-15].

II.MATHEMATICAL FORMULATION AND SOLUTION

Let us consider a FGM shallow spherical shell, as shown in Fig.1. The shell is of the radius of curvature *R*, thickness *h*, span 2*a* and apex height *H*. It is assumed that the shell is made of mixed materials of both metal and ceramic layers, and the former is placed at bottom layer while the latter at upper layer. Material property is assumed to be changed in the direction of thickness of the shell based on a power law associated with volume fractions of the material constituents expressed by

$$\begin{cases} V_m = \left(\dfrac{1}{2} + \dfrac{z}{h}\right)^k \\ V_m + V_c = 1 \end{cases} \tag{1}$$

where V_c and V_m are volume fractions of ceramic and metal respectively, and k is constituent parameter. According to Voight model, one can write

$$P_F = P_m V_m + P_c V_c \tag{2}$$

in which P_F is FGM physical property, P_m and P_c are physical properties of metal and ceramic materials, respectively.

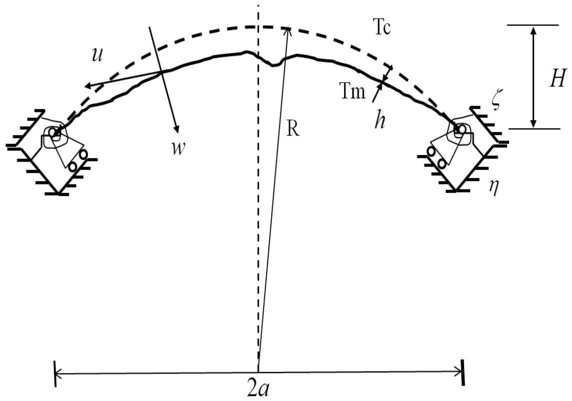

Fig. 1 Geometrical model of FGM shallow spherical shells

According to Eqs.(1) and (2), elastic modulus and coefficient of thermal expansion in FGM, $E(z)$ $\alpha(z)$ can be written by

$$\begin{cases} E(z) = E_c + \left(\dfrac{1}{2} + \dfrac{z}{h}\right)^k (E_m - E_c) \\ \alpha(z) = \alpha_c + \left(\dfrac{1}{2} + \dfrac{z}{h}\right)^k (\alpha_m - \alpha_c) \end{cases} \tag{3}$$

where E_c, E_m and α_c, α_m are elastic moduli and coefficients of thermal expansion for ceramic and metal, respectively. It is seen that $k = 0$ and $k \to \infty$ correspond to pure metal and ceramic, respectively.

For the case of axisymmetric deformation, the strain components at any point in the shell is written by

$$\varepsilon_r = \varepsilon_r^0 - z\chi_r,\ \varepsilon_\theta = \varepsilon_\theta^0 - z\chi_\theta \tag{4}$$

where $\varepsilon_r^0, \varepsilon_\theta^0; \chi_r, \chi_\theta$ are the strain components at mid-plane and curvature variations expressed by radial displacement *u* and deflection *w* as

$$\varepsilon_r^0 = \frac{du}{dr} - \frac{w}{R} + \frac{1}{2}\left(\frac{dw}{dr}\right)^2,\ \varepsilon_\theta^0 = \frac{u}{r} - \frac{w}{R} \tag{5}$$

and

$$\chi_r = \frac{d^2 w}{dr^2}, \chi_\theta = \frac{1}{r}\frac{dw}{dr} \tag{6}$$

Constitutive equations are

$$\sigma_r = \frac{E(z)}{1-\nu^2}\left(\varepsilon_r + \nu\varepsilon_\theta\right) - \frac{E(z)\alpha(z)t(z)}{1-\nu}$$
$$\sigma_\theta = \frac{E(z)}{1-\nu^2}\left(\varepsilon_\theta + \nu\varepsilon_r\right) - \frac{E(z)\alpha(z)t(z)}{1-\nu} \tag{7}$$

Internal forces and moments are expressed as

$$N_r = \int_{-h/2}^{h/2} \sigma_r dz \quad N_\theta = \int_{-h/2}^{h/2} \sigma_\theta dz \tag{8}$$

$$M_r = \int_{-h/2}^{h/2} z\sigma_r dz \quad M_\theta = \int_{-h/2}^{h/2} z\sigma_\theta dz \tag{9}$$

Using Eqs.(4)-(7), Eqs.(8) and (9) change to

$$N_r = \frac{E_1}{1-\nu^2}\left(\varepsilon_r^0 + \nu\varepsilon_\theta^0\right) - \frac{E_2}{1-\nu^2}\left(\chi_r + \nu\chi_\theta\right) - \frac{\Phi_1}{1-\nu}$$
$$N_\theta = \frac{E_1}{1-\nu^2}\left(\varepsilon_\theta{}^0 + \nu\varepsilon_r{}^0\right) - \frac{E_2}{1-\nu^2}\left(\chi_\theta + \nu\chi_r\right) - \frac{\Phi_1}{1-\nu}$$
$$M_r = \frac{E_2}{1-\nu^2}\left(\varepsilon_r^0 + \nu\varepsilon_\theta^0\right) - \frac{E_3}{1-\nu^2}\left(\chi_r + \nu\chi_\theta\right) - \frac{\Phi_2}{1-\nu} \tag{10}$$
$$M_\theta = \frac{E_2}{1-\nu^2}\left(\varepsilon_\theta^0 + \nu\varepsilon_r^0\right) - \frac{E_3}{1-\nu^2}\left(\chi_\theta + \nu\chi_r\right) - \frac{\Phi_2}{1-\nu}$$

where

$$E_1 = \int_{-h/2}^{h/2} E(z)dz = E_c h + \frac{E_{mc}h}{k+1}, E_2 = \int_{-h/2}^{h/2} zE(z)dz = E_{mc}h^2\left(\frac{1}{k+2} - \frac{1}{2k+2}\right)$$
$$E_3 = \int_{-h/2}^{h/2} z^2 E(z)dz = \frac{E_c h^3}{12} + E_{mc}h^3\left(\frac{1}{k+3} - \frac{1}{k+2} + \frac{1}{4+4k}\right), E_{mc} = E_m - E_c \tag{11}$$
$$\Phi_1 = \int_{-h/2}^{h/2} E(z)\alpha(z)t(z)dz, \Phi_2 = \int_{-h/2}^{h/2} E(z)\alpha(z)zt(z)dz$$

The equilibrium equations for the shell are written by

$$\frac{d}{dr}\left(rN_r\right) - N_\theta = 0 \tag{12}$$

$$\frac{d}{dr}\left(rM_r\right) - M_\theta - rQ_r = 0 \tag{13}$$

$$\frac{d}{dr}\left(rN_r\left(\frac{r}{R} + \frac{dw}{dr}\right) + rQ_r\right) = 0 \tag{14}$$

where Q_r is transverse shear force. Eliminating u in Eq.(5), a compatibility equation is derived by

$$\varepsilon_r^0 = \frac{r}{R}\frac{dw}{dr} + \varepsilon_\theta^0 + r\frac{d\varepsilon_\theta^0}{dr} + \frac{1}{2}\left(\frac{dw}{dr}\right)^2 \tag{15}$$

According to Eq.(10), the strain components are expressed by the internal forces below

$$\varepsilon_r^0 = \frac{1}{E_1}\left(N_r - \nu N_\theta + E_2\chi_r + \Phi_1\right)$$
$$\varepsilon_\theta^0 = \frac{1}{E_1}\left(N_\theta - \nu N_r + E_2\chi_\theta + \Phi_1\right) \tag{16}$$

Introduce a force function ϕ defined by

$$N_r = \frac{1}{r}\frac{d\phi}{dr}, N_\theta = \frac{d^2\phi}{dr^2} \tag{17}$$

then Eq.(12) is satisfied automatically. Substituting Eqs.(17) and (16) into Eq.(15), the compatibility equation is expressed in terms of the force function ϕ and deflection w as

$$r\frac{d\phi}{dr}\frac{1}{r}\frac{d}{dr}r\frac{d}{dr}\phi = -E_1\frac{dw}{dr}\left(\frac{r}{R} + \frac{1}{2}\frac{dw}{dr}\right) \tag{18}$$

Further, eliminating Q_r by using Eqs.(13) and (14), then using Eqs.(10), (16) and (17), the equilibrium equation in the direction of thickness is also written by the force function and deflection in the following

$$E_4\left(\frac{d^4w}{dr^4}+\frac{2}{r}\frac{d^3w}{dr^3}-\frac{1}{r^2}\frac{d^2w}{dr^2}+\frac{1}{r^3}\frac{dw}{dr}\right)=\frac{1}{r}\frac{d}{dr}\left(\frac{d\phi}{dr}\left(\frac{r}{R}+\frac{dw}{dr}\right)\right) \tag{19}$$

where $E_4=\dfrac{E_1E_3-E_2^2}{E_1\left(1-\nu^2\right)}$. The equations (18) and (19) form fundamental governing equations for nonlinear deformation of the shell. For the elastic constrained edge, i.e., radial displacement and rotational constraints, the boundary conditions are written by [12]

$$w=0\,,\ M_r=\xi\frac{dw}{dr}\,,\ N_r=-\eta u\,,\ r=a \tag{20}$$

and

$$w=\text{finite}\,,\ \frac{dw}{dr}=0\,,\ rN_r=0\ ,\ r=0 \tag{21}$$

where ξ,η are coefficients of the radial and rotational constraints, respectively. Introducing the following nondimensional variables and quantities

$$\begin{gathered}\rho=\frac{r}{a}\,,W=\frac{w}{h}\,,B_1=\frac{\zeta a}{E_ch^3}\,,B_2=\frac{\eta a}{E_ch}\,,\\ M=\frac{a^2}{Rh}\approx\frac{2H}{h}\,,T=\frac{a}{E_ch^3}rN_r=\frac{a}{E_ch^3}\frac{d\phi}{dr}\,,\\ A_1=\frac{a^2\beta\Phi_1}{E_4\alpha\left(1-\nu\right)}\,,A_2=\frac{a^2\Phi_2}{E_4\left(1-\nu\right)h}\,,Q=A_2-A_1\\ \alpha=\frac{E_1}{E_ch}\,,\beta=\frac{E_2}{E_ch^2}\,,\gamma=\frac{E_4}{E_ch^3}\end{gathered} \tag{22}$$

where Q is nondimensional temperature variation, and A_1, A_2 are associated with the temperature t. Nondimensional governing equations and boundary conditions are expressed by

$$\frac{1}{\rho}\frac{d}{d\rho}\rho\frac{d}{d\rho}\frac{1}{\rho}\frac{d}{d\rho}\rho\frac{d}{d\rho}W=\frac{1}{\gamma}\left(\frac{1}{\rho}\frac{d}{\rho}\left(T\left(M\rho+\frac{dW}{d\rho}\right)\right)\right) \tag{23}$$

$$\rho\frac{d}{d\rho}\frac{1}{\rho}\frac{d}{d\rho}\left(\rho T\right)=-\alpha\frac{dW}{d\rho}\left(M\rho+\frac{1}{2}\frac{dW}{d\rho}\right) \tag{24}$$

and

$$W=0\ ,\ \frac{d^2W}{d\rho^2}+b_1\frac{dW}{d\rho}=\frac{\beta}{\alpha\gamma}T-Q\ ,\frac{dT}{d\rho}-b_2T=-\beta\frac{dW}{d\rho}-\frac{A_1\gamma\alpha\left(1-\nu\right)}{\beta}\,,\ \rho=1 \tag{25}$$

$$\frac{dW}{d\rho}=0\,,\ T=0\,,\ \rho=0 \tag{26}$$

where

$$b_1=\frac{B_1}{\gamma}+\nu\ ,\ b_2=\nu-\frac{\alpha}{B_2} \tag{27}$$

In preceding analysis, it is assumed that the temperature variation is uniform, i.e., $t\left(z\right)=t$ (constant). The asymptotic iteration method is use to solve Eqs.(23) and (24) in connection with Eq.(25) and (26). For the first iteration process, nonlinear terms in Eq.(23) is neglected, the reduced linear problem can be expressed by using the following differential equations

$$\frac{1}{\rho}\frac{d}{d\rho}\rho\frac{d}{d\rho}\frac{1}{\rho}\frac{d}{d\rho}\rho\frac{d}{d\rho}W_1=0 \tag{28}$$

$$\rho\frac{d}{d\rho}\frac{1}{\rho}\frac{d}{d\rho}\left(\rho T_1\right)=-\alpha\frac{dW_1}{d\rho}\left(M\rho+\frac{1}{2}\frac{dW_1}{d\rho}\right) \tag{29}$$

and

$$W_1 = 0\,,\; \frac{d^2W_1}{d\rho^2} + b_1\frac{dW_1}{d\rho} = -Q\;,\; \frac{dT_1}{d\rho} - b_2T_1 = -\beta\frac{dW_1}{d\rho} - \frac{a_1\gamma\alpha(1-\nu)Q}{(a_2-a_1)\beta}\,,\; \rho = 1 \tag{30}$$

$$\frac{dW_1}{d\rho} = 0\;,\; T_1 = 0\,,\; \rho = 0 \tag{31}$$

where subscript 1 represents the first iteration for W and T. Denote the central deflection

$$W_1\,|_{\rho=0} = W_m \tag{32}$$

The solution for Eq.(28) is

$$Q = 2(1+b_1)W_m \tag{33}$$

$$W_1 = W_m - W_m\rho^2 \tag{34}$$

Substituting Eq.(34) into Eq.(29), Using the corresponding condition equations in Eqs.(30) and (31) for T_1, the analytical expression for it can be obtained as

$$T_1 = T_{11}W_m + T_{12}{W_m}^2 \tag{35}$$

in which

$$\begin{aligned} T_{11} &= \left(\frac{3M\alpha}{4(b_2-1)} - \frac{2\beta}{(b_2-1)} + \frac{2\alpha\gamma a_1(\nu-1)(b_1+1)}{\beta(a_1-a_2)(b_2-1)} - \frac{M\alpha b_2}{4(b_2-1)}\right)\rho + \frac{M\alpha}{4}\rho^3 \\ T_{12} &= \left(\frac{\alpha\beta b_2}{4\beta(b_2-1)} - \frac{3\alpha\beta}{4\beta(b_2-1)}\right)\rho - \frac{\alpha}{4}\rho^3 \end{aligned} \tag{36}$$

Applying the resulting W_1 and T_1 for the first iteration, resuming the coupled terms in Eq.(23), the corresponding equation for the second iteration process is written by

$$\frac{1}{\rho}\frac{d}{d\rho}\rho\frac{d}{d\rho}\frac{1}{\rho}\frac{d}{d\rho}\rho\frac{d}{d\rho}W_2 = \frac{1}{\gamma}\left(\frac{1}{\rho}\frac{d}{\rho}\left(T_1\left(M\rho + \frac{dW_1}{d\rho}\right)\right)\right) \tag{37}$$

and

$$\begin{aligned} &W_2 = 0\;,\; \frac{d^2W_2}{d\rho^2} + b_1\frac{dW_2}{d\rho} = \frac{\beta}{\alpha\gamma}T_1 - Q\,,\; \rho = 1 \\ &\frac{dW_2}{d\rho} = 0\;,\; \rho = 0 \end{aligned} \tag{38}$$

where subscript 2 stands for the second iteration. The nonlinear characteristic relation between Q and W_m can be derived by solution for Eq.(37) in the following

$$Q = \lambda_{11}W_m + \lambda_{22}W_m^2 + \lambda_{33}W_m^3 \tag{39}$$

where $\lambda_{11}, \lambda_{22}, \lambda_{33}$ are coefficients associated with geometrical and material parameters of the FGM shell. Based on the extremum condition, $dQ/dW_m = 0$, the critical temperature variations Q_{cr}, t_{cr} can be determined finally.

III. NUMERICAL EXAMPLES

In computation the geometrical sizes are $R = 500mm, a = 200mm, h = 10mm$. The alumina ceramics and steel are chosen as material constituents, their properties are listed as: $E_c = 380GPa, \alpha_c = 7.2\times10^{-6}$ and $E_m = 200GPa, \alpha_m = 11.7\times10^{-6}$. The Poisson' ratio is taken as $\nu = 0.3$. In numerical simulation, S4R element in ABAQUS code is adopted to construct a FEM model of the shell with 16565 elements, as show in Fig. 2. The shell is divided into *n* isotropic layers along the direction of thickness with parameters given by

$$E_i = E_c + (E_m - E_c)\left(\frac{i}{n}\right)^k,\; i = 0,1,2...,n$$

$$\alpha_i = E_c + (E_m - E_c)\left(\frac{i}{n}\right)^k,\; i = 0,1,2...,n$$

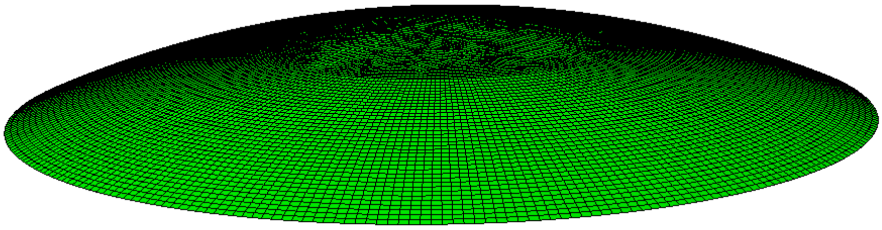

Fig. 2 Finite element model of FGM shallow spherical shells

Snap-through buckling deformation of FGM shallow spherical shells is displayed in Fig. 3. For the case of movable simply-supported shell, a comparison of central deflection induced by different temperature variations is listed in Table 1. A corresponding comparison of critical buckling temperature is presented in Table 2. The results show that the theoretical prediction agrees with the FEM simulation. It should be pointed that the numerical model is comparative approximate one by dividing finite isotropic layers to reflect the change in physical properties in the form of power exponential function along the direction of thickness.

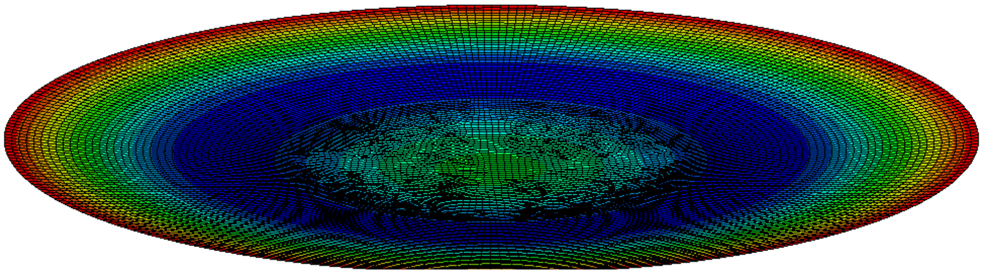

Fig. 3 Snap-through buckling deformation of FGM shallow spherical shells

Table1 Comparison of bending deformation

Temperature variation $\left({}^{\circ}C\right)$	Central deflection (mm) present model	Central deflection (mm) FEM	Error
102	0.089	0.098	10%
202	0.180	0.197	9%
302	0.264	0.297	12%
402	0.352	0.398	13%
502	0.441	0.490	11%
602	0.532	0.593	11%
702	0.622	0.693	10%
802	0.713	0.805	13%
902	0.804	0.912	13%
1002	0.896	1.012	13%

Table 2 Comparison of critical buckling temperature of FGM shallow spherical shells

Critical buckling temperature $\left({}^{\circ}C\right)$ present model	Critical buckling temperature $\left({}^{\circ}C\right)$ FEM	Error
9805	8300	16%

IV. CONCLUSIONS

This paper presents snap-through buckling of shallow spherical shells made of functionally graded material (FGM) under temperature loading. The governing equations for nonlinear deformation of FGM shallow spherical shells are given for the axisymmetrical case. The volume fractions of the material constituents are assumed to be in the form of power exponential function of the coordinate along the thickness direction, and the physical property of FGM is obtained by Voight mixing rate model. The analytical solution for the nonlinear relation between the temperature variation and central deflection of the shell is obtained by using the asymptotic iteration method. In the process of solution a mode of buckling needs not to be prescribed. The theoretical solution is verified by a comparison with finite element simulation. The proposed model can be used to do parametric analysis in evaluation of nonlinear deformation and buckling behaviors of such FGM shell structures.

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