

## Generalized Semi Generalized Closed Sets in Intuitionistic Fuzzy Topological Spaces

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**ABSTRACT:** In 1970, Levine introduced the concept of generalized closed sets in general topology. He observed that the family of all closed sets in a topological space  $X$  is a subfamily of the family of all generalized closed sets. He generalized some of well-known results of general topology replacing closed set by generalized closed sets, for instance, generalized closed subset of a compact space is compact and generalized closed subspace of a normal space is normal. Many authors utilized  $g$ -closed sets for the generalization of various topological concepts in general topology. The concept of intuitionistic fuzzy sets was introduced by Atanassov in 1983, as a generalization of fuzzy sets. In 1997 Coker introduced the concept of intuitionistic fuzzy topological spaces. In 2008, Thakur and Chaturvedi introduced the notion of intuitionistic fuzzy generalized closed set in intuitionistic fuzzy topological space. After that different mathematicians worked and studied in different forms of intuitionistic fuzzy  $g$ -closed set and related topological properties. The aim of this paper is to introduce the new class of intuitionistic fuzzy closed sets called intuitionistic fuzzy generalized semi generalized closed set (briefly intuitionistic fuzzy  $gsg$ -closed sets) in intuitionistic fuzzy topological space. The class of all intuitionistic fuzzy  $gsg$ -closed sets lies between the class of all intuitionistic fuzzy closed sets and class of all intuitionistic fuzzy  $g$ -closed sets. We also introduce the concepts of intuitionistic fuzzy  $gsg$ -open sets in intuitionistic fuzzy topological spaces. As an application of this set we introduce intuitionistic fuzzy  $gsg$ - $T_{1/2}$ -space.

**KEYWORDS:** Intuitionistic fuzzy  $g$ -closed sets, Intuitionistic fuzzy  $sg$ -closed sets, Intuitionistic fuzzy  $gsg$ -closed sets and intuitionistic fuzzy  $gsg$ -open set, intuitionistic fuzzy  $gsg$ - $T_{1/2}$ -space.

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### I. INTRODUCTION

After the introduction of fuzzy sets by Zadeh [24] in 1965 and fuzzy topology by Chang [4] in 1967, several researches were conducted on the generalizations of the notions of fuzzy sets and fuzzy topology. The concept of intuitionistic fuzzy sets was introduced by Atanassov [1] as a generalization of fuzzy sets. In the last 25 years various concepts of fuzzy mathematics have been extended for intuitionistic fuzzy sets. In 1997 Coker [5] introduced the concept of intuitionistic fuzzy topological spaces. In 2008 Thakur and Chaturvedi introduced the concepts of intuitionistic fuzzy generalized closed sets [15] in intuitionistic fuzzy topology. After that many weak and strong forms of intuitionistic fuzzy  $g$ -closed sets such as intuitionistic fuzzy  $rg$ -closed sets [16], intuitionistic fuzzy  $sg$ -closed sets [17], intuitionistic fuzzy  $ag$ -closed sets [14], intuitionistic fuzzy  $g\alpha$ -closed sets [8], intuitionistic fuzzy  $w$ -closed sets [18], intuitionistic fuzzy  $rw$ -closed sets [19], intuitionistic fuzzy  $gpr$ -closed sets [20], intuitionistic fuzzy  $rg\alpha$ -closed sets [21], intuitionistic fuzzy  $gsp$ -[12] closed sets, intuitionistic fuzzy  $gp$  [10], intuitionistic fuzzy strongly  $g^*$ -closed sets [3] and intuitionistic fuzzy  $sgp$ -closed sets [2] have been appeared in the literature.

In the present paper we extend the concepts of fuzzy  $gsg$ -closed sets due to Kalaiselvi S. and Seenivasan V. [9] in intuitionistic fuzzy topological spaces. The class of intuitionistic fuzzy  $gsg$ -closed sets is properly placed between the class of intuitionistic fuzzy closed sets and intuitionistic fuzzy  $g$ -closed sets. We also introduced the concepts of intuitionistic fuzzy  $gsg$ -open sets, intuitionistic fuzzy  $gsg$   $T_{1/2}$ -space and obtain some of their characterization and properties.

### II. PRELIMINARIES

Let  $X$  be a nonempty fixed set. An intuitionistic fuzzy set  $A$  [1] in  $X$  is an object having the form  $A = \{ \langle x, \mu_A(x), \gamma_A(x) \rangle : x \in X \}$ , where the functions  $\mu_A : X \rightarrow [0,1]$  and  $\gamma_A : X \rightarrow [0,1]$  denotes the degree of membership  $\mu_A(x)$  and the degree of non membership  $\gamma_A(x)$  of each element  $x \in X$  to the set  $A$  respectively and  $0 \leq \mu_A(x) + \gamma_A(x) \leq 1$  for each  $x \in X$ . The intuitionistic fuzzy sets  $\mathbf{0} = \{ \langle x, 0, 1 \rangle : x \in X \}$  and  $\mathbf{1} = \{ \langle x, 1, 0 \rangle : x \in X \}$

$\in X$  } are respectively called empty and whole intuitionistic fuzzy set on  $X$ . An intuitionistic fuzzy set  $A = \{ \langle x, \mu_A(x), \gamma_A(x) \rangle : x \in X \}$  is called a subset of an intuitionistic fuzzy set  $B = \{ \langle x, \mu_B(x), \gamma_B(x) \rangle : x \in X \}$  (for short  $A \subseteq B$ ) if  $\mu_A(x) \leq \mu_B(x)$  and  $\gamma_A(x) \geq \gamma_B(x)$  for each  $x \in X$ . The complement of an intuitionistic fuzzy set  $A = \{ \langle x, \mu_A(x), \gamma_A(x) \rangle : x \in X \}$  is the intuitionistic fuzzy set  $A^c = \{ \langle x, \gamma_A(x), \mu_A(x) \rangle : x \in X \}$ . The intersection (resp. union) of any arbitrary family of intuitionistic fuzzy sets  $A_i = \{ \langle x, \mu_{A_i}(x), \gamma_{A_i}(x) \rangle : x \in X, (i \in \Lambda) \}$  of  $X$  be the intuitionistic fuzzy set  $\bigcap A_i = \{ \langle x, \wedge \mu_{A_i}(x), \vee \gamma_{A_i}(x) \rangle : x \in X \}$  (resp.  $\bigcup A_i = \{ \langle x, \vee \mu_{A_i}(x), \wedge \gamma_{A_i}(x) \rangle : x \in X \}$ ). Two intuitionistic fuzzy sets  $A = \{ \langle x, \mu_A(x), \gamma_A(x) \rangle : x \in X \}$  and  $B = \{ \langle x, \mu_B(x), \gamma_B(x) \rangle : x \in X \}$  are said to be  $q$ -coincident ( $A_q B$  for short) if and only if  $\exists$  an element  $x \in X$  such that  $\mu_A(x) > \gamma_B(x)$  or  $\gamma_A(x) < \mu_B(x)$ . A family  $\mathfrak{T}$  of intuitionistic fuzzy sets on a non empty set  $X$  is called an intuitionistic fuzzy topology [3] on  $X$  if the intuitionistic fuzzy sets,  $\theta, I \in \mathfrak{T}$ , and  $\mathfrak{T}$  is closed under arbitrary union and finite intersection. The ordered pair  $(X, \mathfrak{T})$  is called an intuitionistic fuzzy topological space and each intuitionistic fuzzy set in  $\mathfrak{T}$  is called an intuitionistic fuzzy open set. The compliment of an intuitionistic fuzzy open set in  $X$  is known as intuitionistic fuzzy closed set. The intersection of all intuitionistic fuzzy closed sets which contains  $A$  is called the closure of  $A$ . It denoted  $cl(A)$ . The union of all intuitionistic fuzzy open subsets of  $A$  is called the interior of  $A$ . It is denoted  $int(A)$  [5]

**Lemma 2.1** [5]: Let  $A$  and  $B$  be any two intuitionistic fuzzy sets of an intuitionistic fuzzy topological space  $(X, \mathfrak{T})$ . Then:

- (a)  $(A_q B) \Leftrightarrow A \subseteq B^c$ .
- (b)  $A$  is an intuitionistic fuzzy closed set in  $X \Leftrightarrow cl(A) = A$
- (c)  $A$  is an intuitionistic fuzzy open set in  $X \Leftrightarrow int(A) = A$ .
- (d)  $cl(A^c) = (int(A))^c$ .
- (e)  $int(A^c) = (cl(A))^c$ .

**Definition 2.1**[5]: An intuitionistic fuzzy set  $A$  of an intuitionistic fuzzy topological space  $(X, \mathfrak{T})$  is called:

- (a) An intuitionistic fuzzy semi open of  $X$  if there is an intuitionistic fuzzy set  $O$  such that  $O \subseteq A \subseteq cl(O)$ .
- (b) An intuitionistic fuzzy semi closed if the compliment of  $A$  is an intuitionistic fuzzy semi open set.
- (c) An intuitionistic fuzzy regular open of  $X$  if  $int(cl(A)) = A$ .
- (d) An intuitionistic fuzzy regular closed of  $X$  if  $cl(int(A)) = A$ .
- (e) An intuitionistic fuzzy pre open if  $A \subseteq int(cl(A))$ .
- (f) An intuitionistic fuzzy pre closed if  $cl(int(A)) \subseteq A$
- (g) An intuitionistic fuzzy  $\alpha$ -open  $A \subseteq int(cl(int(A)))$
- (h) An intuitionistic fuzzy  $\alpha$ -closed if  $cl(int(cl(A))) \subseteq A$

**Definition 2.2**[5] If  $A$  is an intuitionistic fuzzy set in intuitionistic fuzzy topological space  $(X, \mathfrak{T})$  then

- (a)  $scl(A) = \bigcap \{ F : A \subseteq F, F \text{ is intuitionistic fuzzy semi closed} \}$
- (b)  $pcl(A) = \bigcap \{ F : A \subseteq F, F \text{ is intuitionistic fuzzy pre closed} \}$
- (c)  $\alpha cl(A) = \bigcap \{ F : A \subseteq F, F \text{ is intuitionistic fuzzy } \alpha \text{ closed} \}$
- (d)  $spcl(A) = \bigcap \{ F : A \subseteq F, F \text{ is intuitionistic fuzzy semi pre-closed} \}$

**Definition 2.3:** An intuitionistic fuzzy set  $A$  of an intuitionistic fuzzy topological space  $(X, \mathfrak{T})$  is called:

- (a) Intuitionistic fuzzy  $g$ -closed if  $cl(A) \subseteq O$  whenever  $A \subseteq O$  and  $O$  is intuitionistic fuzzy open.[15]
- (b) Intuitionistic fuzzy  $rg$ -closed if  $cl(A) \subseteq O$  whenever  $A \subseteq O$  and  $O$  is intuitionistic fuzzy regular open.[16]
- (c) Intuitionistic fuzzy  $sg$ -closed if  $scl(A) \subseteq O$  whenever  $A \subseteq O$  and  $O$  is intuitionistic fuzzy semi open.[17]
- (d) Intuitionistic fuzzy  $\alpha g$ -closed if  $\alpha cl(A) \subseteq O$  whenever  $A \subseteq O$  and  $O$  is intuitionistic fuzzy - open.[14]
- (e) Intuitionistic fuzzy  $g\alpha$ -closed if  $\alpha cl(A) \subseteq O$  whenever  $A \subseteq O$  and  $O$  is intuitionistic fuzzy  $\alpha$ - open.[8]
- (f) Intuitionistic fuzzy  $w$ -closed if  $cl(A) \subseteq O$  whenever  $A \subseteq O$  and  $O$  is intuitionistic fuzzy semi open.[18]
- (g) Intuitionistic fuzzy  $rw$ -closed if  $cl(A) \subseteq O$  whenever  $A \subseteq O$  and  $O$  is intuitionistic fuzzy regular semi open.[19]
- (h) Intuitionistic fuzzy  $gpr$ -closed if  $pcl(A) \subseteq O$  whenever  $A \subseteq O$  and  $O$  is intuitionistic fuzzy regular open.[20]
- (i) Intuitionistic fuzzy  $rg\alpha$ -closed if  $\alpha cl(A) \subseteq O$  whenever  $A \subseteq O$  and  $O$  is intuitionistic fuzzy regular  $\alpha$ - open.[21]
- (j) Intuitionistic fuzzy  $gsp$ -closed if  $spcl(A) \subseteq O$  whenever  $A \subseteq O$  and  $O$  is intuitionistic fuzzy - open.[12]

- (k) Intuitionistic fuzzy gp-closed if  $\text{pcl}(A) \subseteq O$  whenever  $A \subseteq O$  and  $O$  is intuitionistic fuzzy open.[10]
- (l) Intuitionistic fuzzy gs-closed if  $\text{scl}(A) \subseteq O$  whenever  $A \subseteq O$  and  $O$  is intuitionistic fuzzy open.[13]
- (m) Intuitionistic fuzzy sgp closed set if  $\text{pcl} \subseteq O$  whenever  $A \subseteq O$  and  $O$  is intuitionistic fuzzy semi-open in  $X$ . [2].
- (n) Intuitionistic fuzzy ags-closed set if  $\alpha\text{cl}(A) \subseteq O$  whenever  $A \subseteq O$  and  $O$  is intuitionistic fuzzy semi-open.[7]
- (o) Intuitionistic fuzzy gspr-closed set if  $\text{spcl}(A) \subseteq O$  whenever  $A \subseteq O$  and  $O$  is intuitionistic fuzzy – regular open.[11]

The complements of the above mentioned closed set are their respective open sets.

**Definition 2.4** [6]: Let  $X$  is a nonempty set and  $c \in X$  a fixed element in  $X$ . If  $\alpha \in (0, 1]$  and  $\beta \in [0, 1)$  are two real numbers such that  $\alpha + \beta \leq 1$  then:

- (a)  $c(\alpha, \beta) = \langle x, c_\alpha, c_{1-\beta} \rangle$  is called an intuitionistic fuzzy point in  $X$ , where  $\alpha$  denotes the degree of membership of  $c(\alpha, \beta)$ , and  $\beta$  denotes the degree of non membership of  $c(\alpha, \beta)$ .
- (b)  $c(\beta) = \langle x, 0, 1 - c_{1-\beta} \rangle$  is called a vanishing intuitionistic fuzzy point in  $X$ , where  $\beta$  denotes the degree of non membership of  $c(\beta)$ .

**Definition 2.5:** [5] Let  $X$  and  $Y$  are two nonempty sets and  $f: X \rightarrow Y$  is a function. :

- (a) If  $B = \{ \langle y, \mu_B(y), \gamma_B(y) \rangle : y \in Y \}$  is an intuitionistic fuzzy set in  $Y$ , then the pre image of  $B$  under  $f$  denoted by  $f^{-1}(B)$ , is the intuitionistic fuzzy set in  $X$  defined by
 
$$f^{-1}(B) = \{ \langle x, f^{-1}(\mu_B)(x), f^{-1}(\gamma_B)(x) \rangle : x \in X \}.$$
- (b) If  $A = \{ \langle x, \lambda_A(x), \nu_A(x) \rangle : x \in X \}$  is an intuitionistic fuzzy set in  $X$ , then the image of  $A$  under  $f$  denoted by  $f(A)$  is the intuitionistic fuzzy set in  $Y$  defined by
 
$$f(A) = \{ \langle y, f(\lambda_A)(y), f(\nu_A)(y) \rangle : y \in Y \}$$
 Where  $f(\nu_A) = 1 - f(1 - \nu_A)$ .

**Definition 2.6**[5]: Let  $(X, \mathfrak{S})$  and  $(Y, \sigma)$  be two intuitionistic fuzzy topological spaces and let  $f: X \rightarrow Y$  be a function. Then  $f$  is said to be

- (a) Intuitionistic fuzzy continuous if the pre image of each intuitionistic fuzzy open set of  $Y$  is an intuitionistic fuzzy open set in  $X$ .
- (b) Intuitionistic fuzzy closed if the image of each intuitionistic fuzzy closed set in  $X$  is an intuitionistic fuzzy closed set in  $Y$ .
- (c) Intuitionistic fuzzy open if the image of each intuitionistic fuzzy open set in  $X$  is an intuitionistic fuzzy open set in  $Y$

**Definition 2.7:** Let  $(X, \mathfrak{S})$  and  $(Y, \sigma)$  be two intuitionistic fuzzy topological spaces and let  $f: X \rightarrow Y$  be a function. Then  $f$  is said to be:

- (a) Intuitionistic fuzzy g-continuous if the pre image of every intuitionistic fuzzy closed set in  $Y$  is intuitionistic fuzzy g –closed in  $X$ . [15]
- (b) Intuitionistic fuzzy w-continuous if the pre image of every intuitionistic fuzzy closed set in  $Y$  is intuitionistic fuzzy w –closed in  $X$ . [18]
- (c) Intuitionistic fuzzy rw-continuous if the pre image of every intuitionistic fuzzy closed set in  $Y$  is intuitionistic fuzzy rw –closed in  $X$ . [19].
- (d) Intuitionistic fuzzy sg-continuous if the pre image of every intuitionistic fuzzy closed set in  $Y$  is intuitionistic fuzzy sg –closed in  $X$ . [ 23]
- (e) Intuitionistic fuzzy rg-continuous if the pre image of every intuitionistic fuzzy closed set in  $Y$  is intuitionistic fuzzy rg –closed in  $X$ . [22]
- (f) Intuitionistic fuzzy gpr-continuous if the pre image of every intuitionistic fuzzy closed set in  $Y$  is intuitionistic fuzzy gpr –closed in  $X$ . [20]
- (g) Intuitionistic fuzzy gs-continuous if the pre image of every intuitionistic fuzzy closed set in  $Y$  is intuitionistic fuzzy gs –closed in  $X$ . [13]

**Remark 2.1**

- (a) Every intuitionistic fuzzy continuous mapping is intuitionistic fuzzy g-continuous, but the converse may not be true [15].
- (b) Every intuitionistic fuzzy w- continuous mapping is intuitionistic fuzzy g-continuous, but the converse may not be true [18].
- (c) Every intuitionistic fuzzy w- continuous mapping is intuitionistic fuzzy rw-continuous, but the converse may not be true [19].

- (d) Every intuitionistic fuzzy w- continuous mapping is intuitionistic fuzzy sg-continuous, but the converse may not be true [19].
- (e) Every intuitionistic fuzzy g-continuous mapping is intuitionistic fuzzy rg-continuous, but the converse may not be true [22].
- (f) Every intuitionistic fuzzy g- continuous mapping is intuitionistic fuzzy gpr-continuous, but the converse may not be true [20].

**Definition 2.7[5]:** Let  $(X, \mathfrak{S})$  and  $(Y, \sigma)$  be two intuitionistic fuzzy topological spaces and let  $f: X \rightarrow Y$  be a function. Then  $f$  is said to be Intuitionistic fuzzy irresolute if the pre image of each intuitionistic fuzzy semi open set in  $Y$  is an intuitionistic fuzzy semi open set in  $X$ .

**Definition 2.8:** An intuitionistic fuzzy topological space  $(X, \mathfrak{S})$  is said to be :

- (a) Intuitionistic fuzzy  $T_{1/2}$  space if intuitionistic fuzzy g-closed set is closed in  $(X, \mathfrak{S})$ . [15]
- (b) Intuitionistic fuzzy w- $T_{1/2}$  space if intuitionistic fuzzy w-closed set is closed in  $(X, \mathfrak{S})$ . [18]
- (c) Intuitionistic fuzzy rw- $T_{1/2}$  space if intuitionistic fuzzy rw-closed set is closed in  $(X, \mathfrak{S})$ . [19]
- (d) Intuitionistic fuzzy regular  $T_{1/2}$  space if intuitionistic fuzzy gg-closed set is regular closed in  $(X, \mathfrak{S})$ . [16]
- (e) Intuitionistic fuzzy semi pre regular  $T^*_{1/2}$  space if intuitionistic fuzzy gspr-closed set is closed in  $(X, \mathfrak{S})$ . [11]

### III. INTUITIONISTIC FUZZY GENERALIZED SEMI GENERALIZED CLOSED SET

**Definition 3.1:** An intuitionistic fuzzy set  $A$  of an intuitionistic fuzzy topological space  $(X, \mathfrak{S})$  is called an intuitionistic fuzzy generalized semi generalized closed set (briefly intuitionistic fuzzy gsg-closed) if  $cl(A) \subseteq O$  whenever  $A \subseteq O$  and  $O$  is intuitionistic fuzzy sg open in  $X$ .

First we prove that the class of intuitionistic fuzzy gsg- closed sets properly lies between the class of intuitionistic fuzzy closed sets and the class of intuitionistic fuzzy g-closed sets.

**Theorem 3.1:** Every intuitionistic fuzzy closed set is intuitionistic fuzzy gsg-closed.

**Proof:** Let  $A$  is intuitionistic fuzzy closed set. Let  $A \subseteq U$  and  $U$  is intuitionistic fuzzy sg-open sets in  $X$ . Since  $A$  is intuitionistic fuzzy closed set we have  $A = cl(A)$ . Hence  $cl(A) \subseteq U$ , whenever  $A \subseteq U$  and  $U$  is intuitionistic fuzzy sg-open in  $X$ . Therefore  $A$  is intuitionistic fuzzy gsg-closed set.

**Remark 3.1:** The converse of above theorem need not be true as from the following example.

**Example 3.1:** Let  $X = \{a, b, c\}$  and intuitionistic fuzzy sets  $O$  and  $U$  are defined as follows:

$$O = \{ \langle a, 0.9, 0.1 \rangle, \langle b, 0, 1 \rangle, \langle c, 0, 1 \rangle \}$$

$$U = \{ \langle a, 0, 1 \rangle, \langle b, 0.8, 0.1 \rangle, \langle c, 0.6, 0.3 \rangle \}$$

$\mathfrak{S} = \{ \emptyset, O, U, I \}$  be an intuitionistic fuzzy topology on  $X$ . Then the intuitionistic fuzzy set  $A = \{ \langle a, 0.7, 0.2 \rangle, \langle b, 0.6, 0.3 \rangle, \langle c, 0, 1 \rangle \}$  is intuitionistic fuzzy gsg -closed but it is not intuitionistic fuzzy closed .

**Theorem 3.2:** Every intuitionistic fuzzy regular -closed set is intuitionistic fuzzy gsg-closed.

**Proof:** It follows from the fact that every intuitionistic fuzzy regular closed set is intuitionistic fuzzy closed set and Theorem 3.1.

**Remark 3.2:** The converse of above theorem need not be true as from the following example.

**Example 3.2:** Let  $X = \{a, b, c\}$  and intuitionistic fuzzy sets  $O$  and  $U$  are defined as follows:

$$O = \{ \langle a, 0.2, 0.8 \rangle, \langle b, 0.1, 0.9 \rangle, \langle c, 0, 1 \rangle \}$$

$$U = \{ \langle a, 0.5, 0.4 \rangle, \langle b, 0, 1 \rangle, \langle c, 0.4, 0.3 \rangle \}$$

Let  $\mathfrak{S} = \{ \emptyset, O, U, I \}$  be an intuitionistic fuzzy topology on  $X$ .

Then the intuitionistic fuzzy set  $A = \{ \langle a, 0.9, 0.1 \rangle, \langle b, 0.6, 0.3 \rangle, \langle c, 0, 1 \rangle \}$  is intuitionistic fuzzy gsg -closed but it is not intuitionistic fuzzy regular-closed .

**Theorem 3.3:** Every intuitionistic fuzzy gsg-closed set is g-closed.

**Proof :** Let  $A$  be any intuitionistic fuzzy gsg-closed set and  $U$  be any intuitionistic fuzzy open set such that  $A \subseteq U$  . Since any intuitionistic fuzzy open set is intuitionistic fuzzy sg-open, we have  $cl(A) \subseteq U$  Hence  $A$  is intuitionistic fuzzy g-closed .

**Remark 3.3:** The converse of above theorem need not be true as from the following example.

**Example 3.3:** Let  $X = \{a, b, c, d\}$  and intuitionistic fuzzy sets  $O, U, V, W$  defined as follows:

$$\begin{aligned} O &= \{ \langle a, 0.9, 0.1 \rangle, \langle b, 0, 1 \rangle, \langle c, 0, 1 \rangle, \langle d, 0, 1 \rangle \} \\ U &= \{ \langle a, 0, 1 \rangle, \langle b, 0.8, 0.1 \rangle, \langle c, 0, 1 \rangle, \langle d, 0, 1 \rangle \} \\ V &= \{ \langle a, 0.9, 0.1 \rangle, \langle b, 0.8, 0.1 \rangle, \langle c, 0, 1 \rangle, \langle d, 0, 1 \rangle \} \\ W &= \{ \langle a, 0.9, 0.1 \rangle, \langle b, 0.8, 0.1 \rangle, \langle c, 0.7, 0.2 \rangle, \langle d, 0, 1 \rangle \} \end{aligned}$$

$\mathfrak{I} = \{\emptyset, O, U, V, W, I\}$  be an intuitionistic fuzzy topology on  $X$ . Then the intuitionistic fuzzy set  $A = \{\langle a, 0, 1 \rangle, \langle b, 0, 1 \rangle, \langle c, 0.7, 0.2 \rangle, \langle d, 0, 1 \rangle\}$  is intuitionistic fuzzy  $g$ -closed but it is not intuitionistic fuzzy  $gsg$ -closed.

**Theorem 3.4:** Every intuitionistic fuzzy  $gsg$ -closed set is intuitionistic fuzzy  $w$ -closed.

**Proof :** Let  $A$  be any intuitionistic fuzzy  $gsg$ -closed set . Let  $A \subseteq U$  and  $U$  is intuitionistic fuzzy semi open sets in  $X$ . Since every intuitionistic fuzzy semi open set is intuitionistic fuzzy  $sg$ -open, we have  $cl(A) \subseteq U$  because  $A$  is intuitionistic fuzzy  $gsg$ -closed set Hence  $A$  is intuitionistic fuzzy  $w$ -closed.

**Remark 3.4:** The converse of above theorem need not be true as from the following example.

**Example 3.4:** Let  $X = \{a, b\}$  and  $\mathfrak{I} = \{\emptyset, U, I\}$  be an intuitionistic fuzzy topology on  $X$ , where  $U = \{\langle a, 0.7, 0.2 \rangle, \langle b, 0.6, 0.3 \rangle\}$ .

Then the intuitionistic fuzzy set  $A = \{\langle a, 0.7, 0.2 \rangle, \langle b, 0.8, 0.1 \rangle\}$  is intuitionistic fuzzy  $w$ -closed but it is not intuitionistic fuzzy  $gsg$ -closed.

**Theorem 3.5:** Every intuitionistic fuzzy  $gsg$ -closed set is intuitionistic fuzzy  $rw$ -closed.

**Proof :** Let  $A$  be any intuitionistic fuzzy  $gsg$ -closed set . Let  $A \subseteq U$  and  $U$  is intuitionistic fuzzy regular semi open sets in  $X$ . Since every intuitionistic fuzzy regular semi open set is intuitionistic fuzzy semi open and therefore intuitionistic fuzzy  $sg$ -open, we have  $cl(A) \subseteq U$  because  $A$  is intuitionistic fuzzy  $gsg$ -closed set Hence  $A$  is intuitionistic fuzzy  $rw$ -closed.

**Remark 3.5:** The converse of above theorem need not be true as from the following example.

**Example 3.5:** Let  $X = \{a, b, c, d\}$  and intuitionistic fuzzy sets  $O, U, V$  defined as follows:

$$\begin{aligned} O &= \{ \langle a, 0.9, 0.1 \rangle, \langle b, 0, 1 \rangle, \langle c, 0, 1 \rangle, \langle d, 0, 1 \rangle \} \\ U &= \{ \langle a, 0, 1 \rangle, \langle b, 0.8, 0.1 \rangle, \langle c, 0, 1 \rangle, \langle d, 0, 1 \rangle \} \\ V &= \{ \langle a, 0.9, 0.1 \rangle, \langle b, 0.8, 0.1 \rangle, \langle c, 0, 1 \rangle, \langle d, 0, 1 \rangle \} \end{aligned}$$

$\mathfrak{I} = \{\emptyset, O, U, V, I\}$  be an intuitionistic fuzzy topology on  $X$ . Then the intuitionistic fuzzy set  $A = \{\langle a, 0, 1 \rangle, \langle b, 0, 1 \rangle, \langle c, 0.6, 0.2 \rangle, \langle d, 0, 1 \rangle\}$  is intuitionistic fuzzy  $rw$ -closed but it is not intuitionistic fuzzy  $gsg$ -closed.

**Theorem 3.6:** Every intuitionistic fuzzy  $gsg$ -closed set is intuitionistic fuzzy  $sg$ -closed.

**Proof:** Let  $A$  be any intuitionistic fuzzy  $gsg$ -closed set . Let  $A \subseteq U$  and  $U$  is intuitionistic fuzzy semi open sets in  $X$ . Since any intuitionistic fuzzy semi-open set is intuitionistic fuzzy  $sg$ -open, we have  $scl(A) \subseteq cl(A) \subseteq U$ . Hence  $A$  is intuitionistic fuzzy  $sg$ -closed.

**Remark 3.6:** The converse of above theorem need not be true as from the following example

**Example 3.6:** Let  $X = \{a, b\}$  and  $\mathfrak{I} = \{\emptyset, U, I\}$  be an intuitionistic fuzzy topology on  $X$ , where  $U = \{\langle a, 0.5, 0.5 \rangle, \langle b, 0.4, 0.6 \rangle\}$ . Then the intuitionistic fuzzy set  $A = \{\langle a, 0.5, 0.5 \rangle, \langle b, 0.3, 0.7 \rangle\}$  is intuitionistic fuzzy  $sg$ -closed but it is not intuitionistic fuzzy  $gsg$ -closed.

**Theorem 3.7:** Every intuitionistic fuzzy  $gsg$ -closed set is intuitionistic fuzzy  $gs$ -closed,.

**Proof :** Let  $A$  be any intuitionistic fuzzy  $gsg$ -closed and Let  $A \subseteq U$  and  $U$  is intuitionistic fuzzy open sets in  $X$ . Since any intuitionistic fuzzy open set is intuitionistic fuzzy  $sg$ -open, we have  $scl(A) \subseteq cl(A) \subseteq U$ . Hence  $A$  is intuitionistic fuzzy  $gs$ -closed.

**Remark 3.7:** The converse of above theorem need not be true as from the following example.

**Example 3.7:** Let  $X = \{a, b\}$  and  $\mathfrak{I} = \{\emptyset, U, I\}$  be an intuitionistic fuzzy topology on  $X$ , where  $U = \{\langle a, 0.5, 0.3 \rangle, \langle b, 0.6, 0.4 \rangle\}$ . Then the intuitionistic fuzzy set  $A = \{\langle a, 0.6, 0.4 \rangle, \langle b, 0.7, 0.3 \rangle\}$  is intuitionistic fuzzy  $gs$ -closed but it is not intuitionistic fuzzy  $gsg$ -closed.

**Theorem 3.8:** Every intuitionistic fuzzy  $gsg$ -closed set is intuitionistic fuzzy  $gsp$ -closed,.



**Proof :** Let  $A$  be any intuitionistic fuzzy gsg-closed and Let  $A \subseteq U$  and  $U$  is intuitionistic fuzzy open sets in  $X$ . Since any intuitionistic fuzzy open set is intuitionistic fuzzy sg-open, we have  $\text{spcl}(A) \subseteq \text{cl}(A) \subseteq U$ . Hence  $A$  is intuitionistic fuzzy gsp-closed.

**Remark 3.8:** The converse of above theorem need not be true as from the following example.

**Example 3.8:** Let  $X = \{a, b\}$  and  $\mathfrak{T} = \{\mathbf{0}, U, \mathbf{1}\}$  be an intuitionistic fuzzy topology on  $X$ , where  $U = \{ \langle a, 0.5, 0.3 \rangle, \langle b, 0.2, 0.3 \rangle \}$ . Then the intuitionistic fuzzy set  $A = \{ \langle a, 0.2, 0.4 \rangle, \langle b, 0.6, 0.1 \rangle \}$  is intuitionistic fuzzy gsp-closed but it is not intuitionistic fuzzy gsg-closed.

**Theorem 3.9:** Every intuitionistic fuzzy gsg-closed set is intuitionistic fuzzy gp-closed.

**Proof :** Let  $A$  be any intuitionistic fuzzy gsp-closed and Let  $A \subseteq U$  and  $U$  is intuitionistic fuzzy open sets in  $X$ . Since any intuitionistic fuzzy open set is intuitionistic fuzzy sg-open, we have  $\text{pcl}(A) \subseteq \text{cl}(A) \subseteq U$ . Hence  $A$  is intuitionistic fuzzy gp-closed.

**Remark 3.9:** The converse of above theorem need not be true as from the following example.

**Example 3.9:** Let  $X = \{a, b, c\}$  and intuitionistic fuzzy sets  $O$  is defined as follows:

$$O = \{ \langle a, 0.6, 0.2 \rangle, \langle b, 0, 1 \rangle, \langle c, 0, 1 \rangle \}$$

$\mathfrak{T} = \{ \{\mathbf{0}, O, \mathbf{1}\} \}$  be an intuitionistic fuzzy topology on  $X$ . Then the intuitionistic fuzzy set  $A = \{ \langle a, 0.7, 0.2 \rangle, \langle b, 0.7, 0.3 \rangle, \langle c, 0, 1 \rangle \}$  is intuitionistic fuzzy gp-closed but it is not intuitionistic fuzzy gsg-closed.

**Theorem 3.10:** Every intuitionistic fuzzy gsg-closed set is intuitionistic fuzzy gpr-closed.

**Proof :** Let  $A$  be any intuitionistic fuzzy gsg-closed and Let  $A \subseteq U$  and  $U$  is intuitionistic fuzzy regular open sets in  $X$ . Since every fuzzy regular open set is intuitionistic fuzzy open set and therefore intuitionistic fuzzy sg-open, we have  $\text{pcl}(A) \subseteq \text{cl}(A) \subseteq U$ . Hence  $A$  is intuitionistic fuzzy gpr-closed.

**Remark 3.10:** The converse of above theorem need not be true as from the following example

**Example 3.10:** Let  $X = \{a, b, c, d, e\}$  and intuitionistic fuzzy sets  $O, U, V$  defined as follows:

$$O = \{ \langle a, 0.9, 0.1 \rangle, \langle b, 0.8, 0.1 \rangle, \langle c, 0, 1 \rangle, \langle d, 0, 1 \rangle, \langle e, 0, 1 \rangle \}$$

$$U = \{ \langle a, 0, 1 \rangle, \langle b, 0, 1 \rangle, \langle c, 0.8, 0.1 \rangle, \langle d, 0.7, 0.2 \rangle, \langle e, 0, 1 \rangle \}$$

$$V = \{ \langle a, 0.9, 0.1 \rangle, \langle b, 0.8, 0.1 \rangle, \langle c, 0.8, 0.1 \rangle, \langle d, 0.7, 0.2 \rangle, \langle e, 0, 1 \rangle \}$$

Let  $\mathfrak{T} = \{ \mathbf{0}, O, U, V, \mathbf{1} \}$  be an intuitionistic fuzzy topology on  $X$ . Then the intuitionistic fuzzy set  $A = \{ \langle a, 0.9, 0.1 \rangle, \langle b, 0, 1 \rangle, \langle c, 0, 1 \rangle, \langle d, 0, 1 \rangle, \langle e, 0, 1 \rangle \}$  is intuitionistic fuzzy gpr-closed but it is not intuitionistic fuzzy gsg-closed.

**Theorem 3.11:** Every intuitionistic fuzzy gsg-closed set is intuitionistic fuzzy rg-closed.

**Proof:** Let  $A$  be any intuitionistic fuzzy gsg-closed and Let  $A \subseteq U$  and  $U$  is intuitionistic fuzzy regular open sets in  $X$ . Since every regular open set is intuitionistic fuzzy open set and therefore intuitionistic fuzzy sg-open, we have  $\text{cl}(A) \subseteq U$ . Hence  $A$  is intuitionistic fuzzy rg-closed.

**Remark 3.11:** The converse of the above theorem need not be true, as seen from the following example

**Example 3.11:** Let  $X = \{a, b, c\}$  and intuitionistic fuzzy sets  $O, U, V$  defined as follows:

$$O = \{ \langle a, 0.9, 0.1 \rangle, \langle b, 0, 1 \rangle, \langle c, 0, 1 \rangle \},$$

$$U = \{ \langle a, 0, 1 \rangle, \langle b, 0.8, 0.1 \rangle, \langle c, 0, 1 \rangle \}$$

$$V = \{ \langle a, 0.9, 0.1 \rangle, \langle b, 0.8, 0.1 \rangle, \langle c, 0, 1 \rangle \}$$

Let  $\mathfrak{T} = \{ \mathbf{0}, O, U, V, \mathbf{1} \}$  be an intuitionistic fuzzy topology on  $X$ . Then intuitionistic fuzzy set  $A = \{ \langle a, 0.9, 0.1 \rangle, \langle b, 0.8, 0.1 \rangle, \langle c, 0, 1 \rangle \}$  is intuitionistic fuzzy rg-closed set in  $X$ , but it is not intuitionistic fuzzy gsg-closed.

**Theorem 3.12:** Every intuitionistic fuzzy gsg-closed set is intuitionistic fuzzy  $\alpha$ -closed.

**Proof:** Let  $A$  be any intuitionistic fuzzy gsg-closed and Let  $A \subseteq U$  and  $U$  is intuitionistic fuzzy  $\alpha$ -open sets in  $X$ . Since every intuitionistic fuzzy  $\alpha$ -open is intuitionistic fuzzy open set which is intuitionistic fuzzy sg-open, we have  $\alpha\text{cl}(A) \subseteq \text{cl}(A) \subseteq U$ . Hence  $A$  is intuitionistic fuzzy  $\alpha$ -closed.

**Remark 3.12:** The converse of above theorem need not be true as from the following example.

**Example 3.12:** Let  $X = \{a, b, c\}$  and intuitionistic fuzzy sets  $O$  is defined as follows:

$$O = \{ \langle a, 0.2, 0.8 \rangle, \langle b, 0.1, 0.9 \rangle, \langle c, 0, 1 \rangle \}$$

$\mathfrak{T} = \{ \mathbf{0}, O, \mathbf{1} \}$  be an intuitionistic fuzzy topology on  $X$ . Then the intuitionistic fuzzy set  $A = \{ \langle a, 0.8, 0.1 \rangle, \langle b, 0.5, 0.3 \rangle, \langle c, 0, 1 \rangle \}$  is intuitionistic fuzzy  $\alpha$ -closed but it is not intuitionistic fuzzy  $\alpha$ -gsg-closed.

**Theorem 3.13:** Every intuitionistic fuzzy  $\alpha$ -gsg-closed set is intuitionistic fuzzy  $\alpha$ -g-closed.

**Proof :** Let  $A$  be any intuitionistic fuzzy  $\alpha$ -gsg-closed and Let  $A \subseteq U$  and  $U$  is intuitionistic fuzzy open sets in  $X$ . Since any intuitionistic fuzzy open set is intuitionistic fuzzy  $\alpha$ -g-open, we have  $\alpha \text{cl}(A) \subseteq \text{cl}(A) \subseteq U$ . Hence  $A$  is intuitionistic fuzzy  $\alpha$ -g-closed.

**Remark 3.13:** The converse of above theorem need not be true as from the following example

**Example 3.13:** Let  $X = \{a, b\}$  and  $\mathfrak{T} = \{ \mathbf{0}, U, \mathbf{1} \}$  be an intuitionistic fuzzy topology on  $X$ , where  $U = \{ \langle a, 0.5, 0.5 \rangle, \langle b, 0.4, 0.6 \rangle \}$ . Then the intuitionistic fuzzy set  $A = \{ \langle a, 0.5, 0.5 \rangle, \langle b, 0.5, 0.5 \rangle \}$  is intuitionistic fuzzy  $\alpha$ -g-closed but it is not intuitionistic fuzzy  $\alpha$ -gsg-closed.

**Theorem 3.14:** Every intuitionistic fuzzy  $\alpha$ -gsg-closed set is intuitionistic fuzzy  $\alpha$ -gs-closed.

**Proof:** Let  $A$  is intuitionistic fuzzy  $\alpha$ -gsg-closed set. Let  $A \subseteq U$  and  $U$  is intuitionistic fuzzy  $\alpha$ -semi open sets in  $X$ . Since every intuitionistic fuzzy semi open is intuitionistic fuzzy  $\alpha$ -g-open,  $U$  is intuitionistic fuzzy  $\alpha$ -g-open, therefore by definition of intuitionistic fuzzy  $\alpha$ -gsg-closed set,  $\text{cl}(A) \subseteq U$ . Note that  $\alpha \text{cl}(A) \subseteq \text{cl}(A)$  is always true. Therefore  $\alpha \text{cl}(A) \subseteq U$ . Hence  $A$  is intuitionistic fuzzy  $\alpha$ -gs-closed set.

**Remark 3.14:** The converse of the above theorem need not be true, as seen from the following example

**Example 3.14:** Let  $X = \{a, b, c, d\}$  and intuitionistic fuzzy sets  $O, U, V$  defined as follows:

$$O = \{ \langle a, 0.9, 0.1 \rangle, \langle b, 0.8, 0.1 \rangle, \langle c, 0, 1 \rangle, \langle d, 0, 1 \rangle \}$$

$$U = \{ \langle a, 0, 1 \rangle, \langle b, 0, 1 \rangle, \langle c, 0.8, 0.1 \rangle, \langle d, 0.7, 0.2 \rangle \}$$

$$V = \{ \langle a, 0.9, 0.1 \rangle, \langle b, 0.8, 0.1 \rangle, \langle c, 0.8, 0.1 \rangle, \langle d, 0.7, 0.2 \rangle \}$$

Let  $\mathfrak{T} = \{ \mathbf{0}, O, U, V, \mathbf{1} \}$  be an intuitionistic fuzzy topology on  $X$ . Then the intuitionistic fuzzy set  $A = \{ \langle a, 0.9, 0.1 \rangle, \langle b, 0, 1 \rangle, \langle c, 0, 1 \rangle, \langle d, 0, 1 \rangle \}$  is intuitionistic fuzzy  $\alpha$ -gs-closed but it is not intuitionistic fuzzy  $\alpha$ -gsg-closed.

**Theorem 3.15:** Every intuitionistic fuzzy  $\alpha$ -gsg-closed set is intuitionistic fuzzy  $\alpha$ -sgp-closed.

**Proof:** Let  $A$  is intuitionistic fuzzy  $\alpha$ -gsg-closed set. Let  $A \subseteq U$  and  $U$  is intuitionistic fuzzy  $\alpha$ -semi open sets in  $X$ . Since every intuitionistic fuzzy semi open is intuitionistic fuzzy  $\alpha$ -g-open,  $U$  is intuitionistic fuzzy  $\alpha$ -g-open, therefore by definition of intuitionistic fuzzy  $\alpha$ -gsg-closed set,  $\text{cl}(A) \subseteq U$ . Note that  $\alpha \text{pcl}(A) \subseteq \text{cl}(A)$  is always true. Therefore  $\alpha \text{pcl}(A) \subseteq U$ . Hence  $A$  is intuitionistic fuzzy  $\alpha$ -sgp-closed set.

**Remark 3.15:** The converse of above theorem need not be true as from the following example.

**Example 3.15:** Let  $X = \{a, b, c, d, e\}$  and intuitionistic fuzzy sets  $O$  and  $U$  defined as follows:

$$O = \{ \langle a, 0.9, 0.1 \rangle, \langle b, 0.8, 0.1 \rangle, \langle c, 0, 1 \rangle, \langle d, 0, 1 \rangle, \langle e, 0, 1 \rangle \}$$

$$U = \{ \langle a, 0, 1 \rangle, \langle b, 0, 1 \rangle, \langle c, 0.8, 0.1 \rangle, \langle d, 0.7, 0.2 \rangle, \langle e, 0, 1 \rangle \}$$

Let  $\mathfrak{T} = \{ \mathbf{0}, O, U, \mathbf{1} \}$  be an intuitionistic fuzzy topology on  $X$ . Then the intuitionistic fuzzy set  $A = \{ \langle a, 0.9, 0.1 \rangle, \langle b, 0, 1 \rangle, \langle c, 0, 1 \rangle, \langle d, 0, 1 \rangle, \langle e, 0, 1 \rangle \}$  is intuitionistic fuzzy  $\alpha$ -sgp-closed but it is not intuitionistic fuzzy  $\alpha$ -gsg-closed.

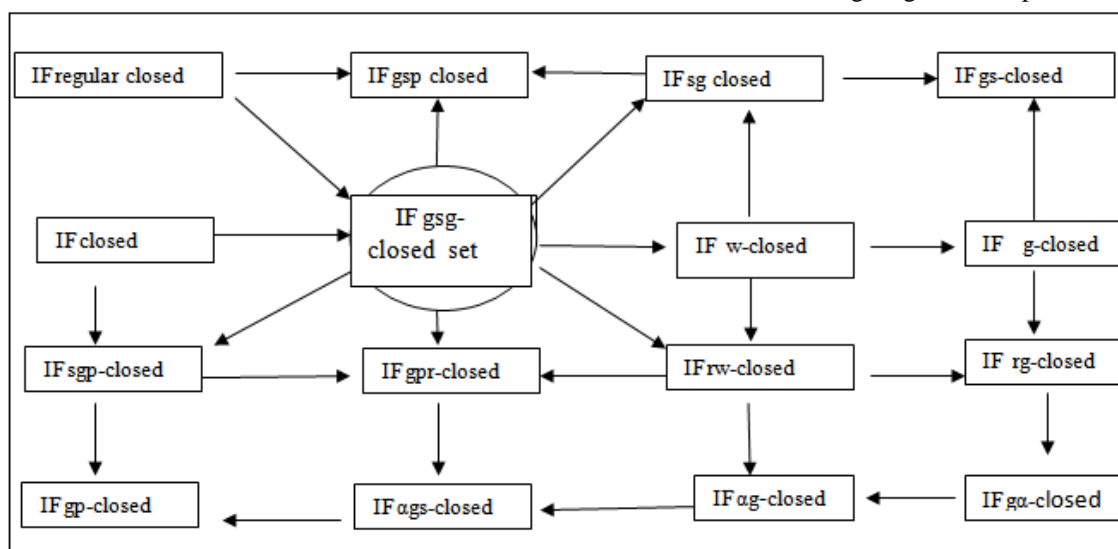
**Theorem 3.16:** Every intuitionistic fuzzy  $\alpha$ -gsg-closed set is intuitionistic fuzzy  $\alpha$ -gspr-closed.

**Proof:** Let  $A$  is intuitionistic fuzzy  $\alpha$ -gsg-closed set. Let  $A \subseteq U$  and  $U$  is intuitionistic fuzzy  $\alpha$ -regular open sets in  $X$ . Since every regular open set is intuitionistic fuzzy open set and therefore intuitionistic fuzzy  $\alpha$ -g-open,  $U$  is intuitionistic fuzzy  $\alpha$ -g-open, therefore by definition of intuitionistic fuzzy  $\alpha$ -gsg-closed set,  $\text{cl}(A) \subseteq U$ . Note that  $\alpha \text{spcl}(A) \subseteq \text{cl}(A)$  is always true. Therefore  $\alpha \text{spcl}(A) \subseteq U$ . Hence  $A$  is intuitionistic fuzzy  $\alpha$ -gspr-closed set.

**Remark 3.16:** The converse of above theorem need not be true as from the following example

**Example 3.16:** Let  $X = \{a, b\}$  and  $\mathfrak{T} = \{ \mathbf{0}, U, \mathbf{1} \}$  be an intuitionistic fuzzy topology on  $X$ , where  $U = \{ \langle a, 0.3, 0.7 \rangle, \langle b, 0.2, 0.8 \rangle \}$ . Then the intuitionistic fuzzy set  $A = \{ \langle a, 0.5, 0.5 \rangle, \langle b, 0.5, 0.4 \rangle \}$  is intuitionistic fuzzy  $\alpha$ -gspr-closed but it is not intuitionistic fuzzy  $\alpha$ -gsg-closed.

**Remark 3.17:** From the above discussion and known results we have the following diagram of implications:



**Fig.1** Relations between intuitionistic fuzzy gsg-closed set and other existing intuitionistic fuzzy closed sets

**Theorem 3.16:** Let  $(X, \mathfrak{F})$  be an intuitionistic fuzzy topological space and  $A$  is an intuitionistic fuzzy set of  $X$ . Then  $A$  is intuitionistic fuzzy gsg-closed if and only if  $\neg(A_q F) \Rightarrow \neg(\text{cl}(A)_q F)$  for every intuitionistic fuzzy sg-closed set  $F$  of  $X$ .

**Proof: Necessity:** Let  $F$  be an intuitionistic fuzzy sg-closed set of  $X$  and  $\neg(A_q F)$ . Then by Lemma 2.1(a),  $A \subseteq F^c$  and  $F^c$  is intuitionistic fuzzy sg-open in  $X$ . Therefore  $\text{cl}(A) \subseteq F^c$  by Def. 3.1 because  $A$  is intuitionistic fuzzy gsg-closed. Hence by lemma 2.1(a),  $\neg(\text{cl}(A)_q F)$ .

**Sufficiency:** Let  $O$  be an intuitionistic fuzzy sg open set of  $X$  such that  $A \subseteq O$  i.e.  $A \subseteq (O^c)^c$ . Then by Lemma 2.1(a),  $\neg(A_q O^c)$  and  $O^c$  is an intuitionistic fuzzy sg closed set in  $X$ . Hence by hypothesis  $\neg(\text{cl}(A)_q O^c)$ . Therefore by Lemma 2.1(a),  $\text{cl}(A) \subseteq ((O^c)^c)^c$  i.e.  $\text{cl}(A) \subseteq O$ . Hence  $A$  is intuitionistic fuzzy gsg-closed in  $X$ .

**Theorem 3.17:** Let  $A$  be an intuitionistic fuzzy gsg-closed set in an intuitionistic fuzzy topological space  $(X, \mathfrak{F})$  and  $c(\alpha, \beta)$  be an intuitionistic fuzzy point of  $X$  such that  $c(\alpha, \beta)_q \text{cl}(\text{int}(A))$  then  $\text{cl}(\text{int}(c(\alpha, \beta)))_q A$ .

**Proof:** If  $\neg(\text{cl}(\text{int}(c(\alpha, \beta)))_q A)$  then by Lemma 2.1(a),  $\text{cl}(\text{int}(c(\alpha, \beta))) \subseteq A^c$  which implies that  $A \subseteq (\text{cl}(\text{int}(c(\alpha, \beta))))^c$  and so  $\text{cl}(A) \subseteq (\text{cl}(\text{int}(c(\alpha, \beta))))^c \subseteq (c(\alpha, \beta))^c$ , because  $A$  is intuitionistic fuzzy gsg-closed in  $X$ . Hence by Lemma 2.1(a),  $\neg(c(\alpha, \beta)_q \text{cl}(\text{int}(A)))$ , a contradiction.

**Theorem 3.18:** Let  $A$  be an intuitionistic fuzzy gsg-closed set in an intuitionistic fuzzy topological space  $(X, \mathfrak{F})$  and  $A \subseteq B \subseteq \text{cl}(A)$ . Then  $B$  is intuitionistic fuzzy ssg-closed in  $X$ .

**Proof:** Let  $O$  be an intuitionistic fuzzy sg-open set in  $X$  such that  $B \subseteq O$ . Then  $A \subseteq O$  and since  $A$  is intuitionistic fuzzy gsg-closed,  $\text{cl}(A) \subseteq O$ . Now  $B \subseteq \text{cl}(A) \Rightarrow \text{cl}(B) \subseteq \text{cl}(A) \subseteq O$ . Consequently  $B$  is intuitionistic fuzzy gsg-closed.

**Theorem 3.19:** Let  $A$  and  $B$  are two intuitionistic fuzzy gsg-closed sets in an intuitionistic fuzzy topological space  $(X, \mathfrak{F})$ , then  $A \cup B$  is intuitionistic fuzzy gsg-closed.

**Proof:** Let  $O$  be an intuitionistic fuzzy sg-open set in  $X$ , such that  $A \cup B \subseteq O$ . Then  $A \subseteq O$  and  $B \subseteq O$ . So,  $\text{cl}(A) \subseteq O$  and  $\text{cl}(B) \subseteq O$ . Therefore  $\text{cl}(A) \cup \text{cl}(B) = \text{cl}(A \cup B) \subseteq O$ . Hence  $A \cup B$  is intuitionistic fuzzy gsg-closed.

**Remark 3.18:** The intersection of two intuitionistic fuzzy gsg-closed sets in an intuitionistic fuzzy topological space  $(X, \mathfrak{F})$  may not be intuitionistic fuzzy gsg-closed. For,

**Example 3.17:** Let  $X = \{a, b, c, d\}$  and intuitionistic fuzzy sets  $O, U, V, W$  defined as follows:

- $O = \{ \langle a, 0.9, 0.1 \rangle, \langle b, 0, 1 \rangle, \langle c, 0.1 \rangle, \langle d, 0.1 \rangle \}$
- $U = \{ \langle a, 0.1 \rangle, \langle b, 0.8, 0.1 \rangle, \langle c, 0.1 \rangle, \langle d, 0.1 \rangle \}$
- $V = \{ \langle a, 0.9, 0.1 \rangle, \langle b, 0.8, 0.1 \rangle, \langle c, 0.1 \rangle, \langle d, 0.1 \rangle \}$
- $W = \{ \langle a, 0.9, 0.1 \rangle, \langle b, 0.8, 0.1 \rangle, \langle c, 0.7, 0.2 \rangle, \langle d, 0.1 \rangle \}$



$\mathfrak{T} = \{0, O, U, V, W, 1\}$  be an intuitionistic fuzzy topology on  $X$ . Then the intuitionistic fuzzy set  $A = \{<a,0.9,0.1>, <b,0.8,0.1>, <c,0,1>, <d,0,1>\}$  and  $B = \{<a, 0.9, 0.1 >, < b, 0, 1>, < c, 0.7, 0.2>, <d,0.9,0.1>\}$  are intuitionistic fuzzy gsg-closed in  $(X, \mathfrak{T})$  but  $A \cap B$  is not intuitionistic fuzzy gsg-closed.

**Theorem 3.20:** If  $A$  is intuitionistic fuzzy semi open and intuitionistic fuzzy gsg-closed set, then  $A$  is intuitionistic fuzzy closed.

**Proof:** Suppose  $A$  is intuitionistic fuzzy semi open and intuitionistic fuzzy gsg-closed set. As every intuitionistic fuzzy semi open set is intuitionistic fuzzy sg-open and  $A \subseteq A$ , we have  $cl(A) \subseteq A$ . because  $A$  is intuitionistic fuzzy gsg-closed set. Also  $A \subseteq cl(A)$ . Therefore  $cl(A) = A$ . That means  $A$  is intuitionistic fuzzy closed.

**Theorem 3.21:** If  $A$  is both intuitionistic fuzzy open and intuitionistic fuzzy g-closed in intuitionistic fuzzy topological space  $(X, \mathfrak{T})$ . Then  $A$  is intuitionistic fuzzy gsg-closed set in  $X$ .

**Proof:** Let  $A$  is both intuitionistic fuzzy open and intuitionistic fuzzy g-closed in  $X$ . Let  $A \subseteq U$ , where  $U$  is intuitionistic fuzzy sg-open in  $X$ . Now  $A \subseteq U$ . By hypothesis  $cl(A) \subseteq A$ . That is  $cl(A) \subseteq A \subseteq U$ . Hence  $cl(A) \subseteq U$ . whenever  $A \subseteq U$  and  $U$  is intuitionistic fuzzy sg-open in  $X$ . Thus  $A$  is intuitionistic fuzzy gsg-closed in  $X$ .

**Definition 3.2:** An intuitionistic fuzzy set  $A$  of an intuitionistic fuzzy topological space  $(X, \mathfrak{T})$  is called intuitionistic fuzzy gsg--open if and only if its complement  $A^c$  is intuitionistic fuzzy gsg-closed.

**Remark 3.19:** Every intuitionistic fuzzy open set is intuitionistic fuzzy gsg-open but its converse may not be true.

**Example 3.18:** Let  $X = \{a, b\}$  and  $\mathfrak{T} = \{0, U, 1\}$  be an intuitionistic fuzzy topology on  $X$ , where  $U = \{<a, 0.7, 0.2 >, <b, 0.6, 0.3 >\}$ .

Then the intuitionistic fuzzy set  $A = \{<a, 0.2, 0.7 >, <b, 0.1, 0.8 >\}$  is intuitionistic fuzzy gsg in  $(X, \mathfrak{T})$  but it is not intuitionistic fuzzy open in  $(X, \mathfrak{T})$ .

**Theorem 3.22:** An intuitionistic fuzzy set  $A$  of an intuitionistic fuzzy topological space  $(X, \mathfrak{T})$  is intuitionistic fuzzy gsg-open if  $F \subseteq cl(A)$  whenever  $F$  is intuitionistic fuzzy sg-closed and  $F \subseteq A$ .

**Proof:** Follows from definition 3.1 and Lemma 2.1.

**Theorem 3.23:** Let  $A$  be an intuitionistic fuzzy gsg-open set of an intuitionistic fuzzy topological space  $(X, \mathfrak{T})$  and  $int(A) \subseteq B \subseteq A$ . Then  $B$  is intuitionistic fuzzy gsg-open.

**Proof:** Suppose  $A$  is an intuitionistic fuzzy gsg-open in  $X$  and  $int(A) \subseteq B \subseteq A \Rightarrow A^c \subseteq B^c \subseteq (int(A))^c \Rightarrow A^c \subseteq B^c \subseteq cl(A^c)$  by Lemma 2.1(d) and  $A^c$  is intuitionistic fuzzy gsg-closed it follows from theorem 3.18 that  $B^c$  is intuitionistic fuzzy gsg-closed. Hence  $B$  is intuitionistic fuzzy gsg-open.

**Theorem 3.24:** Let  $(X, \mathfrak{T})$  be an intuitionistic fuzzy topological space and  $IFSGO(X)$  is the family of all intuitionistic fuzzy sg-open and  $IFC(X)$  be the family of all intuitionistic fuzzy closed sets of  $X$ . Then  $IFRSO(X) \subseteq IFC(X)$  if and only if every intuitionistic fuzzy set of  $X$  is intuitionistic fuzzy gsg-closed.

**Proof : Necessity :** Suppose that  $IFSGO(X) \subseteq IFC(X)$  and let  $A$  is any intuitionistic fuzzy set of  $X$  such that  $A \subseteq U \in IFSGO(X)$  i.e.  $U$  is intuitionistic fuzzy sg-open. Then  $cl(A) \subseteq cl(U) = U$  because  $U \in IFRSO(X) \subseteq IFC(X)$ . Hence  $cl(A) \subseteq U$  whenever  $A \subseteq U$  and  $U$  is intuitionistic fuzzy sg-open. Hence  $A$  is gsg-closed set.

**Sufficiency:** Suppose that every intuitionistic fuzzy set of  $X$  is intuitionistic fuzzy gsg-closed. Let  $U \in IFSGO(X)$  then since  $U \subseteq U$  and  $U$  is intuitionistic fuzzy gsg-closed,  $cl(U) \subseteq U$  then  $U \in IFC(X)$ . Thus  $IFRSO(X) \subseteq IFC(X)$ .

#### IV. INTUITIONISTIC FUZZY gsg $T_{1/2}$ - SPACE

In this section we introduce intuitionistic fuzzy gsg  $T_{1/2}$ -space as an application of intuitionistic fuzzy gsg-closed set.

**Definition 4.1:** An intuitionistic fuzzy topological space  $(X, \mathfrak{T})$  is called a intuitionistic Fuzzy gsg $T_{1/2}$ -Space if every intuitionistic fuzzy gsg-closed set is intuitionistic fuzzy closed.

**Theorem 4.1:** Every intuitionistic fuzzy  $T_{1/2}$ -space is intuitionistic fuzzy gsg $T_{1/2}$  space.

**Proof:** Let  $(X, \mathfrak{T})$  be an intuitionistic fuzzy  $T_{1/2}$  space and let  $A$  be intuitionistic fuzzy gsg-closed set in  $(X, \mathfrak{T})$ . Then  $A$  is intuitionistic fuzzy g-closed, by theorem 3.3, Since intuitionistic fuzzy topological space  $(X, \mathfrak{T})$  is intuitionistic fuzzy  $T_{1/2}$  space,  $A$  is intuitionistic fuzzy closed in  $(X, \mathfrak{T})$ . Hence  $(X, \mathfrak{T})$  is intuitionistic fuzzy gsg $T_{1/2}$  - space.

**Remark 4.1:** The converse of the above theorem need not be true, as seen from the following example.

**Example 4.1:** Let  $X = \{a, b\}$  and Let  $\mathfrak{T} = \{\mathbf{0}, \mathbf{O}, \mathbf{I}\}$  be an intuitionistic fuzzy topology on  $X$ , where  $\mathbf{O} = \{\langle a, 0.5, 0.5 \rangle, \langle b, 0.4, 0.6 \rangle\}$ . Then intuitionistic fuzzy topological space  $(X, \mathfrak{T})$  is intuitionistic fuzzy  $\text{gsgT}_{1/2}$  space but not intuitionistic fuzzy  $\text{T}_{1/2}$ -space.

**Theorem 4.2:** Every intuitionistic fuzzy  $\text{wT}_{1/2}$  space is intuitionistic fuzzy  $\text{gsgT}_{1/2}$  space.

**Proof:** Let  $(X, \mathfrak{T})$  be an intuitionistic fuzzy  $\text{wT}_{1/2}$ -space and let  $A$  be intuitionistic fuzzy  $\text{gsg}$ -closed set in  $(X, \mathfrak{T})$ . Then  $A$  is intuitionistic Fuzzy  $w$ -closed, by theorem 3.4, Since intuitionistic fuzzy topological space  $(X, \mathfrak{T})$  is intuitionistic fuzzy  $\text{wT}_{1/2}$ - space,  $A$  is intuitionistic fuzzy closed in  $(X, \mathfrak{T})$ . Hence  $(X, \mathfrak{T})$  is intuitionistic fuzzy  $\text{gsgT}_{1/2}$ - space.

**Remark 4.2:** The converse of the above theorem need not be true, as seen from the following example.

**Example 4.2:** Let  $X = \{a, b, c\}$  and Let  $\mathfrak{T} = \{\mathbf{0}, \mathbf{A}, \mathbf{B}, \mathbf{I}\}$  be an intuitionistic fuzzy topology on  $X$ , where  $\mathbf{A} = \{\langle a, 0.7, 0.5 \rangle, \langle b, 0.3, 0.6 \rangle, \langle c, 1, 0 \rangle\}$  and  $\mathbf{B} = \{\langle a, 0.7, 0.3 \rangle, \langle b, 0.0, 0.1 \rangle, \langle c, 0, 1 \rangle\}$ . Then intuitionistic fuzzy topological space  $(X, \mathfrak{T})$  is intuitionistic fuzzy  $\text{gsgT}_{1/2}$ - space but not intuitionistic fuzzy  $\text{wT}_{1/2}$ -space.

**Theorem 4.3:** Every intuitionistic fuzzy-  $\text{rwT}_{1/2}$ - space is intuitionistic fuzzy  $\text{gsgT}_{1/2}$  space.

**Proof:** Let  $(X, \mathfrak{T})$  be an intuitionistic fuzzy  $\text{rwT}_{1/2}$  -space and let  $A$  be intuitionistic fuzzy  $\text{gsg}$ -closed set in  $(X, \mathfrak{T})$ . Then  $A$  is intuitionistic Fuzzy  $\text{rw}$ -closed, by theorem 3.5, Since intuitionistic fuzzy topological space  $(X, \mathfrak{T})$  is intuitionistic fuzzy  $\text{rwT}_{1/2}$  -space,  $A$  is intuitionistic fuzzy closed in  $(X, \mathfrak{T})$ . Hence  $(X, \mathfrak{T})$  is intuitionistic fuzzy  $\text{gsgT}_{1/2}$ - space.

**Remark 4.3:** The converse of the above theorem need not be true, as seen from the following example.

**Example 4.3:** Let  $X = \{a, b, c\}$  and Let  $\mathfrak{T} = \{\mathbf{0}, \mathbf{A}, \mathbf{I}\}$  be an intuitionistic fuzzy topology on  $X$ , where  $\mathbf{A} = \{\langle a, 0.6, 0.4 \rangle, \langle b, 0.3, 0.6 \rangle, \langle c, 1, 0 \rangle\}$ . Then intuitionistic fuzzy topological space  $(X, \mathfrak{T})$  is intuitionistic fuzzy  $\text{gsgT}_{1/2}$  -space but not intuitionistic fuzzy  $\text{rwT}_{1/2}$ -space.

**Theorem 4.4:** Every intuitionistic fuzzy regular-  $\text{T}_{1/2}$  space is intuitionistic fuzzy  $\text{gsgT}_{1/2}$  space.

**Proof:** Let  $(X, \mathfrak{T})$  be an intuitionistic fuzzy regular  $\text{T}_{1/2}$  space and let  $A$  be intuitionistic fuzzy  $\text{gsg}$ -closed set in  $(X, \mathfrak{T})$ . Then  $A$  is intuitionistic Fuzzy  $\text{rg}$ -closed, by theorem 3.11. Since intuitionistic fuzzy topological space  $(X, \mathfrak{T})$  is intuitionistic fuzzy regular  $\text{T}_{1/2}$  space,  $A$  is intuitionistic fuzzy regular closed in  $(X, \mathfrak{T})$ . But every intuitionistic fuzzy regular closed is intuitionistic fuzzy closed therefore  $A$  is intuitionistic fuzzy closed in  $(X, \mathfrak{T})$ . Hence  $(X, \mathfrak{T})$  is intuitionistic fuzzy  $\text{gsgT}_{1/2}$  space.

**Remark 4.4:** The converse of the above theorem need not be true, as seen from the following example

**Example 4.4:** Let  $X = \{a, b\}$  and Let  $\mathfrak{T} = \{\mathbf{0}, \mathbf{A}, \mathbf{I}\}$  be an intuitionistic fuzzy topology on  $X$ , where  $\mathbf{A} = \{\langle a, 0.6, 0.4 \rangle, \langle b, 0.3, 0.6 \rangle\}$ . Then intuitionistic fuzzy topological space  $(X, \mathfrak{T})$  is intuitionistic fuzzy  $\text{gsgT}_{1/2}$ - space but not intuitionistic fuzzy regular  $\text{T}_{1/2}$ -space.

**Theorem 4.5:** Every intuitionistic fuzzy semi regular- $\text{T}^*_{1/2}$  space is intuitionistic fuzzy  $\text{gsgT}_{1/2}$ - space.

**Proof:** Let  $(X, \mathfrak{T})$  be an intuitionistic fuzzy semi regular  $\text{T}^*_{1/2}$  space and let  $A$  be intuitionistic fuzzy  $\text{gsg}$ -closed set in  $(X, \mathfrak{T})$ . Then  $A$  is intuitionistic Fuzzy  $\text{gspr}$  closed, by theorem 3.16. Since intuitionistic fuzzy topological space  $(X, \mathfrak{T})$  is intuitionistic fuzzy semi regular  $\text{T}^*_{1/2}$  space,  $A$  is intuitionistic fuzzy closed in  $(X, \mathfrak{T})$ . Hence  $(X, \mathfrak{T})$  is intuitionistic fuzzy  $\text{gsgT}_{1/2}$ - space.

**Remark 4.5:** The converse of the above theorem need not be true, as seen from the following example

**Example 4.5:** Let  $X = \{a, b, c, d\}$  and Let  $\mathfrak{T} = \{\mathbf{0}, \mathbf{A}, \mathbf{B}, \mathbf{I}\}$  be an intuitionistic fuzzy topology on  $X$ , where  $\mathbf{A} = \{\langle a, 0.7, 0.5 \rangle, \langle b, 0.3, 0.6 \rangle, \langle c, 1, 0 \rangle, \langle d, 0, 1 \rangle\}$   
 $\mathbf{B} = \{\langle a, 0.7, 0.3 \rangle, \langle b, 0.0, 0.1 \rangle, \langle c, 0, 1 \rangle, \langle d, 0.5, 0.5 \rangle\}$ .

Then intuitionistic fuzzy topological space  $(X, \mathfrak{T})$  is intuitionistic fuzzy  $\text{gsgT}_{1/2}$  - space but not intuitionistic fuzzy semi regular  $\text{T}^*_{1/2}$  space.

**Theorem 4.6:** Let  $(X, \mathfrak{T})$  be an intuitionistic fuzzy topological space. Let  $\text{IFO}(X)$  be the family of all intuitionistic fuzzy open sets of  $X$  and  $\text{IFGSGO}(X)$  be the family of all intuitionistic fuzzy  $\text{gsg}$  open set. Then intuitionistic fuzzy topological space  $(X, \mathfrak{T})$  is intuitionistic fuzzy  $\text{gsgT}_{1/2}$  - space. If and only if  $\text{IFO}(X) = \text{IFGSGO}(X)$ .

**Proof: Necessity:** let  $(X, \mathfrak{T})$  is intuitionistic fuzzy  $\text{gsgT}_{1/2}$  space. Let  $A$  is intuitionistic fuzzy  $\text{gsg}$ -open set. Then  $A^c$  is intuitionistic fuzzy  $\text{gsg}$ -closed set. By hypothesis  $A^c$  is intuitionistic fuzzy closed set in  $(X, \mathfrak{T})$ . Therefore  $A$  is intuitionistic fuzzy open set. Then  $\text{IFGSGO}(X) \subseteq \text{IFO}(X)$ . But by remark 3.17  $\text{IFO}(X) \subseteq \text{IFGSGO}(X)$ . Hence  $\text{IFO}(X) = \text{IFGSGO}(X)$ .

**Sufficiency:** Let  $\text{IFO}(X) = \text{IFGSGO}(X)$ . Let  $A$  is an intuitionistic fuzzy  $\text{gsg}$ -closed set in  $(X, \mathfrak{T})$ . Then  $A^c$  is intuitionistic fuzzy  $\text{gsg}$  open set in  $(X, \mathfrak{T})$ . By hypothesis  $A^c$  is intuitionistic fuzzy open set in  $(X, \mathfrak{T})$ . Therefore  $A$  is intuitionistic fuzzy closed set in  $(X, \mathfrak{T})$ . Hence  $(X, \mathfrak{T})$  is intuitionistic fuzzy  $\text{gsgT}_{1/2}$  space.

### V. INTUITIONISTIC FUZZY gsg- CONTINUOUS MAPPINGS

**Definition 5.1:** A mapping  $f : (X, \mathfrak{F}) \rightarrow (Y, \sigma)$  is intuitionistic fuzzy gsg- continuous if inverse image of every intuitionistic fuzzy closed set of  $Y$  is intuitionistic fuzzy gsg-closed set in  $X$ .

**Theorem 5.1:** A mapping  $f : (X, \mathfrak{F}) \rightarrow (Y, \sigma)$  is intuitionistic fuzzy gsg- continuous if and only if the inverse image of every intuitionistic fuzzy open set of  $Y$  is intuitionistic fuzzy gsg- open in  $X$ .

**Proof:** It is obvious because  $f^{-1}(U^c) = (f^{-1}(U))^c$  for every intuitionistic fuzzy set  $U$  of  $Y$ .

**Theorem 5.2:** Every intuitionistic fuzzy gsg-continuous mapping is intuitionistic fuzzy g-continuous.

**Proof:** Let a mapping  $f : (X, \mathfrak{F}) \rightarrow (Y, \sigma)$  is intuitionistic fuzzy gsg- continuous. Let  $U$  is intuitionistic fuzzy open set in  $Y$ . Then  $f^{-1}(U)$  is intuitionistic fuzzy gsg- open in  $X$ , by definition of intuitionistic fuzzy gsg-continuous mapping. Since every intuitionistic fuzzy gsg -open set is intuitionistic g-open,  $f^{-1}(U)$  is intuitionistic fuzzy g open in  $X$ . Hence  $f : (X, \mathfrak{F}) \rightarrow (Y, \sigma)$  is intuitionistic fuzzy g- continuous.

**Remark 5.1:** The converse of the above theorem need not be true, as seen from the following example.

**Example 5.1** Let  $X = \{a, b\}$ ,  $Y = \{x, y\}$  and intuitionistic fuzzy sets  $U$  and  $V$  are defined as follows :

$$U = \{ \langle a, 0.7, 0.2 \rangle, \langle b, 0.6, 0.3 \rangle \} \quad V = \{ \langle x, 0.7, 0.2 \rangle, \langle y, 0.8, 0.1 \rangle \}$$

Let  $\mathfrak{F} = \{ \mathbf{0}, U, I \}$  and  $\sigma = \{ \mathbf{0}, V, I \}$  be intuitionistic fuzzy topologies on  $X$  and  $Y$  respectively. Then the mapping  $f : (X, \mathfrak{F}) \rightarrow (Y, \sigma)$  defined by  $f(a) = x$  and  $f(b) = y$  is intuitionistic fuzzy g- continuous but not intuitionistic fuzzy gsg continuous.

**Theorem 5.3:** Every intuitionistic fuzzy gsg-continuous mapping is intuitionistic fuzzy sg-continuous.

**Proof:** Let a mapping  $f : (X, \mathfrak{F}) \rightarrow (Y, \sigma)$  is intuitionistic fuzzy gsg- continuous. Let  $U$  is intuitionistic fuzzy open set in  $Y$ . Then  $f^{-1}(U)$  is intuitionistic fuzzy gsg open in  $X$ , by definition of intuitionistic fuzzy gsg continuous mapping. Since every intuitionistic fuzzy gsg open set is intuitionistic sg-open,  $f^{-1}(U)$  is intuitionistic fuzzy sg open in  $X$ . Hence  $f : (X, \mathfrak{F}) \rightarrow (Y, \sigma)$  is intuitionistic fuzzy sg- continuous.

**Remark 5.2:** The converse of the above theorem need not be true, as seen from the following example.

**Example 5.2:** Let  $X = \{a, b, c\}$   $Y = \{p, q, r\}$  and intuitionistic fuzzy sets  $O, U, V$  are defined as follows:

$$O = \{ \langle a, 0.9, 0.1 \rangle, \langle b, 0, 1 \rangle, \langle c, 0, 1 \rangle \} \quad U = \{ \langle p, 0, 1 \rangle, \langle q, 0.8, 0.1 \rangle, \langle r, 0, 1 \rangle \}$$

Let  $\mathfrak{F} = \{ \mathbf{0}, O, I \}$  and  $\sigma = \{ \mathbf{0}, V, I \}$  be intuitionistic fuzzy topologies on  $X$  and  $Y$  respectively. Then the mapping  $f : (X, \mathfrak{F}) \rightarrow (Y, \sigma)$  defined by  $f(a) = p, f(b) = q, f(c) = r$ , is intuitionistic fuzzy sg- continuous but not intuitionistic fuzzy gsg- continuous.

**Theorem 5.4 :** Every intuitionistic fuzzy gsg-continuous mapping is intuitionistic fuzzy w-continuous.

**Proof:** Let a mapping  $f : (X, \mathfrak{F}) \rightarrow (Y, \sigma)$  is intuitionistic fuzzy gsg- continuous. Let  $U$  is intuitionistic fuzzy open set in  $Y$ . Then  $f^{-1}(U)$  is intuitionistic fuzzy gsg open in  $X$ , by definition of intuitionistic fuzzy gsg continuous mapping. Since every intuitionistic fuzzy gsg open set is intuitionistic w-open,  $f^{-1}(U)$  is intuitionistic fuzzy w open in  $X$ . Hence  $f : (X, \mathfrak{F}) \rightarrow (Y, \sigma)$  is intuitionistic fuzzy w- continuous.

**Remark 5.3:** The converse of the above theorem need not be true, as seen from the following example

**Example 5.3** Let  $X = \{a, b\}$ ,  $Y = \{x, y\}$  and intuitionistic fuzzy sets  $U$  and  $V$  are defined as follows :

$$U = \{ \langle a, 0.5, 0.5 \rangle, \langle b, 0.4, 0.6 \rangle \} \quad V = \{ \langle x, 0.5, 0.5 \rangle, \langle y, 0.5, 0.5 \rangle \}$$

Let  $\mathfrak{F} = \{ \mathbf{0}, U, I \}$  and  $\sigma = \{ \mathbf{0}, V, I \}$  be intuitionistic fuzzy topologies on  $X$  and  $Y$  respectively. Then the mapping  $f : (X, \mathfrak{F}) \rightarrow (Y, \sigma)$  defined by  $f(a) = x$  and  $f(b) = y$  is intuitionistic fuzzy w- continuous but not intuitionistic fuzzy gsg-continuous.

**Theorem 5.5:** Every intuitionistic fuzzy gsg-continuous mapping is intuitionistic fuzzy rw-continuous.

**Proof:** Let a mapping  $f : (X, \mathfrak{F}) \rightarrow (Y, \sigma)$  is intuitionistic fuzzy gsg- continuous. Let  $U$  is intuitionistic fuzzy open set in  $Y$ . Then  $f^{-1}(U)$  is intuitionistic fuzzy gsg open in  $X$ , by definition of intuitionistic fuzzy gsg continuous mapping. Since every intuitionistic fuzzy gsg open set is intuitionistic rw-open,  $f^{-1}(U)$  is intuitionistic fuzzy rw open in  $X$ . Hence  $f : (X, \mathfrak{F}) \rightarrow (Y, \sigma)$  is intuitionistic fuzzy rw- continuous.

**Remark 5.4:** The converse of the above theorem need not be true, as seen from the following example

**Example 5.4:** Let  $X = \{a, b\}$ ,  $Y = \{x, y\}$  and intuitionistic fuzzy sets  $U$  and  $V$  are defined as follows:

$$U = \{ \langle a, 0.5, 0.5 \rangle, \langle b, 0.4, 0.6 \rangle \} , \quad V = \{ \langle x, 0.5, 0.5 \rangle, \langle y, 0.5, 0.5 \rangle \}$$

Let  $\mathfrak{I} = \{0, U, I\}$  and  $\sigma = \{0, V, I\}$  be intuitionistic fuzzy topologies on  $X$  and  $Y$  respectively. Then the mapping  $f: (X, \mathfrak{I}) \rightarrow (Y, \sigma)$  defined by  $f(a) = x$  and  $f(b) = y$  is intuitionistic fuzzy rw- continuous but not intuitionistic fuzzy gsg-continuous.

**Theorem 5.6:** Every intuitionistic fuzzy gsg-continuous mapping is intuitionistic fuzzy gpr- continuous.

**Proof:** Let a mapping  $f: (X, \mathfrak{I}) \rightarrow (Y, \sigma)$  is intuitionistic fuzzy gsg- continuous. Let  $U$  is intuitionistic fuzzy open set in  $Y$ . Then  $f^{-1}(U)$  is intuitionistic fuzzy gsg open in  $X$ , by definition of intuitionistic fuzzy gsg- continuous mapping. Since every intuitionistic fuzzy gsg open set is intuitionistic gpr-open,  $f^{-1}(U)$  is intuitionistic fuzzy gpr open in  $X$ . Hence  $f: (X, \mathfrak{I}) \rightarrow (Y, \sigma)$  is intuitionistic fuzzy gpr- continuous.

**Remark 5.5:** The converse of the above theorem need not be true, as seen from the following example

**Example 5.5:** Let  $X = \{a, b, c\}$ ,  $Y = \{x, y, z\}$  and intuitionistic fuzzy sets  $O, U$  and  $V$  are defined as follows

$$O = \{ \langle a, 0.9, 0.1 \rangle, \langle b, 0.1 \rangle, \langle c, 0, 1 \rangle \} \quad U = \{ \langle a, 0, 1 \rangle, \langle b, 0.8, 0.1 \rangle, \langle c, 0, 1 \rangle \}$$

$$V = \{ \langle a, 0.9, 0.1 \rangle, \langle b, 0.8, 0.1 \rangle, \langle c, 0, 1 \rangle \}$$

Let  $\mathfrak{I} = \{0, O, U, V, I\}$  and intuitionistic fuzzy topologies on  $X$  and  $Y$  respectively. Then the mapping  $f: (X, \mathfrak{I}) \rightarrow (Y, \sigma)$  defined by  $f(a) = x$ ,  $f(b) = y$  and  $f(c) = z$  is intuitionistic fuzzy gpr-continuous but it is not intuitionistic fuzzy gsg- continuous.

**Theorem 5.7:** Every intuitionistic fuzzy gsg-continuous mapping is intuitionistic fuzzy rg-continuous.

**Proof:** Let a mapping  $f: (X, \mathfrak{I}) \rightarrow (Y, \sigma)$  is intuitionistic fuzzy gsg- continuous. Let  $U$  is intuitionistic fuzzy open set in  $Y$ . Then  $f^{-1}(U)$  is intuitionistic fuzzy gsg open in  $X$ , by definition of intuitionistic fuzzy gsg continuous mapping. Since every intuitionistic fuzzy gsg open set is intuitionistic rg-open,  $f^{-1}(U)$  is intuitionistic fuzzy rg-open in  $X$ . Hence  $f: (X, \mathfrak{I}) \rightarrow (Y, \sigma)$  is intuitionistic fuzzy rg- continuous.

**Remark 5.6:** The converse of the above theorem need not be true, as seen from the following example.

**Example 5.6:** Let  $X = \{a, b, c, d, e\}$  and  $Y = \{p, q, r, s, t\}$  and intuitionistic fuzzy sets  $O, U, V$  and  $W$  defined as follows:

$$O = \{ \langle a, 0.9, 0.1 \rangle, \langle b, 0.8, 0.1 \rangle, \langle c, 0, 1 \rangle, \langle d, 0, 1 \rangle, \langle e, 0, 1 \rangle \}$$

$$U = \{ \langle p, 0, 1 \rangle, \langle q, 0, 1 \rangle, \langle r, 0.8, 0.1 \rangle, \langle s, 0.7, 0.2 \rangle, \langle t, 0, 1 \rangle \}$$

Let  $\mathfrak{I} = \{0, O, U, V, I\}$  and  $\sigma = \{0, W, I\}$  be intuitionistic fuzzy topologies on  $X$  and  $Y$  respectively. Then the mapping  $f: (X, \mathfrak{I}) \rightarrow (Y, \sigma)$  defined by  $f(a) = p, f(b) = q, f(c) = r, f(d) = s$  and  $f(e) = t$  is intuitionistic fuzzy rg- continuous but not intuitionistic fuzzy gsg- continuous.

**Theorem 5.8:** Every intuitionistic fuzzy gsg-continuous mapping is intuitionistic fuzzy gs-continuous.

**Proof:** Let a mapping  $f: (X, \mathfrak{I}) \rightarrow (Y, \sigma)$  is intuitionistic fuzzy gsg- continuous. Let  $U$  is intuitionistic fuzzy open set in  $Y$ . Then  $f^{-1}(U)$  is intuitionistic fuzzy gsg open in  $X$ , by definition of intuitionistic fuzzy gsg continuous mapping. Since every intuitionistic fuzzy gsg open set is intuitionistic gs-open,  $f^{-1}(U)$  is intuitionistic fuzzy gs -open in  $X$ . Hence  $f: (X, \mathfrak{I}) \rightarrow (Y, \sigma)$  is intuitionistic fuzzy gs- continuous.

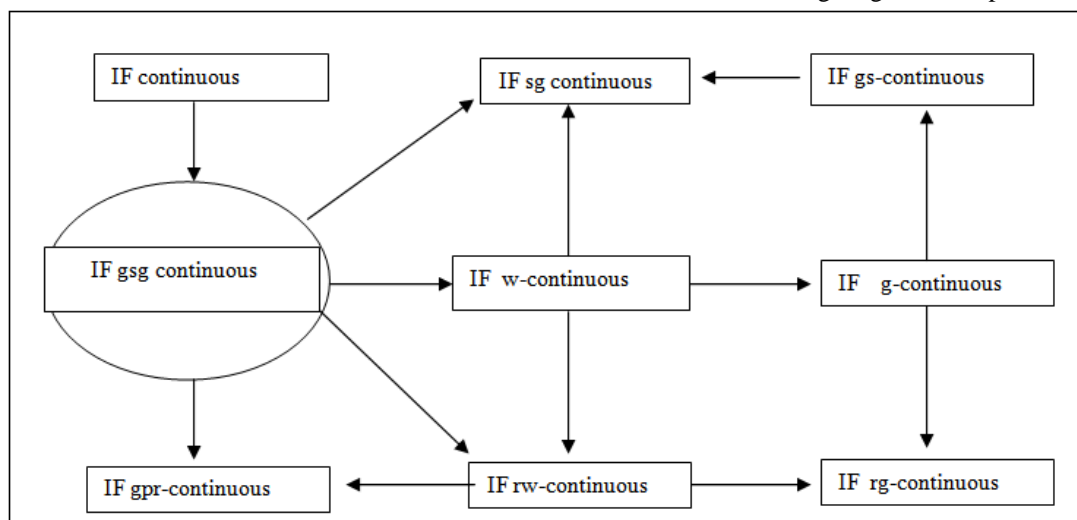
**Remark 5.7:** The converse of the above theorem need not be true, as seen from the following example.

**Example 5.7:** Let  $X = \{a, b\}$ ,  $Y = \{x, y\}$  and intuitionistic fuzzy sets  $U$  and  $V$  are defined as follows :

$$U = \{ \langle a, 0.5, 0.5 \rangle, \langle b, 0.4, 0.6 \rangle \} \quad V = \{ \langle x, 0.5, 0.5 \rangle, \langle y, 0.5, 0.5 \rangle \}$$

Let  $\mathfrak{I} = \{0, U, I\}$  and  $\sigma = \{0, V, I\}$  be intuitionistic fuzzy topologies on  $X$  and  $Y$  respectively. Then the mapping  $f: (X, \mathfrak{I}) \rightarrow (Y, \sigma)$  defined by  $f(a) = x$  and  $f(b) = y$  is intuitionistic fuzzy w- continuous but not intuitionistic fuzzy gsg-continuous.

**Remark 5.8:** From the above discussion and known results we have the following diagram of implication:



**Fig.2** Relations between intuitionistic fuzzy gsg-continuous mappings and other existing intuitionistic fuzzy continuous mappings.

**Theorem 5.8:** If  $f: (X, \mathfrak{I}) \rightarrow (Y, \sigma)$  is intuitionistic fuzzy gsg- continuous then for each intuitionistic fuzzy point  $c(\alpha, \beta)$  of  $X$  and each intuitionistic fuzzy open set  $V$  of  $Y$  such that  $f(c(\alpha, \beta)) \subseteq V$  there exists an intuitionistic fuzzy gsg- open set  $U$  of  $X$  such that  $c(\alpha, \beta) \subseteq U$  and  $f(U) \subseteq V$ .

**Proof :** Let  $c(\alpha, \beta)$  be intuitionistic fuzzy point of  $X$  and  $V$  be an intuitionistic fuzzy open set of  $Y$  such that  $f(c(\alpha, \beta)) \subseteq V$ . Put  $U = f^{-1}(V)$ . since  $f$  is intuitionistic fuzzy gsg-continuous  $f^{-1}(V) = U$  is intuitionistic fuzzy gsg- open set of  $X$  such that  $c(\alpha, \beta) \subseteq U$  and  $f(U) = f(f^{-1}(V)) \subseteq V$ .

**Theorem 5.9:** If  $f: (X, \mathfrak{I}) \rightarrow (Y, \sigma)$  is intuitionistic fuzzy gsg-continuous and  $g: (Y, \sigma) \rightarrow (Z, \mu)$  is intuitionistic fuzzy continuous. Then  $gof: (X, \mathfrak{I}) \rightarrow (Z, \mu)$  is intuitionistic fuzzy gsg-continuous.

**Proof:** Let  $A$  is an intuitionistic fuzzy closed set in  $Z$ . then  $g^{-1}(A)$  is intuitionistic fuzzy closed in  $Y$  because  $g$  is intuitionistic fuzzy continuous. Therefore  $(gof)^{-1}(A) = f^{-1}(g^{-1}(A))$  is intuitionistic fuzzy gsg – closed in  $X$ . Hence  $gof$  is intuitionistic fuzzy gsg – continuous.

**Theorem 5.10:** If  $f: (X, \mathfrak{I}) \rightarrow (Y, \sigma)$  is intuitionistic fuzzy gsg-continuous and  $g: (Y, \sigma) \rightarrow (Z, \mu)$  is intuitionistic fuzzy g-continuous and  $(Y, \sigma)$  is intuitionistic fuzzy  $T_{1/2}$  then  $gof: (X, \mathfrak{I}) \rightarrow (Z, \mu)$  is intuitionistic fuzzy gsg-continuous.

**Proof:** Let  $A$  is an intuitionistic fuzzy closed set in  $Z$ , then  $g^{-1}(A)$  is intuitionistic fuzzy g-closed in  $Y$ . Since  $Y$  is  $T_{1/2}$ , then  $g^{-1}(A)$  is intuitionistic fuzzy closed in  $Y$ . Hence  $(gof)^{-1}(A) = f^{-1}(g^{-1}(A))$  is intuitionistic fuzzy gsg– closed in  $X$ . Hence  $gof$  is intuitionistic fuzzy gsg – continuous.

**Theorem 5.11:** If  $f: (X, \mathfrak{I}) \rightarrow (Y, \sigma)$  is intuitionistic fuzzy rg-irresolute and  $g: (Y, \sigma) \rightarrow (Z, \mu)$  is intuitionistic fuzzy gsg-continuous. Then  $gof: (X, \mathfrak{I}) \rightarrow (Z, \mu)$  is intuitionistic fuzzy rg-continuous.

**Proof:** Let  $A$  is an intuitionistic fuzzy closed set in  $Z$ , then  $g^{-1}(A)$  is intuitionistic fuzzy gsg-closed in  $Y$ , because  $g$  is intuitionistic fuzzy gsg-continuous. Since every intuitionistic fuzzy gsg-closed set is intuitionistic fuzzy rg-closed set, therefore  $g^{-1}(A)$  is intuitionistic fuzzy rg-closed in  $Y$ . Then  $(gof)^{-1}(A) = f^{-1}(g^{-1}(A))$  is intuitionistic fuzzy rg-closed in  $X$ , because  $f$  is intuitionistic fuzzy rg- irresolute. Hence  $gof: (X, \mathfrak{I}) \rightarrow (Z, \mu)$  is intuitionistic fuzzy rg-continuous.

## VI. CONCLUSION

The theory of g-closed sets plays an important role in general topology. Since its inception many weak and strong forms of g-closed sets have been introduced in general topology as well as fuzzy topology and intuitionistic fuzzy topology. The present paper investigated a new form of intuitionistic fuzzy closed sets called intuitionistic fuzzy generalized semi generalized -closed sets ( briefly Intuitionistic fuzzy gsg closed) which contain the classes of intuitionistic fuzzy closed sets and intuitionistic fuzzy regular closed sets and contained in the classes of intuitionistic fuzzy g-closed sets, intuitionistic fuzzy w closed sets. intuitionistic fuzzy rw closed sets, intuitionistic fuzzy sg closed set, intuitionistic fuzzy gs closed sets, intuitionistic fuzzy gp-closed sets, intuitionistic fuzzy gpr-closed sets , intuitionistic fuzzy gspr-closed sets , intuitionistic fuzzy sgp-and class of all intuitionistic fuzzy gsp-closed sets. Several properties and application of intuitionistic fuzzy gsg--closed sets and intuitionistic fuzzy gsg- open set are studied. Many examples are given to justify the result.



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