

## Symmetrical Prime Numbers With Three Prime Digits

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### Abstract

After defining, symmetrical prime numbers will be presented. How many symmetrical prime numbers with three prime digits are there in the interval  $(10^n, 10^{n+1})$ , where  $n$  is a positive integer number and  $n \geq 3$ ? On the one hand, it has been counted by computer ( $T(n)$ ,  $P(n)$ ,  $R(n)$ ,  $S(n)$ ). On the other hand, the function  $F(n)$ ,  $G(n)$ ,  $H(n)$ ,  $Q(n)$  gives the approximate number of symmetrical prime numbers with three prime digits in the interval  $(10^n, 10^{n+1})$ , where  $n \geq 3$  integer.

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### I. INTRODUCTION

The sets of special prime numbers within the set of prime numbers are well-known. For instance, left-truncatable primes (If we leave the initial digits out, the remainder will be prime.), right-truncatable primes (If we leave the last digits out, the remainder will be prime.), the Erdős-primes (the sum of the digits is prime) [8], Fibonacci-primes ( $F_0=0$ ,  $F_1=1$ ,  $F_n=F_{n-1}+F_{n-2}$ ), Gauss-primes (in the form  $4n+3$ ), Leyland-primes (in the form  $x^y+y^x$ , where  $1 \leq x \leq y$ ), Pell-primes ( $P_0=0$ ,  $P_1=1$ ,  $P_n=2P_{n-1}+P_{n-2}$ ), Bölcsföldi-Birkás-Ferenczi primes (all digits are prime and the number of digits is prime), Bölcsföldi-Birkás prime numbers (all digits are prime, the number of digits is prime, the sum of digits is prime), etc. Question: Which further sets of special prime numbers are there within the set of prime numbers? We have found further sets of special prime numbers within the set of prime numbers. These are the sets of symmetrical prime numbers with three prime digits.

### II. SYMMETRICAL PRIME NUMBERS WITH PRIME DIGITS 2,3,5 [3], [9], [10], [11], [12], [13], [14].

Definition: a positive integer number is a symmetrical prime number with prime digits 2,3,5, if

- a/ the positive integer number is prime,
- b/ all digits of number are 2 or 3 or 5,
- c/ the number of digits is prime,
- d/ the sum of digits is prime,
- e/ the number is centrally symmetrical to the middle digit.

**The symmetrical prime numbers with prime digits 2,3,5 are as follows (the last digit can only be 3) :**

{ {353}, {32323, 33533, 35353}, {3223223, 3233323, 3553553},  
{32225352223, 33235553233, 33335353333, 33352525333,  
33532523533, 33552525533, 33553335533, 35332523353, 35533333553,  
35553535553}, {322223222223, 3223535353223, 3225225225223,  
3225533355223, 3225553555223, 3232553552323, 3233325233323,  
323333333323, 3233525253323, ...etc.

$T(n)$  is the factual frequency of symmetrical prime numbers with prime digits 2,3,5 in the interval  $(10^{n-1}, 10^n)$ , where  $n \geq 3$  integer.  $T(3)=1$ ,  $T(5)=3$ ,  $T(7)=3$ ,  $T(11)=10$ ,  $T(13)=35$ ,  $T(17)=204$ ,  $T(19)=577$ ,  $T(23)=4207$  etc.

$F(n)$  function gives the approximate number of symmetrical prime numbers with prime digits 2,3,5 in the interval  $(10^{n-1}, 10^n)$  where  $n \geq 3$  integer.

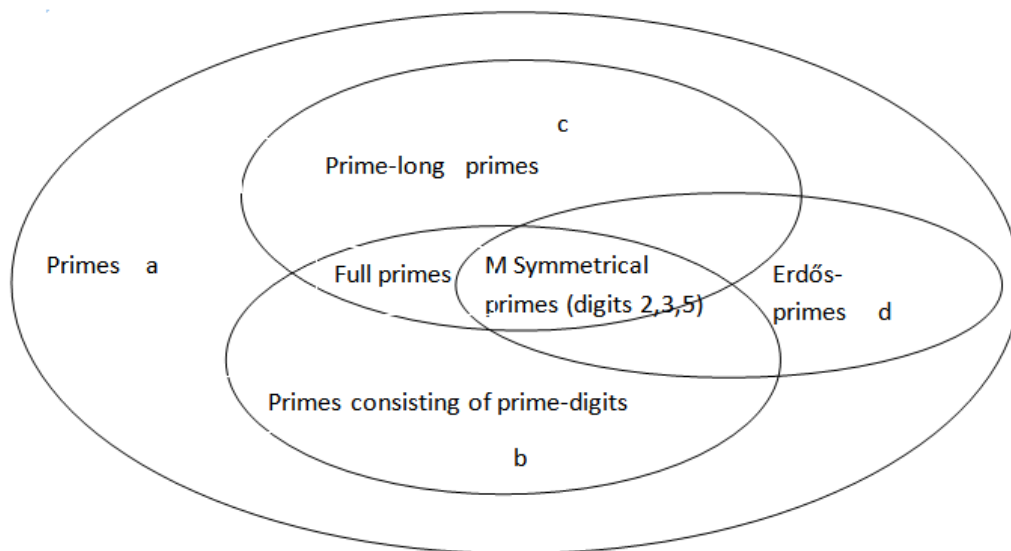
The approximate funktion is

$F(n)=0,065x1,62^n$ , where  $n \geq 3$  integer.

The factual number of symmetrical primes with prime digits 2,3,5 and the number of symmetrical primes with prime digits 2,3,5 calculated according to function  $F(n)$  are as follows:

Number of digits	The factual number by computer of symmetrical primes with digits 2,3,5 in the interval $(10^n, 10^{n+1})$	The number of symmetrical primes (digits 2,3,5) calculated according to funktion $F(n)$	
$n \geq 3$ integer	$T(n)$	$F(n) = 0,065 \times 1,62^n$	$T(n)/F(n)$
3	1	0,28	3,57
5	3	0,72	4,16
7	3	1,90	1,58
11	10	13,11	0,76
13	35	34,40	1,02
17	204	236,96	0,86
19	577	621,19	0,93
23	4270	4283,10	0,99

**Fig.1**



### III. SYMMETRICAL PRIME NUMBERS WITH PRIME DIGITS 2,3,7 [3], [9], [10], [11], [12], [13], [14].

Definition: a positive integer number is a symmetrical prime number with prime digits 2,3,7, if

- a/ the positive integer number is prime,
- b/ all digits of number are 2 or 3 or 7,
- c/ the number of digits is prime,
- d/ the sum of digits is prime,
- e/ the number is centrally symmetrical to the middle digit.

**The symmetrical prime numbers with prime digits 2,3,7 are as follows (the last digit can only be 3 or 7) :**

{ {373}, {32323, 77377}, {3223223, 3233323, 3773773, 7327237, 7733377},  
 {32322722323, 32337773323, 32737373723, 32777777723,  
 33723732733, 33727372733, 33732723733, 72222722227, 72323332327,  
 72332723327, 72733333727, 72777777727, 73232723237, 73327372337,  
 73332723337, 73337773337, 73733733737, ...etc.

$P(n)$  is the factual frequency of symmetrical prime numbers with prime digits 2,3,7 in the interval  $(10^n, 10^{n+1})$ , where  $n \geq 3$  integer.  $P(3)=1, P(5)=2, P(7)=5, P(11)=20, P(13)=58, P(17)=431, P(19)=1077, P(23)=7777$ , etc.  $G(n)$  function gives the approximate number of symmetrical prime numbers with prime digits 2,3,7 in the interval  $(10^n, 10^{n+1})$  where  $n \geq 3$  integer. We think that  $G(n)=0,195 \times 1,585^n$ , where  $n \geq 3$  integer.

The factual number of symmetrical primes with prime digits 2,3,7 and the number of symmetrical primes with prime digits 2,3,7 calculated according to function  $G(n)$  are as follows:

Number of digits	The factual number by computer of symmetrical primes with digits 2,3,7 in the interval $(10^n, 10^{n+1})$	The number of symmetrical primes (digits 2,3,7) calculated according to funktion $G(n)$	
$n \geq 3$ integer	$P(n)$	$G(n)=0,195 \times 1,585^n$	$P(n)/G(n)$
3	1	0,78	1,28
5	2	1,95	1,03
7	5	4,90	1,02
11	20	30,93	0,65
13	58	77,70	0,75
17	431	490,38	0,88
19	1077	1231,94	0,87
23	7777	7775,13	1,00

#### IV. SYMMETRICAL PRIME NUMBERS WITH PRIME DIGITS 2,5,7 [3], [9], [10], [11], [12], [13], [14].

Definition: a positive integer number is a symmetrical prime number with prime digits 2,5,7 if

- a/ the positive integer number is prime,
- b/ all digits of number are 2 or 5 or 7,
- c/ the number of digits is prime,
- d/ the sum of digits is prime,
- e/ the number is centrally symmetrical to the middle digit,

**The symmetrical prime numbers with prime digits 2,5,7 are as follows (the last digit can only be 7) :**

$\{ \{757\}, \{75557\}, \{ \}, \{72222722227, 72225752227, 72775557727, 72777777727, 75222722257, 75225552257, 75255555257, 75522522557, 75555555557, 75572727557, 75577577557, 75772527757, \dots \text{etc.} \}$

$R(n)$  is the factual frequency of symmetrical prime numbers with prime digits 2,5,7 in the interval  $(10^n, 10^{n+1})$ , where  $n \geq 3$  integer.  $R(3)=1, R(5)=1, R(7)=0, R(11)=14, R(13)=19, R(17)=177, R(19)=497, R(23)=3498$ , etc.

$H(n)$  function gives the approximate number of symmetrical prime numbers with prime digits 2,5,7 in the interval  $(10^n, 10^{n+1})$  where  $n \geq 3$  integer..

We think that  $H(n)=0,065 \times 1,6058^n$ , where  $n \geq 3$  integer.

The factual number of symmetrical primes with prime digits 2,5,7 and the number of symmetrical primes with prime digits 2,5,7 calculated according to function H(n) are as follows:

Number of digits	The factual number by computer of symmetrical primes with digits 2,5,7 in the interval $(10^n, 10^{n+1})$	The number of symmetrical primes (digits 2,5,7) calculated according to funktion H(n)	
$n \geq 3$ integer	R(n)	$H(n)=0,065 \times 1,6058^n$	R(n)/H(n)
3	1	0,27	3,70
5	1	0,69	1,45
7	0	1,79	0
11	14	11,90	1,18
13	19	0,68	0,62
17	177	204,08	0,87
19	497	526,08	0,94
23	3498	3497,96	1,00

**V. SYMMETRICAL PRIME NUMBERS WITH PRIME DIGITS 3,5,7 [3], [9], [10], [11], [12], [13], [14].**

Definition: a positive integer number is a symmetrical prime number with prime digits 3,5,7 if

- a/ the positive integer number is prime,
- b/ all digits of number are 3 or 5 or 7,
- c/ the number of digits is prime,
- d/ the sum of digits is prime,
- e/ the number is centrally symmetrical to the middle digit,

**The symmetrical prime numbers with prime digits 3,5,7 are as follows (the last digit can only be 3 or 7) :**

{ {353, 373, 757}, {33533, 35353, 35753, 75557, 77377}, {3553553, 3773773, 7733777}, {33335353333, 33337573333, 33357575333, 33373537333, 33533733533, 33553335533, 33557775533, 33573537533, 33757375733, 35333733353, 35377377353, 35533333553, 35537773553, 35553535553, 3555755553, 35573737553, 35733533753, 35775757753, 37333533373, 37533533573, ...etc.

S(n) is the factual frequency of symmetrical prime numbers with prime digits 3,5,7 in the interval  $(10^n, 10^{n+1})$ , where  $n \geq 3$  integer.  $S(3)=3, S(5)=5, S(7)=3, S(11)=40, S(13)=92, S(17)=520, S(19)=1588, S(23)=9932,$  etc. Q(n) function gives the approximate number of symmetrical prime numbers with prime digits 3,5,7 in the interval  $(10^n, 10^{n+1})$  where  $n \geq 3$  integer. The approximate funktion is  **$Q(n)=0,66 \times 1,519^n$** , where  $n \geq 3$  integer.

The factual number of symmetrical primes with prime digits 3,5,7 and the number of symmetrical primes with prime digits 3,5,7 calculated according to function Q(n) are as follows:

Number of digits	The factual number by computer of symmetrical primes with digits 3,5,7 in the interval $(10^n, 10^{n+1})$	The number of symmetrical primes (digits 3,5,7) calculated according to funktion Q(n)	
n	S(n)	$Q(n)=0,66 \times 1,519^n$	S(n)/Q(n)
3	3	2,31	1,30
5	5	5,33	0,94
7	3	12,32	0,24
11	40	65,57	0,61
13	92	151,28	0,61
17	520	805,43	0,65
19	1588	1858,41	0,85
23	9932	9894,02	1,00

## VI. THE NUMBER OF THE ELEMENTS OF THE SET FOR EXAMPLE OF SYMMETRICAL PRIME NUMBERS WITH PRIME DIGITS 3,5,7 [3], [9],[10], [11], [12], [13], [14].

Let's take the set of Mills' prime numbers!

Definition: The number  $m=[M \text{ ad } 3^n]$  is a prime number, where  $M=1,306377883863080690468614492602$  is the Mills' constant, and  $n=1,2,3,\dots$  is an arbitrary positive integer number. It is already known that the number of the elements of the set of Mills' prime numbers is infinite. The Mills' prime numbers are the following: 2, 11, 1361, 2521008887,...

The connection  $n \rightarrow m$  is the following:  $1 \rightarrow 2, 2 \rightarrow 11, 3 \rightarrow 1361, 4 \rightarrow 2521008887, \dots$

$n \geq 1$ integer	Mills prime number: $m=[M \text{ ad } 3^n]$	Mills-interval: $(10^{m-1}, 10^m)$
1	2	$(10, 10^2)$
2	11	$(10^{10}, 10^{11})$
3	1361	$(10^{1360}, 10^{1361})$
4	2521008887	$(10^{2521008886}, 10^{2521008887})$ , etc.

The Mills' prime number  $m=[M \text{ ad } 3^n]$  corresponds with the Mills-interval  $(10^{m-1}, 10^m)$  and vice versa. For instance:  $2 \rightarrow (10, 10^2)$ ,  $11 \rightarrow (10^{10}, 10^{11})$ ,  $1361 \rightarrow (10^{1360}, 10^{1361})$ , etc. and vice versa. The number of the elements of the set of Mills' prime numbers is infinite. As a consequence, the number of the Mills-intervals  $(10^{m-1}, 10^m)$  that contain at least one Mills' prime number is infinite. The number of symmetrical primes with prime digits 3,5,7 in the Mills-interval

$(10^{m-1}, 10^m)$  is  $Q(m) = 0,66 \times 1,519^m$ .

The number of symmetrical prime numbers with prime digits 3,5,7 is probably infinite:

$\lim_{n \rightarrow \infty} S(n) = \infty$  is probably where  $n \geq 3$  integer.

$n \rightarrow \infty$

## VII. CONCLUSION

Countless different sets of special prime numbers have been known. We have found the following set of special prime numbers within the set of prime numbers. There may be further sets of special prime numbers that we do not know yet. Finding them will be task of researchers of the future.

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