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Lumped Parameter Method for Tip Clearance Control in High Pressure Compressors

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ABSTRACT
The control of tip clearance in high pressure (HP) compressors has been a continuing issue in the gas turbine industry. The gap varies significantly during different operating conditions of the engine due to centrifugal forces on the rotor and differential thermal expansions in the discs and casing. This paperfocuses on the lumped parameter method used in the development of the concept for tip clearance control. The 1-D modelling was used to validate the SC03 model. This is a matching exercise where the finite element thermo-mechanical model results are calibrated against the lumped parameter results. The analysis makes use of a heat transfer and fluid flow based lumped parameter spreadsheet in combination with a full axisymmetric thermo-mechanical finite element High Pressure Compressor (HPC) casing and drum model. The HPC models are used to investigate the effect of various parameters on the closure behaviour of the various HPC stages. The software packages used are SC03, MATLAB and Microsoft Excel. In this report, the validation of the lumped model with data from SC03 model setup was examined. This includes a brief description of the Lumped mass parameter method, mathematical model setup of the model and the approach used for matching simulation results from SC03 with Lumped model data. The validation result of Stage 3 Trent 1000 casing and drum model simulation and Lumped model presented were in good agreement.

KEYWORDS: *Lump, Parameters, TipClearance, Control, High Pressure, Compressor, Aero-engines*

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I. INTRODUCTION

Tip clearance is the radial gap between the stationary compressor casing and the rotating blades. The gap varies significantly during different operating conditions of the engine due to centrifugal forces on the rotor and differential thermal expansions in the discs and casing. In general, as the clearance between the compressor blade tips and the casing increases, the aerodynamic efficiency will decrease and therefore the specific fuel consumption and operating costs will increase, and the clearance is therefore of critical importance to civil airline operators and their customers alike. Compressor clearance control is also an important factor for engine operability and surge margin. The project involves the lumped parameter method used in the development of the concept for tip clearance control. The Lumped (1-D) modelling was used to validate the SC03 model. This is a matching exercise where the finite element thermo-mechanical model results are calibrated against the lumped parameter results.

In this report, consideration is given to the transient analysis of a body in which the temperature is assumed to be constant at any point within and on the surface of the body at a given point. It is assumed that the temperature of the whole body changes uniformly with time. Such an analysis is called a lumped mass parameter method. The lumped parameter scheme divides a thermal system into a number of distinct lumps and assumes that the temperature difference inside each lump is negligible. It is a simple and an approximate procedure in which no spatial variation in temperature is allowed. The change in temperature in such a system varies only with respect to time. It is therefore obvious that the lumped heat capacity is limited to small sized bodies or high thermal conductivity material as the case with rotor/blade of axial compressor in a gas turbine engine (Roland et al. 2004). An example of a gas turbine engine is the Trent 1000 aero engine shown as Figure 1.1. Figure 1.1 shows cutaway Trent 1000 aero engine showing the section of interest of this report. The lumped transient analysis of heat transfer implies that a mass has insignificant internal resistance to heat transfer and is used when a material has relatively low thermal resistance relative to the external thermal resistance.

It is important to note that the transient response of a system to heat transfer process is critical in the aerospace industry. A good example is the heating up of gas turbine compressors during transient operation as they are brought up to speed during take-off. The discs that carry the blades are thick and take a long time to heat up or come to temperature, while the casing is thin thus heat up or comes to temperature rapidly. During this process, the casing expands away from the blade tips, occasionally sufficient to cause serious difficulties with aerodynamic performance of the engine. This may even lead to a phenomenon called surge and may result in increased consumption of aviation fuel. The different expansion by both the casing and the rotor blade resulting from the variation in temperature with time during the various engines operating cycle, such as acceleration and deceleration gives rise to different time constants which is a feature of the lumped system analysis.

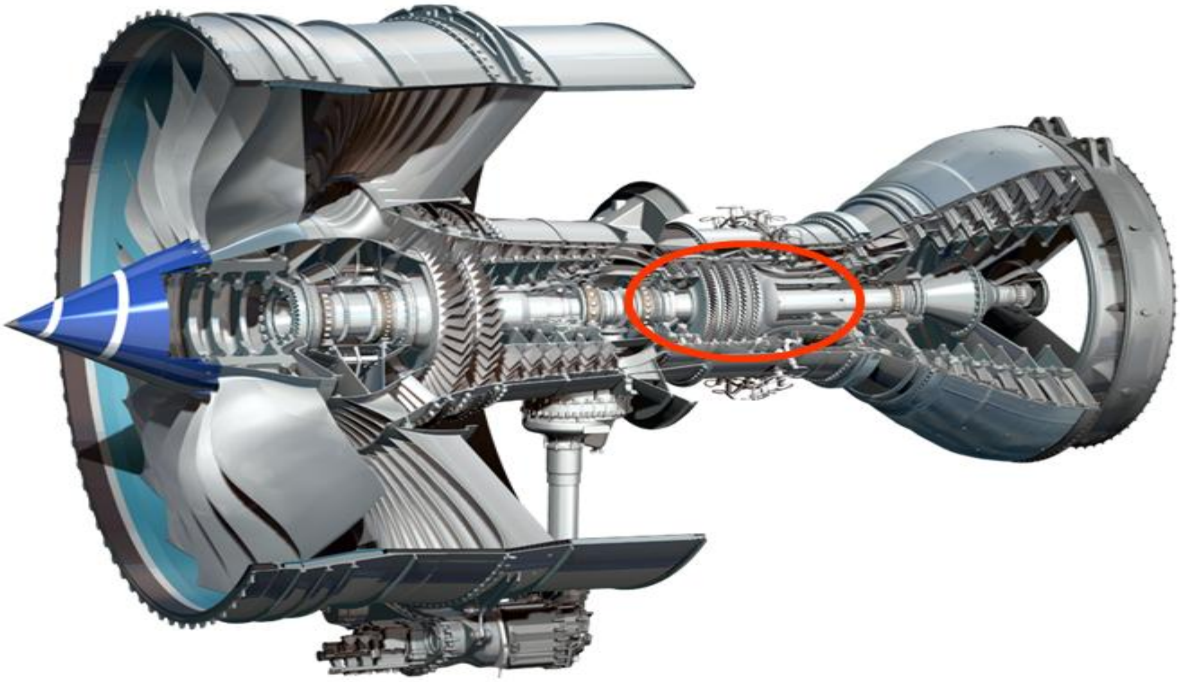

Figure 1.1: Cutaway Trent 1000 aero engine showing the section of interest (Rolls-Royce PLC website)

II. METHODOLOGY

2.1 The Lumped Parameter Concepts for compressor clearance control

In the lumped system analysis for thermal systems, the time constants are a main feature of the analysis. This is employed when objects cool or warm uniformly under the influence of convective cooling or warming as the case in this study during acceleration and deceleration. The time constant is the time required for a physical quantity to rise from zero to $1-\frac{1}{e}$ which is equivalent to 63.2% of its final steady value when it varies with time (t) as $1-e^{-kt}$. And it is also the time required for a physical quantity to fall to $\frac{1}{e}$ which equivalent to 36.8% of its initial value when it varies with time (t) as e^{-kt}.

Bearing in mind, a thermal system when an object cools or warm uniformly under the influence of convective cooling or warming, the heat transfers from the body to the ambient at a given time is proportional to the temperature difference between the body and the ambient. Figure 2.1 shows a solid lumped model for thermal analysis.

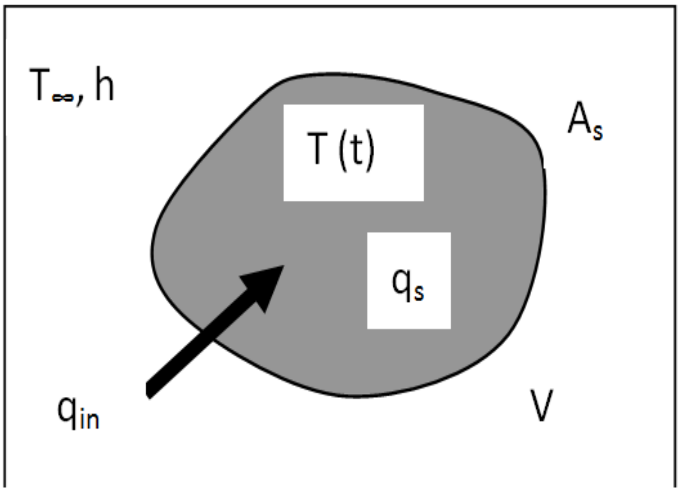

Figure 2.1: A solid lumped model

From Figure 2.1:

Net rate of heat transfer into the solid = $hA_s(T_\infty - T)$ (2.1)

Rate of increase of internal energy of the solid = $\rho C_p V \frac{dT}{dt}$ (2.2)

Therefore, equating Equation 1 and 2 gives:

Net rate of heat transfer into the solid = Rate of increase of internal energy of the solid

$$hA_s(T_\infty - T) = \rho C_p V \frac{dT}{dt} \tag{2.3}$$

From Equation 3, an expression can be obtained for the temperature of the solid as a function of time assuming h and T_∞ are constant as follows:

$$T(t) = T_\infty + (T_0 - T_\infty)e^{\frac{-hA_s}{\rho C_p V}.t} \tag{2.4}$$

Knowing that $\frac{-hA_s}{\rho C_p V} = \frac{1}{\tau}$, Equation 4 becomes,

$$T(t) = T_\infty + (T_0 - T_\infty)e^{\frac{-t_s}{\tau}} \tag{2.5}$$

The time needed to heat a solid from the initial temperature T_0 will be given as:

$$t = \frac{\rho C_p V}{hA_s} = \ln\left(\frac{T_\infty - T_0}{T_\infty - T}\right) \tag{2.6}$$

The time constant is given as:

$$\tau = \frac{\rho C_p V}{hA_s} \tag{2.7}$$

The time constant is the ratio of thermal capacitance to convection thermal resistance and this indicate that the lower the time constant, the faster the solid is heated. It also shows that the greater the masses (ρV) and heat capacities (C_p), the slower the changes in temperature while the larger the surface areas (A_s) and better heat transfer (h), the faster the temperature changes in the solid.

where:

h =heat transfer coefficient, (W/ (m^2K))

A_s = is the surface area, (m^2)

T (t) = body temperature at time t, (K)

T_0 = the constant ambient temperature, (K)

ρ = density, kg/m^3

Cp = specific heat (Jkg^{-1}K^{-1}) and

V = the body volume, (m^3)

The lumped capacitance method is generally said to be valid if the dimensionless Biot number is less than 1, that is $(Bi < 1)$.

Biot Number, $Bi = \frac{hL_c}{k}$ (2.8)

But characteristic length $L_c = \frac{V}{A_s}$ (2.9)

And $\frac{V}{A_s}$ represents a characteristic dimension of the body. The Biot number represents a ratio between conduction resistances within the body to convection resistance at the surface of the hot body as described in Equation 10 as:

$$Bi = \frac{\left(\frac{L_c}{KA_s}\right)}{\left(\frac{1}{hA}\right)} \qquad (2.10)$$

where:

k = thermal conductivity of the solid, (W/ (m.K))

For details on time constants and lumped system analysis for thermal systems, the reader is referred to White (1984), Kreith and Bohn (1993), Lewis et al. (2004) and Incropera*et al.* (2007). In this study, the rotating cavities with radial inflow and axial throughflowconfiguration of Rolls –Royce Trent 1000 casing and drum models which are directly relevant to this paper are not discussed in detail due to the proprietary nature of the work, but the reader is referred to the work by Dorfman (1963), Chew (1982), Pincombe (1983), Long (1984), Farthing and Owen (1988), Firouzian (1986) Owen and Rogers (1989), Tucker (1993), Owen and Rogers (1995), Alexiou (2000), Atkins (2013), Childs *et al.* (2006), Tucker (1993), Ekong(2014), Ekong*et al.* (2013) and Ekong*et al.* (2012). And again, due to the proprietary nature of the work, the inbuilt SC03 heat transfer correlations are not discussed in detail. SC03 is a proprietary finite element analysis program by Rolls-Royce PLC.

In summary, the lumped-mass capacity method shows that if the mass, volume and the area of the solid are constant, that increasing the heat transfer coefficient of the body will result in the reduction of the time constant and that decreasing the heat transfer coefficient of the body will result in the increased of the time constant. The aim of the study is to decrease the time constant of the drum by increasing the heat transfer coefficient of the systems, thereby causing the drum to heat up faster, hence narrowing down the large gap that existed at the beginning of engine transient operation between the casing and the blade. This would cause a reduction in the cruise clearance and a reduction in of clearance at surge point and hence reductions in the overall specific fuel consumption giving rise to higher engine efficiency.

III. RESULTS AND DISCUSSION

A mathematical 1-D model was created in MATLAB to determine the effect of the various parameters on tip clearance in an axial compressor with the relevant equation encoded into the MATLAB program for the analysis. Before a lumped parameter model is set up for tip closure analysis, it is necessary to run a SC03 rotor and casing models through a calibration square cycle. The square cycle models the various phases during engine operation consisting of start, stabilization at idle, acceleration to maximum take-off (MTO), stabilization at maximum take-off (MTO), deceleration to idle and stabilization at idle. In this analysis, an extended square cycle for stage 3 of Trent 1000 casing and drum modelsare used which includes the cruise phase. Figure 3.1 shows the variation of speed with time in the extended Square cycle with indication of the idle, maximum takes off (MTO) and cruise phases in the cycle. The environmental parameters used for the 1D modelling for the lumped parameter analysis are presented in Table 1.

Table 1: Environmental parameters used for the 1D modeling for the lumped parameter analysis

Parameters	Idle	MTO	Cruise
Speed (RPM)	8,467.17	12,359.24	12,016.72
Inlet temperature (T26)	350.86	599.61	582.21
Outlet temperature T30	516.13	909.12	876.20

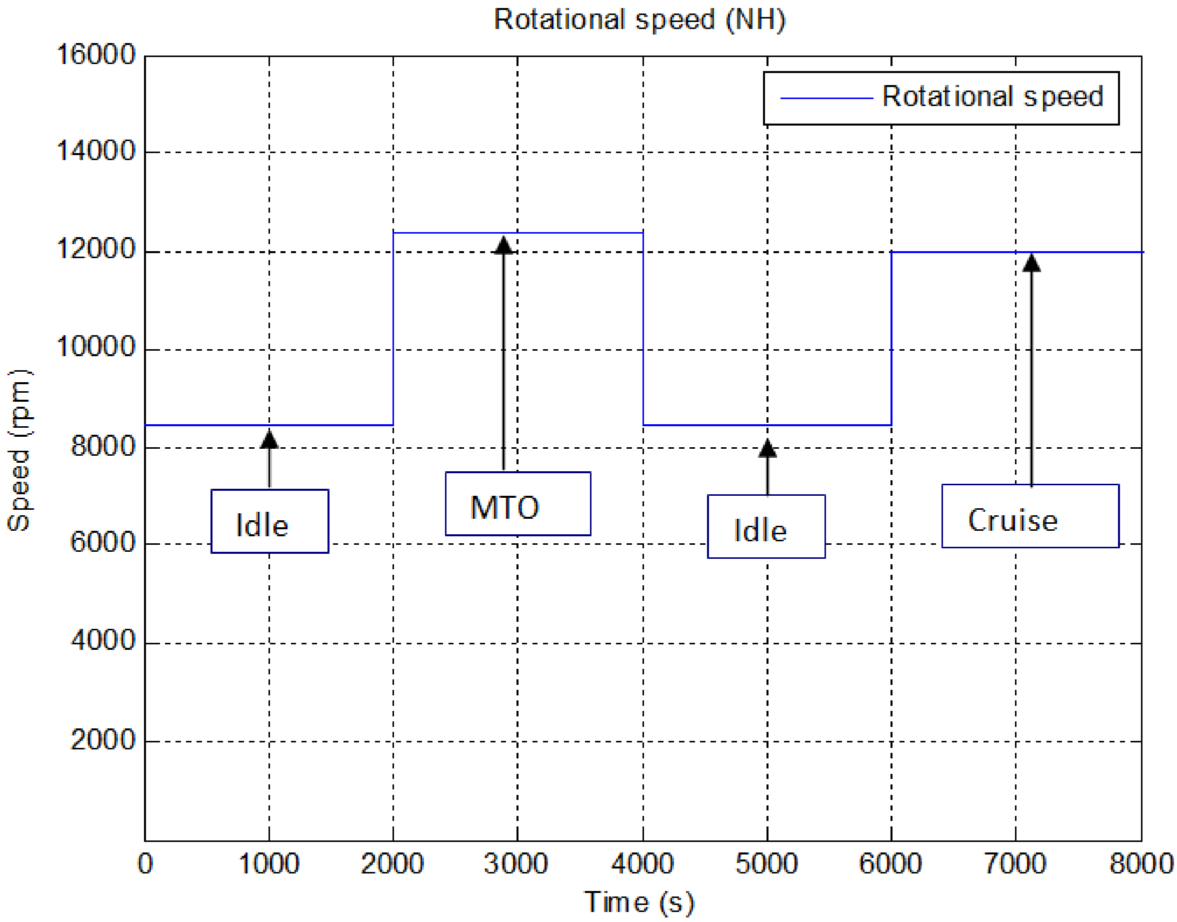

Figure 3.1: The variation of speed with time in the extended Square cycle with indication of the Idle, Maximum take-off (MTO) and Cruise phases in the cycle.

The method for controlling tip clearance indicates that for a given square cycle, tip clearance depends on:

- Casing displacement
- Rotor displacement

Hence, the two essential analyses performed to determine the compressor clearance during transients over a square cycle, are:

- The closure analysis
- The clearance analysis

3.1 Drum displacement calculations

This involves the analysis of drum displacement during engine transient throughout the square cycle. The thermal growths of the drum have to be scaled to different conditions to allow growths to be calculated at any condition throughout any engine cycle. The scaling calculation is performed by taking into consideration the relationship between the stage temperature and drum growth given by the Equation 3.1.

$$\delta_{th} = R\alpha(T_{drum} - 288.15) \tag{3.1}$$

Equation 6.14 shows that the thermal growths of the drum and stage temperature are related together with the coefficient of thermal expansion (α)for the drum material and the radius of the drum (R)(SCO3 Users Guide). If the thermal expansion of the drum (δ_{th}) at maximum take-off and idle are known, the equivalent stage temperatures at idle and maximum take-off can be obtained by from Equation 3.1.

3.1.2Drum centrifugal growth setup (CF growth)

The centrifugal growth (CF) of the rotor is obtained by scaling the CF growth at the square of the maximum rotation speed of the drum (SCO3 Users Guide). This is modelled as the ratio of thermal conductivity of the drum to the young modulus multiply by the square of the speed. This is presented in Equation 3.2 as:

$$CF_{drum} = \frac{k_{drum}}{E} * rpm\hat{}2 \tag{3.2}$$

where:

CF_{drum} = centrifugal growth of drum

K_{drum} = thermal conductivity of drum

E= young modulus of drum

rpm = drum rotational speed in revolution per minute

3.1.3Drum transient thermal growth

The transient thermal growth of the drum has an exponential response. The values of the stage temperature, drum time constant during acceleration and deceleration, acceleration and deceleration time and the thermal growths of the drum are modelled to give the drum transient thermal growth. The governing equation for the modelling of transient thermal growth of the drum is given by Equation 3.3 (SCO3 Users Guide).

$$d_{tran} = 288.15 + (T_{stage} - 288.15) * \left(1 - \exp^{\frac{-\Delta t}{\tau}}\right) \tag{3.3}$$

where:

d_{tran} = transient thermal growth of drum

T_{stage} = stage temperature for Idle, MTO and Cruise

Δt = change in time

τ = thermal time constant

The transient thermal growths are modelled with respect to ramp times over the complete engine cycles.

3.1.4Drum total thermal growth

The total thermal growth of the drum is obtained by the combine effect of the transient thermal growths of the drum with the drum centrifugal growth (CF). That is modelling the combine effect of Equation 3.2 and Equation 3.3. The result gives the drum total thermal growth characteristics over the square cycle as shown in Figure 3.2. Figure 3.2 shows the variation of drum total thermal growth with time over the extended square cycle for stage 3 of the Trent 1000 drum model.

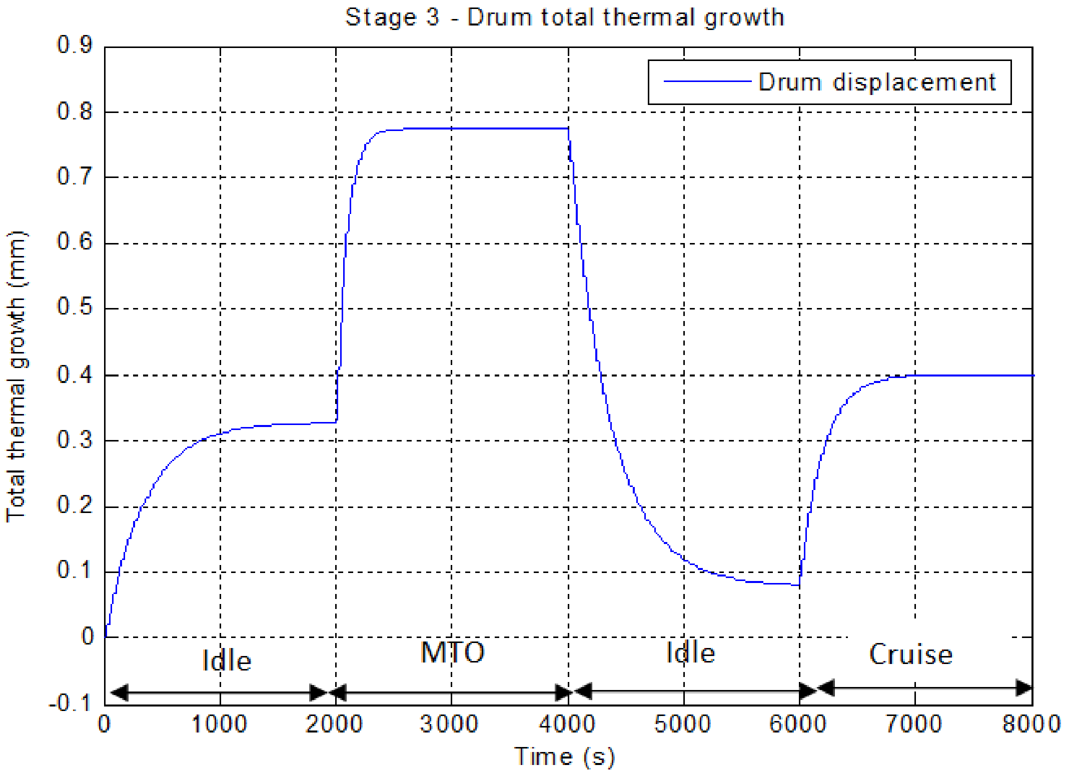

Figure 3.2: The variation of drum total thermal growth with time over the extended square cycle for stage 3 of Trent 1000 drum model.

3.2 Casing displacement calculations

The casing deflections are due to the pressure difference across the shell of the casing. The centrifugal growth (CF) of the casing is obtained by scaling the CF growth at the maximum rotation speed of the rotor to the power of 6 (SCO3 Users Guide). This is modelled as the ratio of the thermal conductivity of the drum to the young modulus multiply by the sixth root of the speed. This is described by the Equation 3.4 as:

$$CF_{ca\sin g} = \frac{k_{drum}}{E} * rpm{}^\wedge 6 \qquad (3.4)$$

where:

CF_{casing}= centrifugal growth of casing
K_drum = thermal conductivity of drum
E= young modulus of drum
rpm = rotor rotational speed in revolution per minute

3.2.1Casing transient thermal growth

The transient thermal growth of the casing has an exponential response as in the case of the drum. The values of the stage temperature, drum time constant during acceleration and deceleration, acceleration and deceleration time and the thermal growths of casing are modelled to give the casing transient thermal growth. The governing equation for the modelling of transient thermal growth is given by Equation 3.5 (SCO3 Users Guide).

$$C_{tran} = 288.15 + \left(T_{stage} - 288.15\right) * \left(1 - \exp^{\frac{-\Delta t}{\tau}}\right) \qquad (3.5)$$

where:

C_{tran} = transient thermal growth of casing
T_{stage} = stage temperature for Idle, MTO and Cruise
Δt = change in time
τ = thermal time constant

The transient thermal growths are modelled with respect to ramp times over the complete engine cycles.

3.2.2 Casing total thermal growth

The total thermal growth of the casing is obtained by the combine effect of the casing centrifugal growth (CF) with the transient thermal growths of the casing. That is modelling the combine effect of Equation 3.4 and Equation 3.5. The result gives the casing total thermal growth characteristic over the square cycle as shown in Figure 3.3. Figure 3.3 shows the variation of casing total thermal growth with time over the extended square cycle for stage 3 of the Trent 1000 casing model.

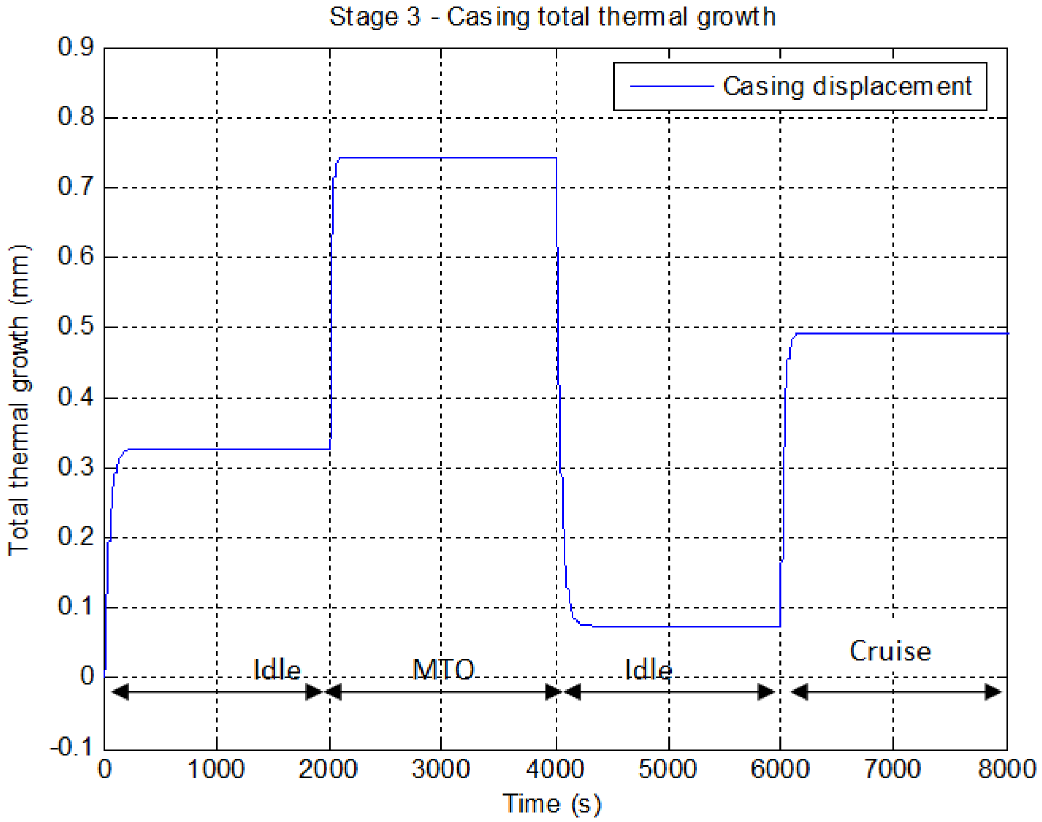

Figure 3.3: The variation of casing total thermal growth with time over the extended square cycle for stage 3 of the Trent 1000 casing model.

3.3 The closure behaviour

In this section, the lumped parameter closure behaviour of stage 3 is presented. This closure behaviour involves the modelling of all the effects associated with Equation 3.1 – Equation 3.5. Figure 3.4 shows the variation of total thermal growth of casing and drum with time over the extended square cycle for stage 3 of Trent 1000 casing and drum models. This shows casing movement relative to drum movement during engine transient over the extended square cycle.

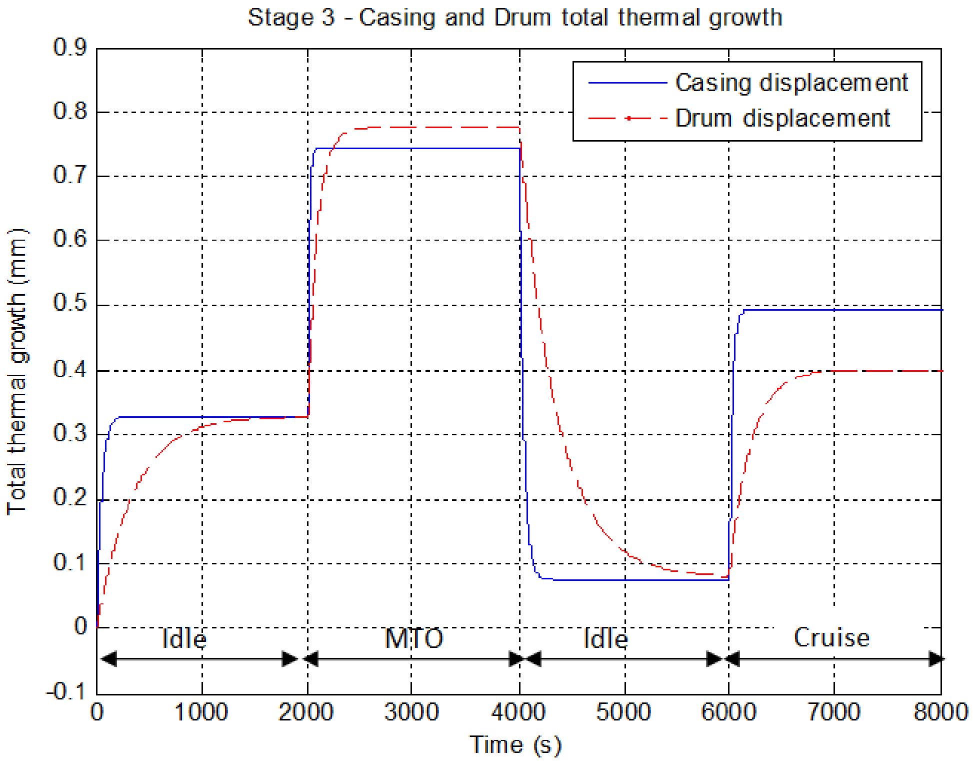

Figure 3.4: The variation of total thermal growth of casing and drum with time over the extended square cycle for stage 3 of Trent 1000 casing and drum models.

The closure behaviour of each stage is given by the relative expansion of the casing to the expansion of the drum. When the casing displacement is subtracted from the drum displacement at any point in the cycle, the result is called the closure as shown in Figure 3.5. Figure 3.5 shows the variation of Lumped model closure with time over the extended square cycle for stage 3 of Trent 1000 casing and drum models.

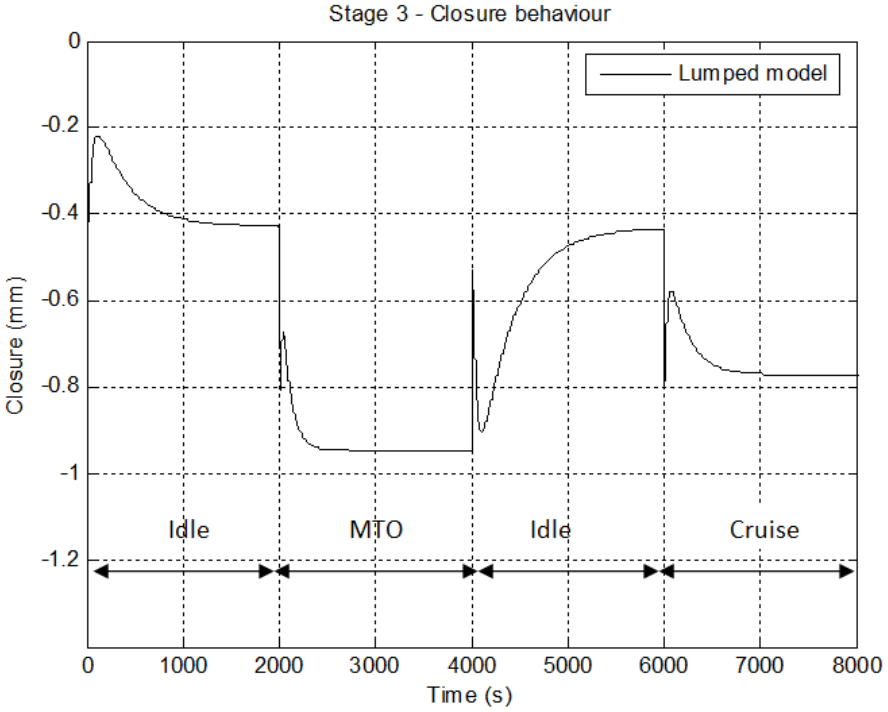

Figure 3.5: The variation of Lumped model closure with time over the extended square cycle for stage 3 of Trent 1000 casing and drum models.

3.4 Thermal matching of the Lumped and SC03 models

This involved the matching of the result from the lumped model with the results of the SC03 model. If the matched well then it is assumed that the validation of the SC03 is correct. Further matching is done between the lumped model, SC03 model and the results obtained from the multiple cavity rig. Figure 3.6 shows matching of the Lumped model closure with SC03 closure over the extended square cycle for stage 3 of Trent 1000 casing and drum models.

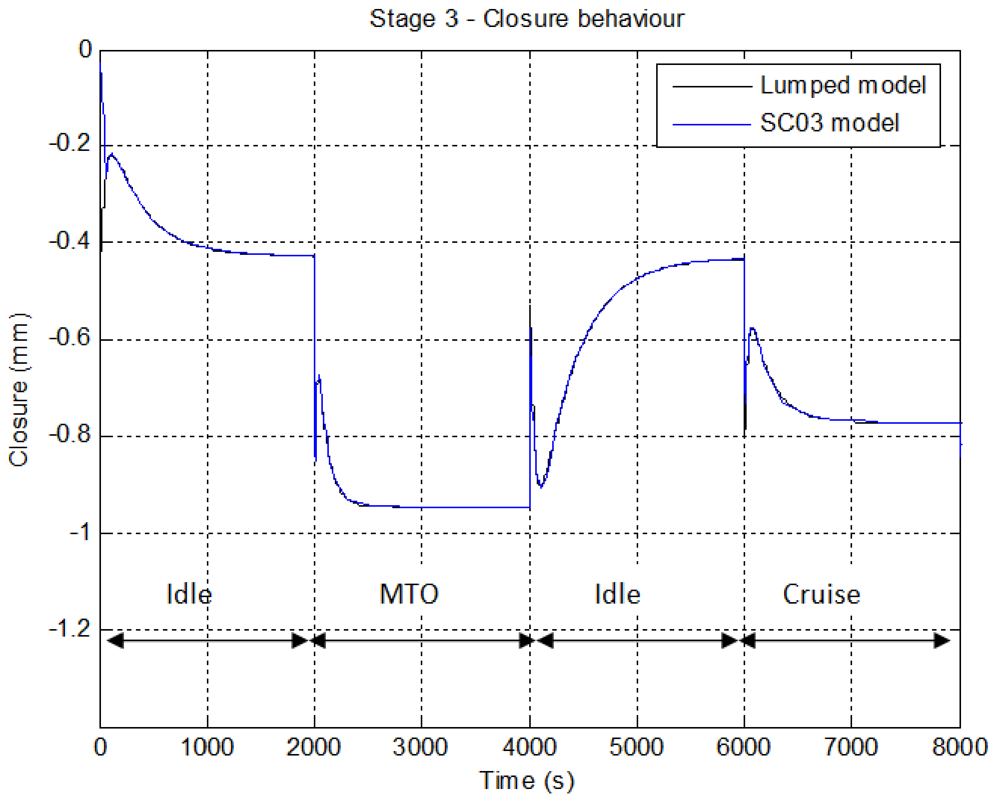

Figure 3.6: The Lumped model closure matched with SC03 closure over the extended square cycle for stage 3 of Trent 1000 casing and drum models.

As shown in Figure 3.5, the matching of the 1-D model (Lumped parameter analysis) closure is in very good agreement with the prediction of the 2-D model from the SC03 over the extended square cycle of stage 3. This is confirmed by the matching profile of SC03 2-D model closure variation over time with the Lumped parameter model closure for the entire cycle. During engine transient, there are no discrepancies during acceleration, deceleration and cruise operation of both models, hence the time constant and closure data at any point in the cycle in both models are the same. Hence, the Lumped parameter model method can be employed in the control of tip clearance concept in HP compressor in jet engines.

IV. CONCLUSION

This paper has reported on the 1D modelling of the of tip clearance control concepts using the lumped parameter method. The lumped parameter data were calibrated against the SC03 HP compressor drum and casing models simulation. The result of stage 3 is presented. The variation of total thermal growth of casing and drum with time over the extended square cycle for stages 3 Trent 1000 casing and drum models were studied. This variation gives the closure characteristics of the system during engine transient. The Lumped parameter model closure was matched with SC03 closure over the extended square cycle for stages 3 of Trent 1000 casing and drum models. The overall result of the matching process shows that, the results of the lumped model are in good agreement with the results the closure data of the SC03 model. Hence, the lumped parameter method is a very method for tip clearance control in HP compressor, since it will give a quick approximation of the clearance during engine transient.

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